On the Formulation of Strain-Space Plasticity with Multiple Loading Surfaces[1]

J. L. Kauschinger.[2] The authors are to be congratulated for their valuable contribution to the knowledge of constitutive modeling of nonlinear materials. It is clear that a formulation in strain space has important numerical advantages over a classical stress space model. It would appear that the authors next line of attack should be to present an approach for obtaining the appropriate material parameters from laboratory tests, and to propose functional relationships for modeling various materials.

While in general, a stress space formulation will require an inversion of the compliance matrix to obtain the stiffness, many times this inversion can be performed algebraically, as in the case of Prevost [4][3], and thereby the computational expense needed to obtain the stiffness is alleviated when performing finite element calculations.

When selecting the nesting rule used to harden the active yield surface, the only strict requirement is that the consistency condition be upheld. Although Prevost has arbitrarily selected a Mroz-type rule for use in his stress space formulation, the possibility of using Prager's Kinematic Rule is not excluded from his model [13]. It is true on the other hand, that when implementing Prager's Rule in a nested surface theory in conjunction with a Von Mises-type failure criterion, there must be a coupling between the translation of the present active surface and the next outer one, thereby increasing the computational effort. It should be emphasized that even in a strain space formulation the user must select some nesting rule. It is hoped that the hardening rule selected will portray the actual behavior of the material as measured in the laboratory. Undoubtedly, this comparison will represent the ultimate test of any functionals selected for use in a model.

Thus, while the advantages of a strain space formulation are many, particularly for monotonic loading, backtracking errors that develop during reversals when modeling an elastoplastic material cannot be eliminated when using either formulation. Therefore, it appears to this writer that the problems associated with implementing a classical stress space model are not as serious as the paper under discussion would seem to indicate. This conclusion was reached by this writer after implementing the Prevost Model [4] in a computer code for the purposes of simulating soil behavior.

Authors' Closure

The authors appreciate Kauschinger's interest in the subject paper and find themselves in general agreement with many of his observations. However, his comments about nesting rules deserve some further discussion, as does the question of backtracking error.

Although it has become customary in stress-space plasticity to require the various surfaces to nest as loading proceeds, this practice does not appear to stem from phenomenological considerations. Given the current stress and stress increment, after all, there is never any difficulty in determining the corresponding strain increment. Even if the stress happens to lie at a point where several yield surfaces cross, one simply finds the plastic contributions from each and sums them. However, when one is given the strain increment and needs to compute the corresponding stress increment, as typically happens in finite element programs, there arises a problem in determining which of the potentially active surfaces are in fact producing plastic strain. This difficulty comes about because the traditional loading criteria are based on the unknown stress increment. It was primarily to sidestep this issue that nesting rules were introduced in the first place. If one somehow contrives to force the yield surfaces to touch tangentially whenever they intersect, then at any given instant either all of the surfaces passing through the stress point will be active or else none of them will.

These considerations led Prévost [4, 13][1] to incorporate a nesting rule into his models for soil mechanics. Based on stress-space yield surfaces, these models lead directly to expressions for the elastoplastic compliance, which in turn are inverted through recourse to the nesting rule. This nesting rule has other ramifications, though, some of which are hard to conceptualize physically. For instance, if one tries to preserve Prager's kinematic hardening law by requiring each surface being loaded to move parallel to its own local normal, the surfaces outside of it must all somehow be made to move out of its way.

An alternative approach, as outlined in the subject paper, is to abandon nesting rules altogether. It should be emphasized that neither the stress nor strain-space models discussed therein make use of any such rule. Even so, it remains possible under fairly general conditions to obtain the elastoplastic stiffness. One particular class of models stands out among all those considered, namely, the ones based on uncoupled, non-nesting, strain-space loading surfaces. For models of this type, the loading surfaces act independently of one another and the stiffness is found quite simply by adding the contributions from each.

An additional advantage of these strain-space models is that the loading criteria are based on strain rather than stress. Thus one can determine whether there has been a loading reversal directly from the strain and strain increment, which can readily be calculated at the close of each time step. Stress-space models, on the other hand, require one to guess whether there has been a loading reversal before updating the stress. For this reason, the strain-space models should be somewhat less prone to backtracking error.

This promise of improved performance under loading reversals, while not compelling in and of itself, does lend support to the arguments in favor of the strain-space formulation. So, too, does the added flexibility that results from being able to use non-nesting loading surfaces. Of course, as Kauschinger intimates, a plasticity model is useful in any particular application only to the extent that its predictions agree with experimental results. The authors anticipate that the strain-space models will indeed prove useful in describing the constitutive behavior of real materials.

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[1] By P. J. Yoder and W. D. Iwan, and published in the December, 1981, issue of the ASME JOURNAL OF APPLIED MECHANICS, Vol. 48, pp. 773–778.

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[3] Numbers in brackets refer to references in Yoder and Iwan's paper.

[1] Numbers in brackets refer to references in the subject paper.