1.4) at an initial Mach number of 3.0 to undergo a normal shock process. The inlet total temperature is 900 deg R; the inlet static pressure is 5 psia. Find the total and static pressures, the total and static temperatures, and the Mach number immediately after the shock.

Solution, by Table 2A:

From

\[ M_{in} = 3.0 \]

we obtain

\[ \frac{p_{in}}{p_{in}} = 0.02722; \quad \Gamma_{in} = \left( \frac{p_{in}^*}{p_{in}} \right) = 0.23615; \quad \frac{p_{in}}{p_{in}} = 3.04558 \]

But

\[ \Gamma_{o2} = \Gamma_{in} \left( \frac{p_{in}}{p_{in}} \right) = 0.23615 \times 3.04558 = 0.71921 = \left( \frac{p_{in}^*}{p_{in}} \right) \]

From

\[ \Gamma_{o2} = 0.71921 \]

we obtain by linear interpolation

\[ \frac{p_{o2}}{p_{o2}} = 0.85621; \quad T_{o2} = 900 \text{ deg R}; \quad M_{o2} = 0.47579 \]

Whence

\[ p_{o2} = 5 \text{ psia} = 183.688 \text{ psia} \]
\[ p_{o2} = 183.688 \text{ psia} = 60.313 \text{ psia} \]
\[ p_{o2} = 0.85621 \times 60.313 \text{ psia} = 51.641 \text{ psia} \]
\[ T_{o2} = T_{in} = 900 \text{ deg R} \]
\[ T_{o2} = 0.95659 \times 900 \text{ deg R} = 860.93 \text{ deg R} \]
\[ M_{o2} = 0.47579 \]

APPENDIX

The Limiting Case of \( \gamma = 1 \)

Equation (3) in this case becomes

\[ \rho = \frac{p}{p_{in}} \]

(3A)

while equation (6) yields

\[ V = \left[ \frac{2}{p_{in}} \ln \frac{p_{in}}{p_{in}} \right]^{1/2} \]

(7A)

and hence the continuity equation, (10), may be expressed as

\[ \left( \frac{T_{in}}{T_{in}} \right)^{1/2} \left( \frac{p_{in}}{p_{in}} \right) \left( \frac{p_{in}}{p_{in}} \right) \left[ \ln \frac{p_{in}}{p_{in}} \right]^{1/2} \]

\[ = \left[ \left( \frac{p_{in}}{p_{in}} \right) \left[ \ln \frac{p_{in}}{p_{in}} \right]^{1/2} \right] \]

(10A)

Now, equations (16) and (7A) may be combined to yield

\[ \frac{p_{*}}{p_{*}} = e^{-1/2} \]

(17A)

which defines the critical pressure ratio for any flow process if \( \gamma = 1 \).

Patterning this development after that in the body of this paper, the reference pressure ratio function for continuity becomes

\[ \left( \frac{p_{*}}{p_{*}} \right) \left[ \ln \frac{p_{*}}{p_{*}} \right]^{1/2} = \left( e^{-1/2} \ln e^{1/2} \right) = \left[ \frac{1}{2e} \right]^{1/2} \]

(19A)

which is seen to be a constant in the \( \gamma = 1 \) case.

Finally, we obtain

\[ \left( \frac{T_{in}}{T_{in}} \right)^{1/2} \left( \frac{p_{in}}{p_{in}} \right) \left( \frac{p_{in}}{p_{in}} \right) \left[ \ln \frac{p_{in}}{p_{in}} \right]^{1/2} \]

(21A)

where

\[ \Gamma' = (2e)^{1/2} \left( \frac{p}{p_{in}} \right) \left[ \ln \frac{p_{in}}{p_{in}} \right]^{1/2} \]

(23A)

and where \( \Gamma' \) varies from 0 to 1 only for any and all flow processes.

DISCUSSION

N. A Carlucci

The theoretical relationships for the Fanno type flow with variable area as presented by the authors has been applied in our analysis of a low pressure turbine exhaust hood scale model quite successfully. The use of these expressions has eliminated the necessity for static pressure taps, with the exception of standard metering instrumentation, since only stagnation conditions are required. This has facilitated testing by reducing the amount of data required for performance evaluation.

In aerodynamic testing, it is always desirable to establish the quantity of airflow. Although this can be obtained from the relationships presented by the authors, it may be more expedient to make use of the relation

\[ \Gamma = \frac{w}{w_{\text{isentropic}}} \]

or

\[ w = \Gamma w_{\text{isentropic}} \]

\[ w = \Gamma A \left[ \left( \frac{p_{in}}{p_{in}} \right) \left[ \ln \frac{p_{in}}{p_{in}} \right]^{1/2} \right] \]

\[ = \left( \frac{2}{\gamma + 1} \right) \left( \frac{2\gamma}{\gamma + 1} \right)^{1/2} \]

(55)

where \( w \) = weight flow per unit of time. This then permits the evaluation of airflow provided that the value of \( \Gamma \) can be established. However, \( \Gamma \) is easily established if the static to total pressure ratio is known at any location. Since static taps are normally employed in the metering section, this is the most convenient location to evaluate the initial \( \Gamma \). This requires the determination of the total pressure. Knowing the inlet static pressure, the static pressure difference, and the velocity of approach factor, the inlet total pressure is given by

\[ p_{in} = p_{in} + \frac{p_{in} - p_{in}}{\left( \sqrt{1 - \beta^2} \right)^2} \]

(56)

where

\[ \beta = D_2/D_1 \]
\[ D = \text{diameter} \]

Having established the initial value of \( \Gamma \), the value of \( \Gamma \) at any other location is easily determined knowing the stagnation conditions at that location.

12 Development Engineer, Steam Division, Westinghouse Electric Corporation, Lester, Pa. Assoc. Mem. ASME.
The authors have done a commendable job. The theoretical expressions and tables presented in this paper are not only of academic interest, but also have practical significance.

A. J. W. Smith

The method elaborated in this paper may be replaced by a greatly simplified analysis based on nondimensional parameters widely used in propulsion literature. The analysis is as follows:

The equation of continuity

\[ \rho Av = m \]  

the definition of Mach number

\[ M = \frac{V}{\sqrt{\gamma RT}} \]  

and the equation of state

\[ \frac{p}{\rho R} = RT \]  

may be combined by eliminating \( V \) and \( \rho \) between equations (57), (58), and (59) to give

\[ m = \sqrt{\frac{T}{RT}} MAp \]  

The total temperature and pressure are defined by

\[ T_t = T \left( 1 + \frac{\gamma - 1}{2} M^2 \right) \]  

\[ p_t = p \left( 1 + \frac{\gamma - 1}{2} M^2 \right)^{\frac{\gamma - 1}{\gamma}} \]  

Substitution of equations (61) and (62) into (60) yields on rearrangement:

\[ \frac{m \sqrt{RT_t}}{Ap_t} = \frac{\sqrt{\frac{T}{RT}} M \left( 1 + \frac{\gamma - 1}{2} M^2 \right)^{\frac{\gamma + 1}{2(\gamma - 1)}}} = X \]  

It should be noted that this equation has been derived without specifying the flow process, and applies to systems in which frictional, area change, and heat exchange effects occur simultaneously or separately.

By substituting \( p/p_t \) in equation (25) of this paper, it is easily shown that

\[ \Gamma = \frac{m \sqrt{RT_t}}{Ap_t} \left( \frac{\gamma + 1}{2} \right) \frac{\gamma + 1}{2(\gamma - 1)} \]  

or

\[ \Gamma = X \frac{1}{\sqrt{\gamma}} \left( \frac{\gamma + 1}{2} \right) \frac{\gamma + 1}{2(\gamma - 1)} \]  

The nondimensional ratios given in Tables 1 and 2 of the paper under discussion (apart from \( \Gamma \)) are all derived and tabulated in many sources, e.g., in Ref. [1]. The function \( X \) is shown graphically in Ref. [3] and is tabulated in a dimensional form for air in Ref. [2].

By virtue of the relationship expressed in equation (65) it is clear that all the examples at the end of the paper may be solved using the parameter \( X \) instead of \( \Gamma \), thereby showing the simplicity of the method outlined and the redundancy of the analysis in the paper under discussion.

References


Authors' Closure

The authors wish to thank Messrs. Carlucci and Smith for their discussions. Mr. Carlucci has called attention to a very practical application of the \( \Gamma \) function to a variable area Fanno type flow process. His equation (54) is quite correct and gives a useful interpretation of the \( \Gamma \) function. Indeed, equation (54) indicates clearly the basis of the \( \Gamma \) function, for note from the paper: Continuity is first invoked; then a general critical state is defined; then expressions describing these two conditions are ratioed to yield a general \( \Gamma \) function; and finally, the specific \( \Gamma \) function of the tables is defined in terms of a definite critical state, namely the isentropic critical state. In Mr. Smith's equation (65), the physical meaning of \( \Gamma \) (i.e., a state condition referred to a specific critical state) is entirely lost.

Note that, in equation (58), \( R \) has the units ft \( ^2 \)/sec \( ^2 \) deg R, as opposed to the conventional ft \( \text{sec}^2 \) deg R units of \( R \) used throughout the paper. In equation (62), the reciprocal of the exponent of the bracketed term is obviously called for. The \( R \) in Mr. Smith's equation (63) is usually called the flow number when the gas constant, \( R_t \), is written on the other side of the equality sign. While it is true that \( X \) or flow number can be derived without reference to a specific flow process since it is merely a state point function, it has no utility of itself. It is the process multipliers, clearly brought out in equation (21) of the paper, which are of vital importance when proceeding from one state point to another, and these can only result from an analysis such as is presented in the paper. Note that equation (64) is simply a rearranged version of Mr. Carlucci's equation (58), and that both follow directly from the paper.

Mr. Smith mentions that his \( X \) function has been presented graphically in one of his references. We wonder if it was applied there to arbitrary flow processes. Usually, the \( p/p_t \) plot is applied specifically and solely to isentropic processes (and then in terms of \( p/p_t \)). Unfortunately, the authors did not have the advantage of studying the unpublished references [2] and [3]. Professor Shapiro's well-known work, [1], has, of course, been widely accepted.

We wish to indicate once more the scope of our paper. A critical state and a generalized \( \Gamma \) function are introduced. Equation (21) is developed to provide a method for proceeding from the \( \Gamma \) of one thermodynamic state to the \( \Gamma \) of another state for any flow process. Various simplified flow processes are discussed and mapped in terms of \( h-s \) and \( p/p_t \) processes. Finally, one generalized compressible flow table is presented to emphasize the common bonds which link all flow processes.