Coulomb-Eikonal Amplitude for the Hydrogen Break-Up Process by Electron Impact

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A method is developed for the evaluation of the $(e, 2e)$ ionization amplitude in the eikonal approximation, where the two outgoing electrons are considered to be in uncorrelated Coulomb continuum states. The triple differential cross section for the process is analyzed in a coplanar geometry by using the Coulomb-projected eikonal approximation. A comparison is made with experimental data and with results obtained by other approximation methods. It is found that the method proposed here provides rather good estimate in the small momentum transfer condition.

Recently the Glauber or eikonal-type approximation has been applied extensively with great success to the analysis of atomic scattering. In particular it gives an interesting result for the break-up process of the hydrogen system. It seems to bring realistically good estimates not only at high energies but also at intermediate energies. However, all these investigations are mainly limited to the integrated cross section, which provides only a one-side test of the theory, because the information extracted from the integrated cross section is related only to some averaged effect of the original process. So the examination of the differential amplitude which is relevant to five kinematic parameters would be worthwhile. In fact, as is well known, the triple differential cross section (TDCS) provides a more detailed information about the mechanism of the break-up process, so that it becomes a sensitive test to the approximation method.

Since the first measurement of TDCS of $He(e, 2e)He^+$ and that of $H(e, 2e)H^+$ many theoretical efforts including the first Born (FBA), Coulomb-projected Born (CBA) and distorted wave approximations (DWA) have been devoted to the interpretation of the experimental results. In these approaches the break-up process is regarded as the first order transition with the distortion considered in the initial and final channels, and then their results failed to reproduce the experimental ones satisfactorily.

On the other hand, there are several works which treat the ionization process from different viewpoints. Among them Byron et al. recently calculated TDCS by the second Born approximation (SBA) and obtained good results in a limited case. For example, they found a higher recoil peak and shifts of both recoil and binary peaks to larger angles than those estimated by FBA. However, SBA cannot show, in general, any improvement over FBA except the case of small scattering angles and small ejected electron energies. Hereby we propose to apply the multiple scattering theory taking account of a distortion effect to the initial and final channels for the present break-up process.

In principle the eikonal approximation can be interpreted in terms of multiple scattering theory, where its phase integral appears in the formalism as a measure of distortion of the initial or final channel due to the interaction potential between both electrons and can be used to describe the channel coupling effect in a multiple scattering process. As a
further test of the validity of the eikonal-type approximation we evaluate TDCS by the present approximation.

Now we formulate the direct scattering amplitude for the break-up process of atomic hydrogen by electron impact in the Coulomb-projected eikonal approximation (CEA) with the post form. Let $k_i$, $k_f$ and $x$ be respectively the momenta of the incident, scattered and ejected electrons, then from the formal collision theory the direct scattering amplitude is given by

$$F^+(k_i, k_f, x) = -(2\pi)^3 \times \langle \Phi_f(r_1, r_2) | \psi^+_i(r_1, r_2) \rangle,$$  

(1)

where $r_1$ and $r_2$ are respectively the coordinates of the incident and bound electrons relative to the nucleus. The atomic units are used here unless otherwise stated. The total scattering states developing out of the free initial state $\psi^+_i(r_1, r_2)$ is represented by

$$\psi^+_i(r_1, r_2) = (2\pi)^{-3/2} \times \left[ \exp\left(i\frac{k_i \cdot r_1}{k_i} - i \int_{-\infty}^{z_1} V_i dz_1 \right) \right] \psi_i(r_2).$$

(2)

In Eqs. (1) and (2) $V_i = -1/r_1 + 1/R$ with $R = |r_1 - r_2|$ is the interaction between the incident electron and the target, $V_f = 1/R$ is the interaction between the final electrons, and $\psi_i(r_2)$ is the initial state of the atomic electron to be given as $\pi^{-1/2} \exp(-r_2)$.

In our formalism the final wave function $\Phi_f(r_1, r_2)$ is chosen as the product of two continuum Coulomb wave functions with the incoming boundary condition, and then we obtain the following three-dimensional integral expression in real space for the direct scattering amplitude:

$$F^+(k_i, k_f, x) = C(\eta_i, \eta_f, \eta_e) \times \int \frac{dA}{|A + q|^2 + \pi |r_i|^2} (2y)^{1/2} H(y)$$

$$\times S_1(k_i, k_f, q, A) S_2(x, A)$$

(3)

with $q = k_i - k_f$, where

$$C(\eta_i, \eta_f, \eta_e) = 2^{1/2} \pi^{-2} \exp\left[\frac{\pi}{2} (\eta_e + \eta_f - \eta_i)\right]$$

$$\times \Gamma(1 + i\eta_i) \Gamma(1 - i\eta_i) \Gamma(1 - i\eta_e) \Gamma(1 - i\eta_e),$$

$$H(y) = \Theta(y) + \exp(y\eta_i) \Theta(-y),$$

$$S_1(k_i, k_f, q, A) = A_f^q + iG_f \exp\left(-i\frac{G_f}{B_f} E_f\right),$$

$$S_2(x, A) = A_k^{\eta_e - 1} B_k^{\eta_e - 1} \left[ \left(1 + i\eta_e\right) F(1) - \left(1 + i\eta_f\right) F(2) \right]$$

$$\times \left[ \left(1 - i\eta_f\right) B_e^{\eta_f} \right]$$

with $\eta_i = Z_i/k_i$, $\eta_f = Z_f/k_f$, $\eta_e = Z_e/k_e$ and $y = (A + q) \cdot k_f$, besides the charge parameters are given by $Z_i = 1$, $Z_f = 1$ and $Z_e = 1$. Here

$$A_x = (A + q)^2 - 2(A + q) \cdot x + x^2 + 1,$$

$$B_x = (A + q)^2 + 1 - x^2 - 2ix,$$

$$A_f = (A^2 + \varepsilon^2)/2,$$

$$B_f = A_f - k_f \cdot A - ik_f \varepsilon,$$

$$E_f = \varepsilon_k + ik_f A,$$

$$G_f = E_f - i(k_f k_f + k_i k_f),$$

$$\xi_f = (1 - i\eta_f)(F(1) - i\eta_f F(2)),$$

$$\xi_e = (1 - i\eta_e)(Z_f F(2),$$

$$\xi_v = (Z_i k_i/k_r - Z_f) F(1) - Z_f (1 - i\eta_f)(k_i/k_f) F(2),$$

and $\Theta(y)$ represents the step function. $F(1)$ and $F(2)$ are the abbreviation of the hypergeometric functions: $F(1)=_{2}F_{1}(i\eta_i, i\eta_f, 1, x)$ and $F(2)=_{2}F_{1}(i\eta_e, 1 + i\eta_f, 2, x)$ with $x = 1 - A_f G_f / B_f E_f$.

In Eq. (3) the term $S_2(x, A)$ shows a similar form to the Born amplitude, and the first two terms in the bracket of $S_1(k_i, k_f, A)$ correspond to the Coulomb-Born contribution, while the latter two terms to the eikonal correction. We note that Eq. (3) with $Z_i = 0$
is reduced to the CBA amplitude and is the same as that given by Geltman and Hidalgo.9)

The method of reduction of Eq. (3) is also applicable to the plane-wave eikonal approximation.

For numerical procedure an infinitesimal positive quantity $\epsilon$ is conveniently introduced in Eq. (3). This numerical method is similar to that adopted by Geltman et al. in calculation of the CBA amplitude for the $H(e, 2e)H^+$ process. We estimate the accuracy of our calculation to be about 5 to 15 percent.

The values of TDCS obtained by Eq. (3) for $E_t = 100$, $E_r = 61.4$ and $E_e = 25$ eV at the scattering angle 20° are shown in Fig. 1. Since

$$d^3\sigma/dk\,d\omega\,dE_e$$

plotted as a function of the ejection angle $\theta_e$.

The calculated cross sections are represented by -- for CEA ($\times 1.12$), --- for CBA ($\times 0.48$), ...... for GA ($\times 0.46$) and - - - - for FBA ($\times 0.67$). The calculated and experimental (○) results are normalized to be the same at the peak in each case. Since the experiment by Weigold et al. is based on relative and independent measurement for each scattering angle, the theoretical TDCS value and experimental data are normalized to give the same height of the peak. The absolute value of TDCS can be obtained by multiplying the graphical results by a factor given in the figure caption. We can see that CEA and CBA give better description for the angular distribution in binary-encounter collisions than that of FBA. There exists large difference in absolute magnitude for different approximations. The absolute measurement would be extremely valuable for TDCS.

Our results are shown also in Fig. 2 for the $E_t = 250$, $E_r = 232.4$ and $E_e = 4$ eV at the scattering angle $\theta_t = 4°$. For comparison different calculated values are normalized to give the same height at the position of each binary peak given by different approximations. It can be seen that the TDCS results in CBA show a rotation of the binary peaks to larger angles of the ejected electron than those in FBA, while the recoil peak is shifted towards

![Fig. 1. The triple differential cross section (in a.u.) for $H(e, 2e)H^+$ at $E_t = 100$ eV, $E_r = 61.4$ eV, $E_e = 25$ eV and $\theta_t = 20°$ ($\phi_t - \phi_t = \pi$) plotted as a function of the ejection angle $\theta_e$. The calculated cross sections are represented by -- for CEA ($\times 1.12$), --- for CBA ($\times 0.48$), ...... for GA ($\times 0.46$) and - - - - for FBA ($\times 0.67$). The calculated and experimental (○) results are normalized to be the same at the peak in each case.](https://academic.oup.com/ptp/article-abstract/68/1/306/1882239)

![Fig. 2. The triple differential cross section $d^3\sigma/dk\,d\omega\,dE_e$ (in a.u.) for $H(e, 2e)H^+$ at $E_t = 250$ eV, $E_r = 232.4$ eV, $E_e = 4$ eV and $\theta_t = 4°$ plotted as a function of the ejection angle $\theta_e$. The calculated results are normalized so that the value of the binary peak is unity in each case: -- for CEA ($\times 8.32$), --- for CBA ($\times 11.2$), - - - - for FBA ($\times 11.3$).](https://academic.oup.com/ptp/article-abstract/68/1/306/1882239)
the forward direction, which is not in harmony with experimental results for a different, but similar process $\text{He}(e, 2e)\text{He}^+$. However, the CEA curve gives a rotation of recoil peak to larger angles of the ejection. At the same time the angle of binary peak remains to be larger than that of the FBA. The ratio of magnitudes at the maxima of the binary peak and the recoil peak are 4.8, 4.5 and 3.9 for the FBA, CBA and CEA, respectively. It can be seen that the relative intensity of the recoil peak is enhanced in CEA.

Up to now every theory can hardly reproduce the experimental results over the entire angular range. For example, DWA and CBA give rather good results only for binary collision part, while SBA only for the recoil part. However, CEA gives some improvement both for binary collision and for the recoil one.

It is interesting to compare the result of CEA with that of the Glauber approximation (GA). Roy et al.\textsuperscript{131} have calculated TDCS by using GA with the choice of $z$-axis perpendicular to $q$ and by taking the final wave function in different footing from CEA. The angular distribution given by GA is more or less identical with that of FBA, while there exists a large difference in magnitude. It can be understood that CEA gives better description of binary angular shape than FBA and GA.

Our method can easily include the exchange effect, because Eq. (1) exactly satisfies the symmetry relation $f(k_f, k) = g(k, k_f)$ given by Peterkop.\textsuperscript{132} Consequently further consideration of exchange effect is necessary for the present work.

Finally, it is emphasized that TDCS for the $(e, 2e)$ process is shown to be very sensitive to the choice of wave functions for the initial and especially for the final state. It can be seen that the eikonal distortion considering Coulomb-projected model in the initial state provides rather good estimate than FBA, CBA and GA.

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