Multi-Quark Hadrons in the Joined-Spring Quark Model

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The level structures of typical multi-quark exotic hadrons, \((q^2 \bar{q}^2)\)-meson, \((q^4 \bar{q})\)-baryon and \((q^6)\)-di-baryon with only nuclear quarks, are investigated systematically in the joined spring quark model, where hadron masses squared are described in terms of harmonic oscillators with various quanta. This model was proposed by us as a simple idealization of the string-junction model so as to make it possible a quantum mechanical treatment of constituent quarks.

An interesting prediction of our model is that there exist orbital Regge trajectories with the steeper slope than the usual one for non-exotic hadrons. Somewhat strong experimental suggestions for existence of these trajectories are seen for all respective cases of exotic hadrons.

§ 1. Introduction

Recently possible existence of multi-quark exotic hadrons has aroused much interest both experimentally and theoretically.

First the existence of several di-nucleon resonances seems to be established experimentally due to many recent efforts, especially the polarized proton-proton scattering experiment by Yokosawa et al.\(^1\) and the phase shift analysis by Hoshizaki et al.\(^2\). Their existence is also supported by the phenomenological analyses of the processes \(\pi d \rightarrow \pi d^3\) and \(pp \rightarrow \pi d\).\(^4\) Also it has been pointed out that their existence as a single hadron with quark-configuration \((q^6)\) is playing some desirable role in the deuteron form factors\(^6\) and in the so-called cumulative particle production\(^6\) in particle-nucleus collisions.

Since the first suggestion from the MIT bag model by Jaffe and Johnson\(^7\) that the known \(J^{PC}=0^{++}\) mesons may be \((q^2 \bar{q}^2)\) states rather than \(P\)-wave \((q \bar{q})\) states, there have appeared many works seeking for the exotic mesons from both experimental and theoretical sides. Recently, a venturous conjecture has been made\(^8\) along the same line that all the low lying baryons with \(J^P=1/2^-\), being thus far considered as \(P\)-wave \((q^3)\) states, are rather \((q^4 \bar{q})\) states. The extensive efforts have been published mainly in calculating the level spectra of exotic hadrons in the bag model.\(^8\)\(^9\) We have also discussed\(^10\) various aspects on exotic hadrons from a different stand-point for these years.

In a previous work\(^11\)^\(^\text{(*)}\) we have investigated the level structures of di-baryons

\(^{(*)}\) In Table I in Ref. 11 the numbers for the 1st trunk-excited levels in the color Q. M. were misprinted, which should be corrected as in Table II(c) of p. 888.
applying the joined-spring quark model. This model was, so as to make it possible a quantum mechanical treatment of constituent quarks, proposed by two of the present authors (S.I. and M.O.) as a simple idealization of the string-junction model,\textsuperscript{12}) which recently attracted much attention as including phenomenologically possible effects due to the duality and the triality constraint.

In this paper we shall investigate\textsuperscript{13,14)} systematically the level structures of typical exotic hadrons with quark configurations \((q^2 \bar{q}^2), (q^4 \bar{q}), (q^6),\) and \((q^8)\). Our consideration will be limited on the exotic hadrons with only nuclear quarks, whose experimental knowledge is relatively rich.

An important aspect of exotic hadrons is that their level structures become, as was discussed previously,\textsuperscript{10,11)} very much different\textsuperscript{14)} depending upon the supposed quark statistics. In this paper all considerations will be made in both of the two general frameworks of hadrons, the conventional confined-color Fermi quark model (color Q.M.) and the Bose quark model\textsuperscript{15)} (Bose Q.M.) without the color freedom. Regardless of various successes of the former it cannot be considered, without solving the confinement problem, as well established. We think it still important to seek for other approaches, and believe that the latter model may be a promising possibility.

\section{2. Joined spring model}

The basic set up for the joined spring quark model\textsuperscript{11)} is as follows:

P1. A hadron is a system of universal oriented massless springs, which are joined so as to produce only triality-zero hadrons and carry quarks at their free ends.

P2. Wave functions for a hadron must have appropriate symmetry properties (implied by Bose (Fermi) statistics of quarks in the case of Bose (color) Q.M.) under the exchange of constituents which is possible without breaking the spring connection.

P3. In the case of color Q.M. each junction has a transformation property of \(\epsilon_{ijk}\) in the color \(SU(3)\) space, while it is independent of the other quantum numbers in both cases.

Thus our relevant non-exotic and exotic hadrons with quark configurations \((q \bar{q}), (q^3), (q^2 \bar{q}^2), (q^4 \bar{q})\) and \((q^6)\), which will be called in this paper as mesons, baryons, di-mesons, meso-baryons and di-baryons, respectively, are schematically represented as in Fig. 1. There open (black) circles denote massive quarks \(q\) (anti-quark \(\bar{q}\)), lines represent massless universal springs, and crosses denote a

\textsuperscript{14)} The content of this paper is a partial reproduction on the general results of the unpublished report (NUP-A-80-14) by the same authors, “Level spectrum of exotic hadrons in the joined spring quark model.” The preliminary result was given in Ref. 13).
Multi-Quark Hadrons in the Joined-Spring Quark Model

I.

Joint of two springs, which are newly introduced to give common Regge slopes to mesons and baryons.

Now we shall explain briefly our general procedure taking an example of meso-baryon system. The classical-mechanical Lagrangian for our system is given as

\[ L = \sum_{i=1}^{5} \frac{1}{2} m \dot{x}_i^2 - \left( \sum_{j=0}^{2} \frac{1}{2} k r_j^2 + \sum_{l=1}^{4} \frac{1}{2} k s_l^2 \right), \]  

(2.1)

where \( m \) is quark mass, \( k \) is an elastic constant of the spring, \( x_i's (\dot{x}_i's) \) are the \( i \)-th quark coordinates (their time derivatives) and \( r_j's \) and \( s_l's \) are the relative coordinates, representing elongation of respective springs (see Fig. 1 as for their definition).

From the Lagrangian (2.1) we derive the Hamiltonian

\[ H = H_{CM} + H_{in} \]  

(2.2)

represented in terms of independent normal coordinates and their conjugate momenta. Here \( H_{CM} = P^2 / (2 \times 5 m) \), \( P \) being the conjugate momentum to the center-of-mass coordinate \( X_G = \sum_{i=1}^{5} x_i / 5 \), and \( H_{in} \) is a sum of internal four (three-dimensional) harmonic oscillators (H.O.); two “trunk” oscillators around the junction \( J_0 \) with respective angular frequencies \( \omega_1 = 1 / \sqrt{3} \cdot \omega_0 \) and \( \omega_2 = \sqrt{5 / 7} \cdot \omega_0 \) (\( \omega_0 = \sqrt{k / m} \)), and two “limb” oscillators around the junctions \( J_1 \) and \( J_2 \) with the frequency \( \omega_1 = \omega_2 = \omega = \omega_0 \). Then following the usual procedure, we can easily derive the non-relativistic Schrödinger equation for our quantum-mechanical five-quark system with the Hamiltonian (2.2).

Instead, however, in order to reproduce the experimental squared-mass regularity heuristically, we set up a Klein-Gordon equation for our system

\[ \left( -\frac{\partial^2}{\partial x_{0,n}^2} + \mathcal{M}^2 \right) \Phi(x_1, x_2, \ldots, x_5) = 0, \]  

(2.3)

where the mass squared term is identified, aside from a dimensional factor \( d \), with the \( H_{in} \) in (2.2), and now the coordinate variables should be regarded as Lorentz four-vectors.\(^\ast\)

\(^\ast\) In the Appendix explicit forms of \( \mathcal{M}^2 \) of our all relevant hadrons

\(^\ast\) Recently we found a consistent formulation for this procedure where the constant is given as \( d = \sum m_i \) (\( m_i: i \)-th quark mass). See Ref. 16).

\(^\ast\) This procedure is now well-known in the relativistic H. O. model.\(^\dagger\) In the Appendix the normalization of wave functions is three-dimensional.
are given. Thus mass squared of our system is given in terms of the four relativistic H.O., whose eigenvalues are given as

\[ M_s^2 = M_0^2 + \sum_i n_i Q_i, \]  

(2.4)

\( n_i \)'s being numbers of respective quanta \( Q_i \)'s (\( Q_i = d \cdot \omega_i \)).

<table>
<thead>
<tr>
<th>Hadron</th>
<th>Meson</th>
<th>Baryon</th>
<th>Di-meson</th>
<th>Meso-baryon</th>
<th>Di-baryon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quark</td>
<td>( q\bar{q} )</td>
<td>( q^3 )</td>
<td>( q^2 \bar{q}^2 )</td>
<td>( q^4 \bar{q} )</td>
<td>( q^6 )</td>
</tr>
<tr>
<td>( \Omega_i )</td>
<td>( \Omega_0 )</td>
<td>( \Omega_0, \Omega_0 )</td>
<td>( \Omega_2, \Omega_0 )</td>
<td>( \Omega_2, \Omega_0, \Omega_0, \Omega_0 )</td>
<td></td>
</tr>
</tbody>
</table>

The oscillator quanta for our general hadrons are given in Table I. It is interesting that, while all of their values are equal for non-exotic mesons and baryons, their values of trunk oscillations for exotic hadrons are generally smaller, even though we have started with all universal springs. Here it may be worthwhile to note that the present kinematical framework for non-exotic hadrons are essentially the same as the usual relativistic H.O. quark model. \(^{17} \)

Now we give attention to the ground-state wave function of (2·3). Since the ground state is non-degenerate, the wave function has the same symmetry, with regard to exchange of coordinates \( x_i \), as the potential energy term. Thus it is symmetric, as easily seen from Fig. 1, for exchange of

limb symmetry; \( x_1 \leftrightarrow x_2 \) and/or \( x_3 \leftrightarrow x_4 \), \hspace{1cm} (2·5a)  
trunk symmetry; \( (x_1, x_2) \leftrightarrow (x_3, x_4) \). \hspace{1cm} (2·5b)

This is confirmed directly by the explicit form of ground state solution given in the Appendix, where the expressions for other hadrons are also given. Then the suppositions (P2) of quark statistics and (P3) of junction transformation property require that our wave function

\[ \Phi_{A_1 \cdots A_6}^A(x_1, \cdots, x_4; x_5) \]  

(2·6)

(\( A_i \) denoting thus far omitted spin and flavor of the \( i \)-th quark) has the symmetry properties with regard to spin- and flavor-suffices as follows: (L) symmetric for exchange of

\( A_1 \leftrightarrow A_2 \) and/or \( A_3 \leftrightarrow A_4 \) in both the models. \hspace{1cm} (2·7a)  
(T) symmetric (anti-symmetric) for exchange of

\[ \Phi_{A_1 \cdots A_6}^A(x_1, \cdots, x_4; x_5) \]  

(2·6)
These symmetry properties play an important role in determination of level structures. We can see that the difference in the trunk symmetry (T) between the two models leads to a decisive difference in the level structures of meso-baryons and di-nucleons; while no difference in those of non-exotic hadrons and di-mesons.

§ 3. Results of our level scheme and comparison with experiments

In this section we shall present the results obtained by applying the formulation described in § 2. We shall also try to compare their general features with experiments, although the present experimental knowledge for exotic hadrons is very poor.

A. Ground and first-excited levels

It is a simple task to determine quantum numbers of each level appearing in our scheme. The problem is mere vector addition in both the iso-spin and the angular momentum space with the symmetry restrictions implied by quark statistics as was discussed in § 2.

The results for di-mesons, meso-baryons and di-baryons are given in Tables II(a), (b) and (c), respectively; where the numbers mean the degeneracy (in the symmetric limit) of corresponding states, and the levels with up to only one oscillator quanta are included. In these tables we have also collected the experimental candidates with our tentative assignment. Here we see that the much richer spectrum is expected in our scheme. This is a natural and common feature to all models based on a multi-quark picture of exotic states. In order to solve this problem we need further investigations on the properties of exotic hadrons.
(b) Level structures of meso-baryons with nuclear quarks.

<table>
<thead>
<tr>
<th></th>
<th>1/2</th>
<th>3/2</th>
<th>5/2</th>
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<tr>
<td>J</td>
<td>1/2</td>
<td>3/2</td>
<td>5/2</td>
</tr>
<tr>
<td>Ground levels (Parity -)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bose Q.M.</td>
<td>4 3 1 0</td>
<td>3 3 1 0</td>
<td>1 1 1 0</td>
</tr>
<tr>
<td>Color Q.M.</td>
<td>3 3 1 0</td>
<td>3 3 1 0</td>
<td>1 1 1 0</td>
</tr>
<tr>
<td>Expl. cand.</td>
<td>≫ N*(1.34)</td>
<td>† N*(1.42)</td>
<td></td>
</tr>
<tr>
<td>1st excited levels (Parity +)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>low trunk (√1/3 · Ωb)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bose Q.M.</td>
<td>6 7 4 1</td>
<td>6 7 4 1</td>
<td>2 2 1 0</td>
</tr>
<tr>
<td>Color Q.M.</td>
<td>7 8 4 1</td>
<td>6 7 4 1</td>
<td>2 3 2 1</td>
</tr>
<tr>
<td>high trunk (√5/7 · Ωb)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bose Q.M.</td>
<td>7 8 4 1</td>
<td>6 7 4 1</td>
<td>2 3 2 1</td>
</tr>
<tr>
<td>Color Q.M.</td>
<td>6 7 4 1</td>
<td>6 7 4 1</td>
<td>2 2 1 0</td>
</tr>
<tr>
<td>limb (Ωb)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bose Q.M.</td>
<td>8 9 5 1</td>
<td>8 9 5 1</td>
<td>2 2 1 0</td>
</tr>
<tr>
<td>Color Q.M.</td>
<td>8 9 5 1</td>
<td>8 9 5 1</td>
<td>2 2 1 0</td>
</tr>
<tr>
<td>Expl. cand.</td>
<td>P1, (1710)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(c) Level structures of di-baryons with nuclear quarks.

<table>
<thead>
<tr>
<th></th>
<th>0 1 2 3 4</th>
<th>0 1 2 3 4</th>
<th>0 1 2 3 4</th>
<th>0 1 2 3 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>J</td>
<td>0 1 2 3 4</td>
<td>0 1 2 3 4</td>
<td>0 1 2 3 4</td>
<td>0 1 2 3 4</td>
</tr>
<tr>
<td>Ground levels (Parity +)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bose Q.M.</td>
<td>3 0 1 0 0</td>
<td>0 4 1 1 0</td>
<td>1 1 2 0 0</td>
<td>0 1 0 0 1</td>
</tr>
<tr>
<td>Color Q.M.</td>
<td>0 2 0 1 0</td>
<td>2 1 2 0 0</td>
<td>0 2 1 0 0</td>
<td>1 0 0 0 1</td>
</tr>
<tr>
<td>NN^{2s+1}L_j</td>
<td>× S × D × G</td>
<td>× S × D × G</td>
<td>× S × D × G</td>
<td></td>
</tr>
<tr>
<td>Expl. cand.</td>
<td>(^3D_1) (^3G_2)</td>
<td>(^3F_2)</td>
<td>(^3B_1(2.14))</td>
<td></td>
</tr>
<tr>
<td>1st excited levels (Parity -)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>trunk (√1/3 · Ωb)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bose Q.M.</td>
<td>2 5 4 2 0</td>
<td>5 10 9 4 1</td>
<td>3 7 6 3 1</td>
<td>1 2 2 1</td>
</tr>
<tr>
<td>Color Q.M.</td>
<td>2 5 4 2 0</td>
<td>5 10 9 4 1</td>
<td>3 7 6 3 1</td>
<td>1 2 2 1</td>
</tr>
<tr>
<td>limb (Ωb)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bose Q.M.</td>
<td>5 6 7 2 1</td>
<td>4 13 8 4 0</td>
<td>4 7 7 3 1</td>
<td>0 2 1 1</td>
</tr>
<tr>
<td>Color Q.M.</td>
<td>2 6 4 2 0</td>
<td>6 11 10 4 1</td>
<td>3 7 5 2 0</td>
<td>1 1 1 0</td>
</tr>
<tr>
<td>NN^{2s+1}L_j</td>
<td>× P × F</td>
<td>× P × F</td>
<td>× P × F</td>
<td></td>
</tr>
<tr>
<td>Expl. cand.</td>
<td>(^3F_2)</td>
<td>(^3H_2)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

states such as decay widths and form factors.
B. Level spacing

In our scheme squared masses of hadrons are, as was described in § 2, given in terms of H.O. with various values of oscillator quanta (see, Table I). The trunk quanta for exotic hadrons are generally smaller than those for non-exotic ones, leading to existence of orbital Regge trajectories with the steeper slope than the usual ones. Thus we expect that the corresponding level spacings between two neighboring states with opposite parities are smaller than those for non-exotic hadrons. This is a general feature of the joined-spring model regardless of supposed quark-statistics.

In Figs. 2(a), (b) and (c) we have shown the present experimental situations on this point for di-mesons, meso-baryons and di-baryons, respectively. There are given the mass positions of levels with up to two quanta which are determined from the symmetrical mass formula (2·4) with \( Q_0 = 1.1 \text{ GeV}^2 \) (obtained from the universal slope for non-exotic hadrons) and by using as an input to determine \( M_0 \) the mass of a plausible candidate for respective hadrons:

**di-mesons** As the input level we have adopted \( \varepsilon(800) \) resonance in the \( \pi \pi \) system with \( (I, J^{PC}) = (0, 0^{++}) \) and mass \( \approx 0.8 \text{ GeV} \). There we have also collected, without any selection, all meson resonances recently reported, except for the ones usually assigned to \( (q\bar{q}) \) states. The existence of \( \varepsilon(800) \) has been controversial for long time. However, as will be shown elsewhere, the rather strange behavior of \( \pi\pi \) phase shift \( \delta_0 \) up to energy \( \sim 1.1 \text{ GeV} \) is explained naturally as a result of existence of two resonances; one with mass \( \sim 800 \text{ MeV} \) and with very broad width \( I' \sim 600 \text{ MeV} \), and the other \( S^*(980) \) with rather small width \( I \sim 40 \text{ MeV} \). In this connection, there is a recent interesting experiment,\(^{19}\) where a broad peak, corresponding to low mass \( \varepsilon \), was observed in an invariant \( \pi^0\pi^0 \) mass plot through the process \( \pi^+p \rightarrow \Delta^{++}\pi^0\pi^0 \). It is also notable that two recent analyses\(^{20,21}\) favor the existence of a very broad resonance corresponding to the low mass \( \varepsilon \). The broad width of \( \varepsilon(800) \) may be a good evidence for its being a \( (q^2q^2) \)-level as was suggested by Jaffe.\(^{22}\) Recently there seems to be a good possibility of existence of two \( A_1 \) resonances with \( (I, J^{PC}) = (1, 1^{++}) \) and of similar mass, \( A_1(1050) \) and \( A_1(1280) \). Either one of them, plausibly \( A_1(1050) \) as was noted,\(^{23}\) may be assigned as a \( (q^2q^2) \)-level.

From Fig. 2(a) we see that the respective positions of experimental candidates are close to our predicted ones. It is especially interesting that several experimental candidates, including \( \rho(1250) \) which seems\(^{23}\) recently to be established, with negative parity exist in the right region corresponding to the first excited levels; and they have too small mass values to be assigned as the second excited levels of the \( (q\bar{q}) \)-system, whose positions are also shown there.

**meso-baryons** As the input level we have adopted the new mass enhancement \( N^*(1.34) \) with \( I = 1/2 \) and mass \( \approx 1.34 \text{ GeV} \), which was observed\(^{14,15}\) in the

\(^{*)\} The quantum number \( J^P \) of \( N^*(1.34) \) and \( N^*(1.45) \) seems to be \( \frac{1}{2}^+ \) and \( \frac{3}{2}^+ \), respectively.\(^{35}\)
Fig. 2.(a) Level spectra for \((q^3\bar{q}^3)\)-states in the symmetrical limit in comparison with experiments: Each level is degenerate as shown in Table II(a). \(l(l')\) denotes trunk (limb) H. O. quanta \(Q_1 = \sqrt{3}/2 \cdot Q_o (Q_o = Q_0)\). The mass regions for the corresponding \((q\bar{q})\)-states, determined from the mass of \(\pi\) and \(\rho\) with \(Q_0 = 1.1\) GeV\(^2\), are also shown. Our experimental candidates refer to, respectively, as \(\epsilon(800)\), \(A_1(1050)\), \(M(1097)\), \(\rho'(1250)\), \(\eta(1275)\), \(M(1342)\), \(M(1395)\) and \(M(1410)\). (continued)

diffractive nucleon system and conjectured\(^{36,37}\) recently as a candidate for the five-quark ground state. From Fig. 2(b) we see that the respective positions of two groups of several experimental candidates with positive and negative parities are corresponding correctly to those for our first and second excited states,
respectively. Here it may be notable that for the non-exotic \( (q^3) \)-system the lowest positive parity states with \( [SU(6), L^P]=[56, 0^+] \) and the lowest negative parity states with \( [70, 1^-] \) are fully occupied; and the next positive- and negative-parity states with two and three oscillator quanta, respectively, are expected to have much larger masses than the experimental candidates, as shown there. However, it should be noted that, in Fig. 2(b) and Table II(b) the candidates with arrows might be assigned to the excited states for \( (q^3) \)-system according to the recent successful Isgur-Karl model. 38)

**di-baryons** As the input level we have adopted the \( p\bar{p} \) resonance \( B_1^e(2.14) \) in the \( ^1D_2 \) state with \( (I, J^P)=(1, 2^+) \) and mass = 2.14 GeV, of which existence seems to be established. 11,12 Among the other experimental candidates shown in Fig. 2(c) the existence of \( B_1^e(2.22; ^3F_3) \) and \( B_0^e(2.22; ^1F_3) \) also seems to be established. 11,12 From this figure we see that the trunk excited states with the smaller quanta seem to be observed experimentally.

From the above comparison we may conclude that present experiments seem to suggest rather strongly the existence of the steeper Regge trajectories for exotic hadrons, predicted by the joined-spring model.

**C. Level structures and quark statistics**

As was discussed in § 2, the difference of supposed quark statistics, Bose or Fermi, in the two approaches leads to the difference in the level structures for meso-baryons and di-baryons, while no difference for di-mesons.

This difference in the level structures is seen in Tables II(b) and (c). In our previous works 10,11,37,41 we already discussed in some detail this point in com-
paring with the present experiments. The essential points are as follows:

**meso-baryons** The difference\(^{(37,41)}\) is that in the Bose Q.M. the possibly lowest level out of several states with \((I, J^P)=(1/2, 1/2^-)\) and a level with \((5/2, 5/2^-)\) exist, while in the color Q.M. they do not. In this connection it is important to know the parity of \(N^*(1.34)\) and the spin-parity of a possible exotic baryon with \(I=5/2\) recently reported.\(^{(42)}\)

**di-baryons** The difference\(^{(10,11)}\) in the level structures is remarkable. The resonance \(B_{12}(2.14)\) in the \(p\cdot p \, ^1D_2\) state, which seems to be only an established candidate for the ground level, has its seat in both of the two approaches. However, in the color Q.M. we expect fine structures in this state because of degeneracy = 2 and existence of other two resonances in the \(p\cdot p \, ^1S_0\) state of a similar mass = 2.1 ~ 2.2 GeV. There seems to be no evidence\(^{(2)}\) for them.

§ 4. Concluding remarks

In this paper we have investigated systematically the level spectra of typical exotic hadrons, di-mesons \((q^2q^2)\), meso-baryons \((q^4q)\) and di-baryons \((q^6)\) with only nuclear quarks, applying the joined spring model in the framework of both the standard colored Fermi quark model and the Bose quark model: We have determined the level structures up to the first excited levels (§ 3). An interesting feature of the joined spring model is that it leads to the existence of orbital Regge trajectories also with the steeper slope \(a' = \sqrt{3}a_0'\) for exotic hadrons, while the one only with universal slope \(a_0' = (1.1 \text{ GeV}^2)^{-1}\) for non-exotic hadrons. Present experiments seem to suggest rather strongly their existence in the respective systems of exotic hadrons (§ 3). Another interesting point is that the level structures in the two approaches become very much different reflecting the difference of supposed quark statistics, although it seems too early to draw any conclusion.

In the present paper we take a standpoint that all our candidates of exotic hadrons are multi-quark states, while there seems to be a somewhat strong belief that all or some candidates (especially for di-baryons\(^{(43)}\)) are **multi-hadron** resonances. In this connection it may be instructive to remember that in the past the physical picture of \(J_{22}(1232)\) was understood as a \(\pi\cdot N\) resonance, while now it is more systematically interpreted as a state of \((q^3)\) system.

The extensive investigations of multi-quark hadrons have been also made in the framework of MIT bag model.\(^{(7,8)}\) Our treatment has the following features compared with the bag model: i) The internal structures of multi-quark states are uniquely determined in our model, while they seem to be, except for the ground level, ambiguous in the bag model. ii) A prescription to make color-singlet states in our color Q.M. is rather special. In this connection it is interesting that recent experiments seem\(^{(23)}\) not to support the existence of mock-type baryonium,\(^{(44)}\)
which, with a simple structure, does not exist in our scheme from the beginning.

iii) The mass squared of hadrons is described in terms of H.O. in our scheme, while the mass itself is treated in the bag model. Reflecting of this situation, the $SU(6) \otimes O(3)$ classification (thus far rather established) of well-known resonances as non-exotic levels is almost kept in our scheme, while it is changed largely in the bag model.

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Appendix

--- Coordinates and Wave Functions of Hadrons ---

(a) Di-meson

Constraints on spring coordinates

$$r_1 = s_1 + s_2, \quad r_2 = s_3 + s_4, \quad r_1 + r_2 = 0.$$ 

Normal coordinates

$$X_0 = \frac{1}{4} \sum_{i=1}^{4} x_i, \quad S = \frac{1}{2} (x_1 + x_2) - \frac{1}{2} (x_3 + x_4) = \frac{3}{2} (r_1 - r_2),$$

$$U_1 = x_1 - x_2 = s_1 - s_2, \quad U_2 = x_3 - x_4 = s_3 - s_4.$$ 

Mass operator

$$M_s^2 = \frac{\lambda}{4} \left( P_s^2 + \frac{4}{3} K_0^2 S^2 \right) + \sum_{i=1}^{2} \frac{\lambda}{2} (P_{s_i}^2 + K_0^2 U_{s_i}^2); \quad \lambda = \frac{2d}{m}, \quad K_0 = \frac{mk}{4}.$$ 

Ground wave function

$$f_G(S, U_1, U_2) = \left( \frac{4}{3} \right)^{3/8} \left( \frac{\pi}{K_0} \right)^{9/4} \exp \left\{ - \frac{K_0}{2} \left( \frac{2}{\sqrt{3}} S^2 + U_1^2 + U_2^2 \right) \right\}.$$ 

(b) Meso-baryon

Constraints on spring coordinates

$$r_1 = s_1 + s_2, \quad r_2 = s_3 + s_4, \quad r_0 + r_1 + r_2 = 0.$$ 

Normal coordinates

$$X_0 = \frac{1}{5} \sum_{i=1}^{4} x_i,$$

$$R = -\frac{1}{4} \sum_{i=1}^{4} x_i + x_5 = -\frac{7}{4} (r_1 + r_2).$$
Mass operator

$$\mathcal{M}_s^2 = \frac{5\lambda}{16} \left( P_s^2 + \frac{64}{35} K_0^2 R^2 \right) + \frac{\lambda}{4} \left( P_s^2 + \frac{4}{3} K_0^2 S^2 \right) + \sum_{i=1}^{2} \frac{\lambda}{2} (P_{0i}^2 + K_0^2 U_i^2).$$

Ground wave function

$$f_0(R, S, U_1, U_2) = \left( \frac{256}{105} \frac{\pi}{K_0} \right)^{12/4} \times \exp \left\{ -\frac{K_0}{2} \left( \frac{8}{\sqrt{35}} R^2 + \frac{2}{\sqrt{3}} S^2 + U_1^2 + U_2^2 \right) \right\}.$$

(c) Di-baryon

Constraints on spring coordinates

$$r_1 = s_1 + s_2, \quad r_2 = s_3 + s_4, \quad r_3 = s_5 + s_6, \quad r_1 + r_2 + r_3 = 0.$$

Normal coordinates

$$X_0 = \frac{1}{6} \sum_{i=1}^{6} x_i, \quad R = -\frac{1}{4} (x_1 + x_2 + x_3 + x_4) + \frac{1}{2} (x_5 + x_6) = -\frac{9}{4} (r_1 + r_2),$$

$$S = \frac{1}{2} (x_1 + x_2) - \frac{1}{2} (x_3 + x_4) = \frac{3}{2} (r_1 - r_2), \quad U_1 = x_1 - x_2 = s_1 - s_2,$$

$$U_2 = x_3 - x_4 = s_3 - s_4, \quad U_3 = x_5 - x_6 = s_5 - s_6.$$

Mass operator

$$\mathcal{M}_s^2 = \frac{3\lambda}{16} \left( P_s^2 + \frac{64}{27} K_0^2 R^2 \right) + \frac{\lambda}{4} \left( P_s^2 + \frac{4}{3} K_0^2 S^2 \right) + \sum_{i=1}^{3} \frac{\lambda}{2} (P_{0i}^2 + K_0^2 U_i^2).$$

Ground wave function

$$f_0(R, S, U_1, U_2, U_3) = \left( \frac{256}{81} \frac{\pi}{K_0} \right)^{15/4} \times \exp \left\{ -\frac{K_0}{2} \left( \frac{8}{3\sqrt{3}} R^2 + \frac{2}{\sqrt{3}} S^2 + U_1^2 + U_2^2 + U_3^2 \right) \right\}.$$

(d) Meson

Constraints on spring coordinates

$$r_1 + r_2 = 0.$$
Normal coordinates

\[ X_0 = \frac{1}{2} \sum_{i=1}^{2} x_i, \quad S = \frac{1}{2} (x_1 - x_2) = \frac{1}{2} (r_1 - r_2). \]

Mass operator

\[ \mathcal{M}_s^2 = \frac{\lambda}{2} (P_s^2 + K_0^2 S^2). \]

Ground wave function

\[ f_0(S) = \left( \frac{K_0}{\pi} \right)^{3/4} \exp \left[ -\frac{K_0}{2} S^2 \right]. \]

(e) **Baryon**

Constraints on spring coordinates

\[ r_1 + r_2 + r_3 = 0. \]

Normal coordinates

\[ X_0 = \frac{1}{3} \sum_{i=1}^{3} x_i, \]

\[ R = \frac{1}{2} (2x_3 - x_1 - x_2) = \frac{3}{2} r_3, \quad S = x_1 - x_2 = r_1 - r_2. \]

Mass operator

\[ \mathcal{M}_s^2 = \frac{\lambda}{2} (P_s^2 + K_0^2 S^2) + \frac{3\lambda}{8} \left( P_R^2 + \frac{16}{9} K_0^2 R^2 \right). \]

Ground wave function

\[ f_0(R, S) = \left( \frac{4}{3} \right)^{3/4} \left( \frac{K_0}{\pi} \right)^{3/2} \exp \left[ -\frac{K_0}{2} \left( S^2 + \frac{4}{3} R^2 \right) \right]. \]

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