Monte Carlo Study of the $SU(2)$ Lattice Gauge Theory with the Quintet Representation

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The $SU(2)$ gauge theory regularized by Wilson's space-time lattice is studied by the Monte Carlo method. The link variables are taken to be in the quintet representation. The result suggests that a first order phase transition occurs at the equivalent inverse temperature $\beta_s \approx 3.7$. Moreover a long-lived false vacuum is found for $\beta_s > 5$.

It is widely believed that the $SU(2)$ lattice gauge theory with the defining representation has the single-phase structure. This situation is not altered in the realistic $SU(3)$ case. Thus it is quite natural to expect that the confinement of quarks may be understood, since the lattice gauge theory explains it in the strong-coupling regime.

Recently the $SU(2)$ theory with the adjoint representation has been found to show the sharp first-order phase transition at $\beta_s \approx 2.5$. The reason is believed to be the singlet nature of the adjoint representation under the $Z_2$ transformation, where $Z_2$ is the center of the group $SU(2)$. Furthermore the mixed action with the defining and the adjoint representations has been studied and found to have the very rich phase structure.

In this letter we shall study the $SU(2)$ lattice gauge theory with the quintet representation in order to confirm the importance of the $Z_2$ degrees of freedom in the $SU(2)$ gauge system. The quintet 5 is trivial under $Z_2$ as the adjoint representation. Thus it may be inferred that the gauge system with 5 must show the same kind of phase structure as that with the adjoint representation.

Our system is defined by the partition function

$$Z(\beta_s) = \int [dU] \exp(-\beta_s S),$$

where the Euclidean action $S$ is defined as

$$S = \sum_{\phi_k} S_{\phi},$$

$$S_{\phi} = 1 - \frac{1}{5} \text{Tr} U_{\phi}.$$  \hspace{1cm} (2a)

The $5 \times 5$ matrix $U_{\phi}$ is a product of four link variables which are elements of the five-dimensional representation. In Eq. (2a) summations are taken over all independent plaquettes. The equivalent inverse temperature $\beta_s$ is related to the bare Yang-Mills coupling $g_0$ as

$$g_0^2 = 2 \beta_s.$$  \hspace{1cm} (3)

Equation (2b) can be conveniently rewritten by using the formula

$$\text{Tr} U_{\phi} = (\text{Tr} U_{\phi})^4 - 3(\text{Tr} U_{\phi})^2 + 1,$$  \hspace{1cm} (4)

where $U_{\phi}$ is the defining two dimensional representation. Therefore we can safely forget the $5 \times 5$ matrix and use the $2 \times 2$ matrix to deal with the system. The classical one-plaquette action is depicted in Fig. 1 as a

$$S_{\phi} = 1.4$$

Fig. 1. Classical one plaquette action.
function of $\cos \theta = (1/2) \text{Tr} U \tau$. Our gauge system will be of interest itself as a toy model of quantized theory, since the classical potential has a typical local minimum.

We parametrize a link variable as

$$U_L = a I + i a \sigma_j,$$

$$a^2 = a_1^2 + a_2^2 + a_3^2 = 1,$$

where $I$ is the $2 \times 2$ unit matrix and $\sigma_j (j = 1, 2, 3)$ are hermitian Pauli matrices. Our Monte Carlo algorithm follows that of Metropolis et al.\cite{4} Each link variable is changed as

$$a_\mu(\text{NEW}) = N [a_\mu(\text{OLD}) + \xi],$$

where $N$ is a normalization factor to satisfy $a_\mu^2(\text{NEW}) = 1$, $\xi$ a random number uniformly between $-1$ and $+1$, and $\alpha$ a suitable positive number. We have used $\alpha = 0.175$. After a new link variable is generated by (7), we compare the new and the old actions for six plaquettes touched to the relevant link ($S_6(\text{NEW})$ and $S_6(\text{OLD})$). If an inequality

$$\eta > \exp[-\beta S(S_6(\text{NEW}) - S_6(\text{OLD}))]$$

is satisfied, we reject the change and try a new change. Here $\eta$ is a uniform random number between 0 and 1. This algorithm is applied ten times to each link for one Monte Carlo iteration. In Fig. 2 we have shown the simplest one-plaquette Wilson loop

$$W = 1 - \langle S_\tau \rangle$$

at $\beta_s = 2.95$ as a function of iterations in order to illustrate the convergence in our method. The initial condition used is the ordered start. As $\beta_s$ approaches 3.7 from below, it needs large number of iterations to realize the canonical distribution in the case of the ordered start. At $\beta_s = 3.1$, for example, the value of $W$ abruptly drops down to $\sim 0.17$ after about 850 iterations. Figure 3 is a thermal graph for cold and hot starts with 100 Monte Carlo iterations. The initial configurations for each $\beta_s$ are fixed by the final configurations for the previous value of $\beta_s$. The lattice used is $4^4$ sites in size. Dots are from the cold start and open circles are from the hot start. The shape of Fig. 3 is very different from that obtained in the case of the adjoint representation.\cite{23,3} It does not form a complete thermal cycle, so that a definite conclusion on the phase structure cannot be extracted. Presumably there exists a first order phase transition at $\beta_s \approx 3.7$, and the lower curve for $\beta_s \approx 5$ in Fig. 3 will be a very long-lived false ground state which may abruptly coincide with the ground state curve (the upper curve), when a sufficient
The number of iterations are performed. The appearance of the long-lived metastable state will be a remnant of the local minimum of the classical potential depicted in Fig. 1. If the above arguments are the case, the gauge field configurations will be trapped long time in the color-confining phase even in the weak-coupling regime when the system cools down from the hot configurations.

Our analyses in this letter are highly incomplete, so that further study is desirable.

Numerical calculations have been carried out with FACOM M-200 at the Computer Center, Kyushu University.

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