DISCUSSION

C. J. Dodds[2]

I'm afraid some of the comments and results in this paper are open to criticism. I will restrict my remarks to the authors' treatment of their problem by the theory of stationary random vibration and to their results. I will not comment on the experimental technique or computational aspects of the work, although I think it is worth noting that a road measurement exercise using a profilometer of similar design to that used by the authors was carried out by MIRA [9][3] in 1969. The problems associated with recording and analyzing the large dynamic range incurred, when measuring road profile displacement, were discussed in this report.

The theory presented at the beginning of this paper inherently assumes (equations (2)—(5)) that the vibrating system under consideration is a *single input* linear system. It is debatable whether a road vehicle can be treated in this way, but many authors, including myself, have often obtained results from a simple mathematical model with one input, which agree favorably with experimental measurements taken from a test vehicle. But we must remember this fact when we evaluate a "transfer function" from experimental data; for the case of a four-wheeled vehicle the transfer function may be defined in terms of the response of the vehicle to each individual input, or to all four inputs in phase, or it may be speed dependent to allow for a phased input at the rear. The transfer function in this paper must be defined as the response of the system due to a unit sinusoidal excitation applied to all four wheels in phase.

The authors claim that the PSD curve does not describe the amplitude distribution on the road. This is true: a stationary random signal requires not only a spectral density, but also an amplitude distribution for complete description. However, it is physically impossible for a 'large PSD to be caused by a few large amplitudes or a combination of many small ones.' If a random Gaussian (or near-Gaussian) signal is passed through a linear filter of narrow bandwidth (as in PSD analysis) the resultant narrow band random signal will have a Gaussian amplitude distribution and a *Rayleigh peak distribution.* (It is the limiting distribution function of peak values for a narrow band Gaussian random signal as the bandwidth approaches zero.)

There is some evidence to suggest that the undulations of road surface roughness may be governed by a Gaussian (or near Gaussian) distribution [10, 11]. It is difficult to check conclusively for a Rayleigh distribution in each bandwidth using the data presented in Figs. 10 and 11, but I suggest that the authors should investigate this possibility.

The transfer function presented by the gain and phase plots of Figs. 8 and 9 is misleading. The frequency scale is speed dependent, and an unsatisfactory method of presenting vehicle (not road) data. Much more serious, however, is the implication that a phase curve which is almost constant (just below 90 deg) can be obtained for a 'linear' system with resonant conditions at around 1, 5, and 12 Hz. I would hope that this graph is in error and refer the authors to any textbook on dynamic systems analysis.

Our own results on cross-spectral analysis between the vehicle response and the road input at one wheel show quite clearly that the phase curves obtained go from 0–180 deg and pass through *90 deg at resonance* (see Fig. 12).

The authors suggest that the linearity of the system under study can be validated by a 'coherence function of 0.75.' It is not clear what they mean by this statement. The coherence function as defined by Bendat and Piersol [12] is *frequency dependent.* If the coherence function is greater than zero but less than unity, one or more of three possible situations exist:

a) Extraneous 'noise' is present in the measurements;
b) The system is nonlinear;
c) The output is due to more than one input.

It is unlikely that conditions (a) and (c) do not exist in the response of the test vehicle, as the response will be due not only to the road input but also to other extraneous sources (e.g., aerodynamic, engine, etc.). In my view a value of 0.75 is low, and to draw conclusions on the linearity of the system from this figure is unjustifiable.

Finally, I would suggest that the AFD technique presented in this paper contributes nothing to the descriptive process for a stationary random signal. If a stationary random signal, $x(t)$, is applied to a linear system with receptance $\alpha(if)$, to produce a response $y(t)$, then complete description of $y(t)$ depends on the form of $x(t)$. The frequency distribution of $y(t)$ can be obtained utilizing the spectral density approach:

$$S_y(f) = |\alpha(if)|^2 S_x(f)$$

If $x(t)$ is governed by a Gaussian amplitude distribution, then so is $y(t)$ with its mean square value obtained from:

$$\sigma_y^2 = \int_0^\infty S_y(f)df$$

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[3] Numbers in brackets designate Additional References at end of discussion.

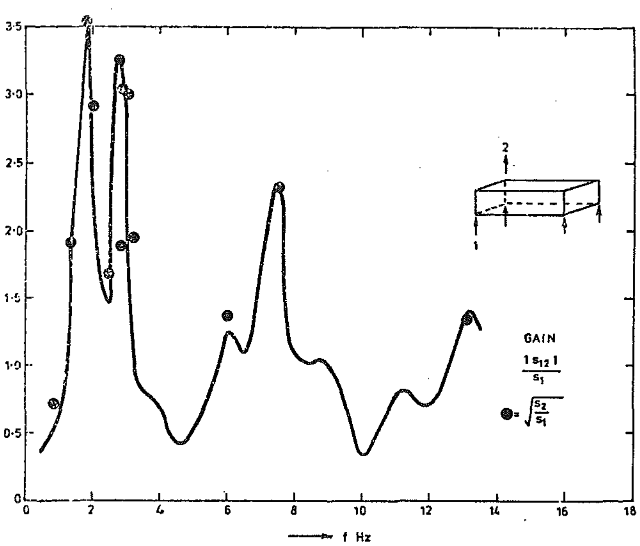

Fig. 12(*a*) Vehicle transfer function—*A*, Gain (other inputs moving)

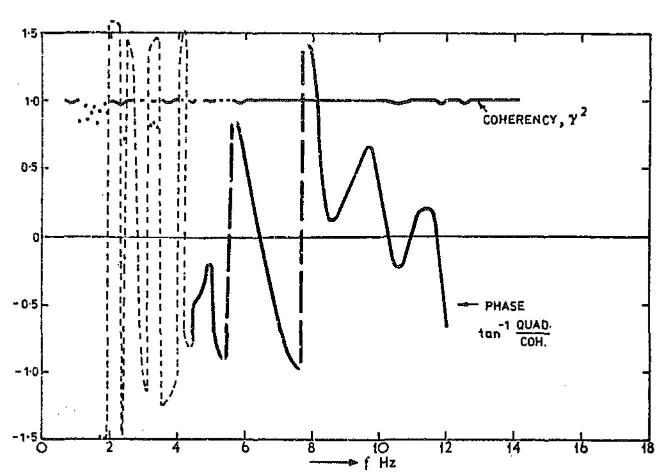

Fig. 12(*b*) *B*, phase and coherency

However, if the amplitude distribution of $x(t)$ is non-Gaussian then to the best of my knowledge no information regarding the distribution of $y(t)$ can be obtained by a simple mathematical approach. It may be that the work in this paper is an attempt to handle non-Gaussian random data, but this is not suggested by the authors.

Therefore, for the case under study in this paper, I would suggest that a Rayleigh distribution will be found to exist governing the data in each analysis bandwidth, making the AFD technique redundant.

Additional References

9 Labarre, R. P., Forbes, R. T., and Andrew, S., "The Measurement and Analysis of Road Surface Roughness," MIRA Report No. 1970/5.

10 Dodds, C. J., "The Response of Vehicle Components to Random Road Surface Undulations," Ph.D. thesis, Department of Mechanical Engineering, University of Glasgow, 1972.

11 Braun, H., "Untersuchungen über Fahrbahnunebenheiten," Deut. Kraft **186**, 1966.

12 Bendat and Piersol, *Random Data: Analysis and Measurement Procedures*, Wiley Interscience, 1971.

Author's Closure

We appreciate receiving the comments made by Dr. Dodds. While our Conclusion 6 points out that the AFD method offers the possibility of using a transfer matrix for nonlinear system analysis (in much the same way that PSD is related to a transfer function for linear systems), it appears we did not emphasize this enough. In addition, we completely neglected to mention that the AFD method was developed to eliminate the restriction that the random data be Gaussian. With these facts in mind, most of Dr. Dodds' criticisms are unfounded.

We further believe that much of the disagreement is also due to differences in approach. Looking at average roads for inputs to a vehicle, as Dr. Dodds is doing, is much different than having an average vehicle look at a specific road, as done in this project. When looking at specific roads from a highway department's viewpoint of maintenance schedules, etc., one cannot eliminate the potholes and other discontinuities present. Under these conditions it is our contention that a section of road does not have Gaussian distributed roughness.

There are, however, three points made by Dr. Dodds that require clarification. First, it is agreed that the phase relationship was questionable. At the time, we briefly looked at it and concluded that the digital smoothing performed on the profile PSD and the profile-acceleration CPSD caused the inaccuracies. Since the research did not require use of the phase information, further work on this aspect could not be justified.

The second point relates to linearity and coherence function. The project report shows a plot of coherence function with frequency; however, this curve was eliminated from the paper at the suggestion of a reviewer. While some may feel an average value of 0.75 is not a good indication that a system can be considered linear, it was found sufficient for the purpose of this project and was verified by the final results.

The third and most disturbing point reacts to the statement "using a profilometer of a similar design." After viewing a picture presented by Dr. Dodds, we question the similarity other than in principle. The vehicles and trailing wheel dynamics appear to be quite different; in fact, the preload used on the PennDOT vehicle might lift one of the car wheels on the vehicle presented by Dr. Dodds. Likewise, the PennDOT profilometer measures both wheel path profiles simultaneously rather than just one. There is a great deal of literature available on the use and accuracy of the profilometer, and a few of these reports are given here for reference.

Additional References

13 Darlington, J. R., "Evaluation and Application Study of the General Motors Corporation Rapid Travel Profilometer," Michigan Dept. of State Highways Research Report No. R-731, Oct. 1970.

14 Brickman, A. D., Wambold, J. C., and Zimmerman, J. C., "An Amplitude-Frequency Description of Road Roughness," *Highway Research Record No. 116,* 1971, pp. 53–67.

15 Peterson, D. E., "A System for Planning Roadway Improvements," Utah State Highway Dept. Paper, Highway Research Board Summer Meeting, Aug. 17–19, 1970, Sacramento, Calif.

16 Spangler, E. B., and Sprangler, W. J., "GMR Road Profilometer Method for Measuring Road Profile," General Motors Research Publication GMR-452, 1964.

17 Hudson, W. R., "High Speed Road Profile Equipment Evaluation," *Highway Research Record No. 189,* 1967, pp. 150–164.

18 Darlington, J. R., and Milliman, P., "A Progress Report on the Evaluation and Application Study of the General Motors Rapid Travel Profilometer," *Highway Research Record No. 214,* 1968, pp. 50–67.