Hydraulic performance analysis of sewer systems with uncertain parameters

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ABSTRACT

In this paper, a flexible fuzzy model is proposed for the hydraulic performance analysis of separate domestic sewer systems. In the proposed model, all modeling outputs such as discharge, velocity and depth are developed as fuzzy numbers by taking into account all the available information and expert knowledge about the basic design/analysis parameters. The fuzzy outputs are then combined with performance assessment curves to calculate the hydraulic performance values. The proposed model was applied to a part of the sewer system of a city in Iran, and performance graphs were plotted. Such graphs can be used by design engineers and operation managers to improve the design quality, reliability and the performance of a system with uncertain parameters. The analysis results can also be used in decision-making and identifying priorities to develop rehabilitation strategies.

Key words | fuzzy logic, hydraulic performance analysis, sanitary sewer system, uncertainty

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INTRODUCTION AND OBJECTIVES

Sewer systems are an essential part of the urban water infrastructure and are of particular importance to the community because they play a major role in protecting public health and safety. Currently, however, many cities have old sewer systems that are deteriorating, leaving communities vulnerable to unexpected catastrophic failures that could disrupt not only the sewer service but also above-ground activities (Ana & Bauwens 2007). Therefore, special care should be taken in the design, operation, maintenance and rehabilitation of sewer systems. To this end, performance analysis has been established as an essential tool.

Performance analysis becomes a key issue in the engineering approach to control the system, both as a natural process of evolution of the modeling and design methods available and as a consequence of an ever increasing and generalized awareness of the quality of the service provided (Coelho 1997; Cardoso *et al*. 1999). In this regard, performance indicators and performance assessment systems are widely used in many industry sectors to support economic and financial decision-making. Performance analysis has been successfully applied to water supply systems (Faria & Alegre 1996; Alegre 1997; Coelho & Alegre 1997), and its application to sewer systems has been gaining popularity.

The main purpose of this paper is to evaluate the hydraulic performance of separate domestic sewer systems, in terms of depth and flow velocity in the pipes, considering uncertainties involved in selecting the design and analysis parameters. The idea originated from the fact that according to the Iranian Standard for Designing Wastewater Collection Systems (ISDWCS) (Ministry of Energy of Iran 1998), in the process of designing separate sanitary sewer systems, the following two main points must be considered:

1. The conveyance efficiency of all the pipes must be sufficiently high to accommodate the effluent discharges at the end of the design period (i.e. the planning period for designing a sewer system). Generally speaking, the slope and diameter of the pipes must be adjusted in such a way that the pipes have the capacity to convey the maximum possible discharges. Satisfying other requirements, such as maximum and minimum allowable water depths in the pipes, maximum and minimum permissible velocities in the pipes and minimum required cover for the

doi: 10.2166/hydro.2011.129

pipes, further complicates the design procedure. The minimum required cover is the minimum required distance from ground surface to the top of the pipe set by the design standards or local authorities. It is practically impossible to satisfy all the requirements considering the daily, seasonal and annual variation of discharges likely to occur during a design period of, say, 25 years. Therefore, as part of the design procedure, a hydraulic performance analysis is required regarding the water depth and flow velocity at the beginning of the design period when discharges are expected to be much smaller than the maximum capacity of the pipes. Some local authorities even require that such a performance analysis be presented every 5 years in order to obtain a better perspective of the performance of the system over time. Small velocities and depths in the pipes may cause sedimentation and solid deposition. Velocities greater than the maximum permissible velocity, which depends on the type of pipe material used, may cause damage to the pipe. As an overall recommendation, water depth greater than 0.8D, where D is the diameter of the pipe, is not allowed due to the conveyance efficiency of the pipe and to ensure good ventilation of the sewer system. Therefore, the performance analysis determines the self-cleaning capacity of the pipes and identifies the need for manual cleaning and maintenance of some of the sewer pipes during the design period.

2. In designing hydraulic systems, design engineers usually select design parameters by referring to the design standards or making engineering judgments using past experience and knowledge. However, uncertainty in the design parameters is inherently present in designing any hydraulic system. This is more pronounced for designing sewer networks where different factors may contribute to the occurrence of such uncertainties. Imprecise estimation of the design discharges and their daily and seasonal variation, and vagueness in selecting hydraulic parameters such as the Manning's roughness coefficient, are the two main sources of uncertainties. Design engineers consider these uncertainties in their design and usually try to deal with the problem of uncertainty by appropriately selecting average values for those parameters. The main problem with this approach, especially in communities where there is no well-established data, is that the recommended ranges for design and performance analysis parameters are very wide and this makes the hydraulic performance analysis very unreliable.

Considering the above two items, the main objective of this study was to provide a model for hydraulic performance analysis, which considers the uncertainties involved in selecting the design and performance analysis parameters.

Among the different sources of uncertainty, such as model uncertainty, parameter uncertainty, etc., the parameter uncertainty is considered as the major source of uncertainty in engineering work and the uncertainty analysis and modeling is aimed at understanding how the uncertainty in model parameters is propagated to the unknowns or outputs of the problem. Various mathematical theories have been proposed to model uncertainty. Probability theory, interval arithmetic, random sets and fuzzy sets are among them (Oberguggenberg 2005).

In engineering, probabilistic models as a traditional way of handling uncertainty have been criticized as requiring more input from the design engineer or the decision maker than could be plausibly provided or that would be reasonably required for a rough estimate. This is particularly true for domestic sewer systems where there is a general agreement among engineers on lack of information and data about the model parameters, especially in societies where there is no well-developed monitoring system for data collection and analysis. Therefore, there is always ambiguity in selecting the design and analysis parameters, which results in imprecision in the selected parameters. A tool well suited to process such data is given by the fuzzy set theory and fuzzy numbers. This method was formulated with the specific purpose of mathematically describing imprecise information and interpreting concepts defined by linguistic expressions into a mathematical model.

The next section of this paper gives a background of the subject. In the background section, a link is also made between the overall structure of the methodology used in this paper and the relevant background knowledge. Then, the methodology is introduced followed by an application of this methodology to a case study. The case study presents the hydraulic performance analysis of part of a domestic network designed for the city of Amole in the northern part of Iran. Finally, the results obtained from this study are discussed.

BACKGROUND

Many studies have been conducted on the performance analysis of sewer networks. The works of Cardoso and colleagues (Cardoso *et al.* 1999, 2001, 2004, 2005, 2006) are among the best references that can be used in defining performance indicators and applying performance assessment systems in this field. Cardoso *et al.* (2005) presented a comprehensive review of the structure of a performance assessment system applicable to combined or separate domestic sewer networks. The structure of such a system is shown in Figure 1.

As an example, the water depth in a sewer pipe is used as a state variable of interest for the hydraulic performance analysis of a sewer system. The values of this state variable, for any given state or scenario of interest, can be either generated using hydraulic simulation models or obtained from reliable records. In step 4 a performance curve, which plots the performance value credited to the state variable or network property at the network element level, over a given range of selected variables is introduced. The performance value varies between a no-service and an optimum-service situation, and the curves are supposed to penalize any deviation from the latter (Cardoso *et al.* 2005). Finally, a mathematical operator must be used that will allow the performance values at element level to be aggregated across the system or parts of it. This might simply be an average, a weighted average, a maximum or a minimum value.

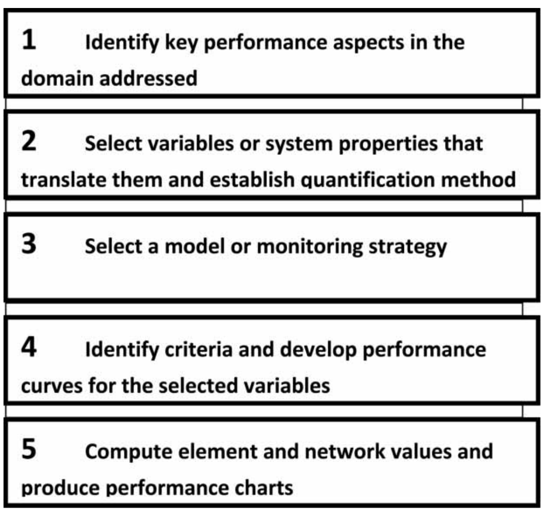

Figure 1 | Structure of a performance assessment system (Cardoso *et al.* 2005).

The type of operator depends on the objective of the analysis. The results of the performance analysis will consist of a set of performance values representing the network and a global aggregated value. These may be plotted against time, as a result of a time-driven simulation such as that generated by a rain or domestic flow event, in order to produce an event graph. This represents the global system performance over time. Another possible type of graph, referred to as a system graph, plots the performance for different but related scenarios corresponding to a range of load factors applied, for example, to the average dry weather flow. System graphs translate the overall system performance as a function of load factor, return period or any set of related scenarios. They can show how a system is performing according to the load variations. Cardoso *et al.* (2005) used percentiles and presented them in various graphs to give a statistical perspective of the performance indicators along the sewer network. Further description and discussion regarding the different steps in this performance assessment system is given in the hydraulic model section because this system was used in the present study.

In another study, Cardoso *et al.* (2006), referring to the APUSS (Assessing infiltration and exfiltration on the Performance of Urban Sewer Systems) project, a project associating universities, small and medium enterprises and municipalities in seven European countries, introduced some performance indicators to assess the impacts of infiltration/exfiltration on combined or separate domestic sewer systems. Infiltration of groundwater is particularly detrimental to the treatment plant's efficiency, while exfiltration of wastewater can lead to groundwater contamination. As reported by the authors, one of the objectives of the APUSS project was to provide end-users with integrated elements for decision support that account for infiltration/exfiltration rates, impact on the sewer functioning, the wastewater treatment plant and on the economic value, and facilitate the comparison of different investment strategies to reduce infiltration and exfiltration. In their work they used four performance indicators for infiltration and three indicators for exfiltration. These indicators were basically defined as the ratio of infiltration or exfiltration flow to different characteristics of the sewer such as, for example, the daily mean dry weather flow or the sewer longitudinal area. The overall and combined use of such performance indicators, which were calculated for three case studies in Italy and France, provided means to assess the performance of the sewers. This performance assessment can support decisions on when and where to rehabilitate, depending on the selection criteria (hydraulic, environmental, economic, etc.). As concluded by Cardoso *et al.* (2006), the application of performance indicators for infiltration and exfiltration was based on available data, which means that the quality and uncertainty of the results of the performance indicators depend on the quality and uncertainty of the data used. They recommended that possible next steps would consist of accounting for uncertainties and establishing a robust classification for performance indicators based on more case studies.

Korving *et al.* (2009) presented a risk-based approach to decision-making regarding sewer system rehabilitation. The approach considers uncertainties in the sewer system dimensions, natural variability in rainfall, and the uncertainty in the cost function that describes environmental damage. In particular, the application of different shapes of cost functions was studied. The optimization method was demonstrated with a case study on the optimization of the storage capacity of a sewer system by balancing the investment cost and the damage due to combined sewer overflows.

Bennis *et al.* (2003) proposed a methodology for evaluating the hydraulic performance for possible rehabilitation of sewer systems. It involves assigning a hydraulic performance index to each pipe section. This hydraulic index reflects both the local surcharge in a pipe and the surcharge induced at upstream sections of the same branch of the sewer network. The hydraulic index also takes into account the vulnerability and the retention capacity of each pipe section. This index may be used directly to establish the rehabilitation priority of different sections to maximize the hydraulic performance of the entire network. This methodology was successfully applied to the sewer system of the city of Laval in Canada. The results showed how pipe dimensions and locations have the effect of surcharging or relieving a pipe network and how the hydraulic performance index adequately rates the contributions of the sewer network components. Bengassem & Bennis (2000) used a hydraulic performance index to build a fuzzy hydraulic system for each pipe of a sewer network. A fuzzy structural system was also built by taking the intrinsic condition, extrinsic condition, and site vulnerability of the pipe as inputs. A fuzzy global system made use of all the involved factors to set up a global performance index for each pipe. The outputs of the systems were expressed as performance indices which range from 0 to 100%, where 0% expresses a perfect state. Such indices can be used in any rehabilitation program. The application of the method to a network of the city of Laval, Canada, gave very good results (Bengassem & Bennis 2000). Hahn *et al.* (2002) presented the development and evaluation of a knowledge base for an expert system that predicts the criticality of sewer pipelines. The expert system considers information regarding the environment and the state of a sewer line through an extensive set of relationships that describe failure impact mechanisms. The paper discusses the constraints faced by sewer utility operators and managers and the tools developed specifically to assist small to medium sized utilities to operate and maintain their systems by focusing their inspection efforts on their most critical pipelines. The knowledge base was evaluated with a series of case studies and was shown to be effective at mimicking the knowledge of experts.

The methodology adopted in this study basically uses the same performance analysis system proposed by Cardoso *et al.* (2005), tailored for the problem at hand. In this paper, a fuzzy model is proposed for the hydraulic performance analysis of separate sanitary sewer networks. In this model, the hydraulic performance is determined on the basis of the satisfactory performance of the flow velocity and depth for each individual pipe or entire network, at any specified time of its operation, such as the beginning and end of the design period. In the proposed model, the average, minimum and maximum design discharges are considered fuzzy variables, which can be affected by many other uncertain parameters such as an overestimation or underestimation of population growth, the rate of connection to the network and the per capita daily water consumption. In the hydraulic model, the Manning roughness coefficient is a fuzzy parameter while the slope and diameters are crisp numbers. In general, if required, in the proposed model any expected or unusual condition can also be included in the analysis by defining related design or operation parameters as fuzzy numbers. Considering uncertainty in the selected parameters, the maximum and minimum fuzzy depths/velocities at any particular time can be calculated using fuzzy arithmetic. Consequently, these fuzzy outputs are used to calculate hydraulic performance values when they are combined with the recommended performance assessment curves. The main advantage of this approach is that the hydraulic performance analysis is conducted in a wider spectrum of selected significant uncertain design or analysis parameters.

METHODOLOGY

Design parameters, discharges and uncertainties

Figure 2 shows the general layout of a sanitary sewer network that has seven pipes and eight manholes. According to ISDWCS, the following four main discharges must be calculated for each pipe of any sanitary sewer network such as pipe number 6, shown in Figure 2. The discharges, which can be calculated for any particular year during the design period, are:

$$Q_{ave} = PW \tag{1}$$

$$Q_{max} = k_{max}PW + Infiltration + Inflow \tag{2}$$

$$Q_{min} = k_{min}PW + Infiltration \tag{3}$$

$$Q_{sc} = k_{max}PW + Infiltration \tag{4}$$

where Q_{ave}, Q_{max}, Q_{min}, and Q_{sc} refer to the average, maximum, minimum and self-cleaning discharge of the pipe, respectively. P is the population that is serviced by the pipe, and W is the per capita daily wastewater produced.

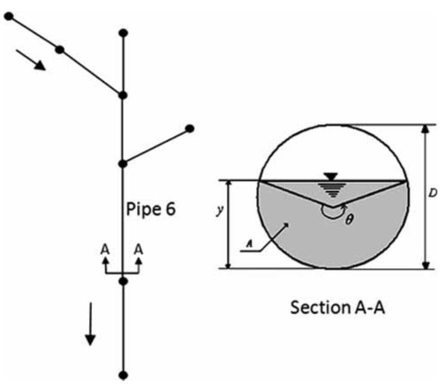

Figure 2 | General layout of a separate domestic sewer system with typical flow section in a circular pipe.

The coefficients k_{max} and k_{min} are the parameters that account for the hourly variation in the discharge during the day. *Infiltration* refers to the quantity of groundwater or soil water surrounding the pipe that enters into the sewer network through pipe structural defects such as pipe cracks, joints and manholes. Sanitary systems, although not designed to carry storm water, carry storm water inflow through system defects, such as manhole covers, illegal drain connections, and unintended cross connections with storm sewers (Hahn *et al*. 2002). *Inflow* refers to all waters draining to the sewer network from such sources. In the above equations, any possible discharge coming from large local industries has been ignored; they can be appropriately added if required. The concept and methodology for the selection or calculation of the above parameters are as follows.

P, which is the population serviced by the pipe or living upstream of the pipe, for any specific year after the sewer network operation, can be estimated based on the population study of the area. However, population numbers resulting from such studies must be multiplied by another factor called the 'connection rate'. The connection rate, which is a number less than or equal to 1, arises from the fact that it is not always true that all domestic sewer systems are connected to the city sewer network. The connection rate depends on many factors such as people's culture, their income and interest in connecting to a new sewer system, difficulties existing with their current system of wastewater disposal and legal regulations. Although there are recommendations for estimating the connection rate, it is considered to be a very uncertain design and analysis parameter, especially in communities with no well-established data. In this paper, P is the population serviced by the pipe considering the connection rate. Therefore, a reasonable uncertainty is included in P, considering the design recommendations and expert knowledge.

The coefficients k_{max} and k_{min}, which account for the hourly variation in discharge during a day, are considered to be a function of the population. The following equations are recommended to estimate k_{max} and k_{min} values for a population of less than one million.

$$k_{max} = \frac{5}{P^{0.167}} \tag{5}$$

$$k_{\min} = \frac{P^{0.167}}{5} \tag{6}$$

where the population, P, is expressed in thousands. Therefore, $k_{\min}$ varies between 0.2 and 0.63 and $k_{\max}$ varies between 1.58 and 5. Although there are uncertainties involved in the model structure and constant parameters of the above equations, in this paper it is assumed that only uncertainty in P is propagated to the process of estimating $k_{\max}$ and $k_{\min}$. Also, it should be considered that although the flow rate variation depends on other socio-economical factors in different parts of Iran, the above equations indicate that the maximum and minimum values of flow rate highly depend on population, and the coefficients in the equations reflect an average effect of the other factors.

W can be estimated by multiplying the per capita daily water consumption by a conversion factor. According to ISDWCS, the conversion factor depends on many complex parameters such as the people culture and climatic conditions in the area. This parameter is difficult to estimate and a value of 0.6–0.8 is recommended for it. Fortunately, in most societies, reliable data are usually available to estimate the per capita daily water consumption. Therefore, in this paper an overall reasonable uncertainty is considered for W, considering the above mentioned factors.

Infiltration is also a very complex uncertain parameter that mainly depends on the local groundwater level and pipe condition. The simplest method recommended by ISDWCS, is that for estimating $Q_{\max}$, a 10–20% of the average daily wastewater production, Q_{ave}, should be considered as infiltration in arid regions where the groundwater table is very low. For $Q_{\min}$ and Q_{sc}, the estimated possible infiltration percentage in arid regions is zero. However, one issue that has not been discussed in the standards for estimating $Q_{\min}$ and Q_{sc} is the exfiltration resulting from wastewater leakage through the pipes, which has a high likelihood of occurrence in arid regions. In this study, the uncertainty considered for *Infiltration* includes all these factors, including possible exfiltration. The recommended *Inflow* value by ISDWCS is similar to *Infiltration* and is about 10–20% of Q_{ave}. As Equations (1)–(4) show, *Inflow* is only considered for estimating $Q_{\max}$.

Q_{sc} is an indication of the self-cleaning discharge of the pipe. It is a momentarily high discharge that does not include possible *Inflow*. The concept is that such a discharge can wash all the sediments and solids deposited in the sewer pipe during the day. It is recommended that the need for any manual cleaning of the pipe at any particular year be predicted on the basis of the normal velocity and depth of flow under the discharge Q_{sc}. These variables can be referred to as self-cleaning velocity and depth. Therefore, Q_{sc} is a very important flow rate element in hydraulic performance analysis of sewer systems.

Hydraulic model

Each pipe in any sanitary sewer network, such as pipe 6 in Figure 2, must be able to convey wastewater at various rates of flow, as described above, under a free flow condition as shown in the figure. Most of the calculations which are used to adjust diameter and slope and to control design criteria are based on the one-dimensional, incompressible, steady and uniform flow which is usually called normal flow. Although various equations such as the Darcy–Weisbach, Chezy and Manning equations can be used to model this kind of flow, present design practice in Iran and most countries tend to use the Manning equation. The equation in the SI system of units is given as follows:

$$V = \frac{1}{n} S_0^{1/2} R^{2/3} \tag{7}$$

where V is the mean velocity of flow (m/s), R is the hydraulic radius (m), S_0 is the longitudinal slope of the pipe, and n is the Manning roughness coefficient. In terms of a general discharge:

$$Q = \frac{1}{n} S_0^{1/2} R^{2/3} A \tag{8}$$

where A is the cross-sectional area and Q can be any flow rate such as those expressed by Equations (1)–(4). With reference to Figure 2, the hydraulic radius R, which is the ratio of the cross-sectional area to the wetted perimeter of the section, can be expressed as follows:

$$R = \frac{D}{4}\left(1 - \frac{\sin\theta}{\theta}\right) \tag{9}$$

where θ, in radians, is the central angle shown in the figure. Another important geometric relationship is the relationship between θ and the depth of flow (y):

$$y = \frac{D}{2}\left(1 - \cos\frac{\theta}{2}\right) \tag{10}$$

In this study, the Manning roughness coefficient is the only parameter which was considered to be subject to uncertainty in the hydraulic model. The pipe diameters and longitudinal slope were assumed to be crisp values with relatively negligible uncertainty.

Hydraulic performance assessment system and curves

Flow velocity and depth in the pipelines are two decision variables or performance indicators considered in this study in order to conduct the hydraulic performance analysis under different flow rates. The performance assessment system is similar to that proposed by Cardoso *et al*. (2005), previously discussed in the background section. A grading system was established, whereby 1 indicates optimum service, 0.75 indicates good performance, and 0.5 indicates an acceptable level of performance. Values below 0.5 indicate the performance is unacceptable, with 0.25 representing poor performance and 0.0 denoting a service failure. Figure 3 shows the performance function or curves used in this study to estimate the performance values for a separate domestic sewer system under different flow rates at specific years of operation during the design period.

The velocity curve in Figure 3 is justified based on the fact that low flow velocities in the pipe can cause sedimentation and solid deposition and also intensify the production of hydrogen sulfide by increasing travel time. These phenomena could happen under a flow velocity of 0.3 m/s. A flow velocity of 0.9 m/s is considered optimum since it can keep solids and sand suspended. A high flow velocity can have a damaging effect on the pipes depending on the abrasive characteristics of the wastewater, flow characteristics and pipe material. To be on the safe side, we set an overall flow velocity greater than 3.6 m/s as representing no service.

The relative depth curve in Figure 3 presents the performance values against the dimensionless relative depth. The following discussion explains the rationale behind using such a curve. Generally, it is not desirable to design sewer networks for full flow even at peak rates. Considerations such as the conveyance of the pipe, aeration in the pipeline and ease of connections during system construction are usually considered when prescribing the maximum allowable relative depth. Relative depths above 0.9–0.95 are considered unstable and may result in a sudden loss of carrying capacity with a surcharge at the manholes. A relative depth of 0.5–0.8 represents a full service in Figure 3. According to ISDWCS, a relative depth less than 0.1 can cause solid deposition and clogging in the system. Therefore, the performance is merely acceptable at a relative depth of 0.1 and increases to 1 at a relative depth of 0.5.

The performance can be estimated for each pipe for different flow rates of interest. For comparing performance of different networks or different parts of a network, a network-wide performance value can be developed by using a generalization function. Equation (11) is the generalization function used in this study. The generalizing function is a weighted average, using $k \cdot Qf \cdot L$ as the

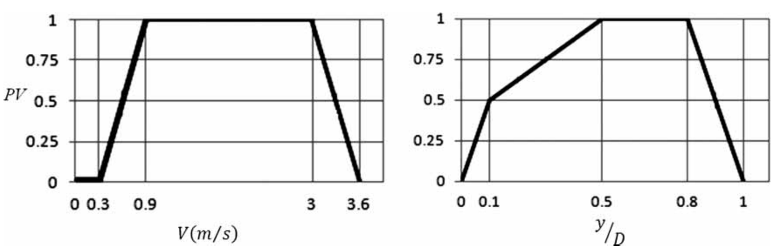

Figure 3 | Performance value vs. flow velocity and relative depth.

weight, where Qf and L are the full discharge capacity and length of the pipe, respectively, and k is a risk coefficient. If all sections are equally vulnerable, they are all assigned a value of $k = 1$.

$$\mathrm{PV_t} = \frac{\sum_1^N k_i Qf_i L_i \mathrm{PV}_i}{\sum_1^N k_i Qf_i L_i} \tag{11}$$

where N is the number of pipes in the network, PV_i is the performance value of each pipe, and $\mathrm{PV_t}$ is the total or overall performance of the network.

Fuzzy logic and fuzzy input parameters

The parameters involved in the design and hydraulic performance analysis of separate domestic sewer systems, within the context of this paper, were introduced in the design parameters, discharges and uncertainties section. The methods for estimating these parameters and the ranges for selecting them recommended by the design standards were also briefly introduced. As discussed, from the design and analysis point of view, it is hard or even impossible to accurately quantify these parameters. Therefore, uncertainty is an inherent ingredient of the involved parameters. In this study, fuzzy approach was adopted to quantify the uncertainties.

The fundamental definitions related to fuzzy set theory and fuzzy numbers can be found in many excellent references (Kaufmann & Gupta 1991; Zimmermann 1991; Hanss 2005). Let X be a set (universe). A is called a fuzzy set that belongs to X if A is a set of ordered pairs, $A = \{[x, \mu_A(x)], x \in X, \mu_A(x) \in [0, 1]\}$ where $\mu_A(x)$ is the grade of membership of x in A. The closer $\mu_A(x)$ is to one, the more x belongs to A; the closer it is to zero, the less it belongs to A. The α-level cut (set) of a fuzzy set A is the set of those elements which have the least α membership. In engineering practice, fuzzy sets are often understood as fuzzy numbers. A fuzzy set A is called a fuzzy number if A is a normal, convex fuzzy subset of the set of real numbers (Reveli & Ridolfi 2002). Figure 4 shows a triangular fuzzy number, typically used in the uncertainty analysis of engineering problems. A triangular fuzzy number can be expressed in the form of a triplet as $A = [a, b, c]$, where a and c are the lower and upper bounds of the triangle at $\alpha = 0$, respectively, and b is the value that corresponds to $\alpha = 1$. The $(c - a)$ value is called the support of A at $\alpha = 0$. The wider the support of the membership function, the higher the uncertainty.

In this paper, the alpha-cut technique is used to conduct the uncertainty analysis in order to quantify how the uncertainty in model parameters is propagated to the model outputs. The method is based on the extension principle, which implies that functional relationships can be extended to involve fuzzy arguments and can be used to map the dependent variable as a fuzzy set (Abebe *et al.* 2000). In simple arithmetic operations, this principle can be used analytically. However, in most practical modeling applications, the functional relationships involve complex structures that make analytical application of the principle difficult. Therefore, interval arithmetic is used to carry out the analysis. The so-called DSW algorithm (Ross 2004) is one method that can be used for continuous valued functions and mappings. This algorithm uses full α-level cuts in standard interval analysis and consists of four steps: (1) select an α value where $0 \leq \alpha \leq 1$, (2) find the interval(s) in the input membership function(s) that correspond to this α, (3) using standard binary interval operations, compute the interval for the output membership function for the selected α-level cut, and (4) repeat steps 1–3 for different values of α to complete the α-level representation of the solution (Ross 2004). This information is then directly used to construct the corresponding fuzziness (membership function) of the output which is used as a measure of uncertainty. If the output is monotonic with respect to the fuzzy

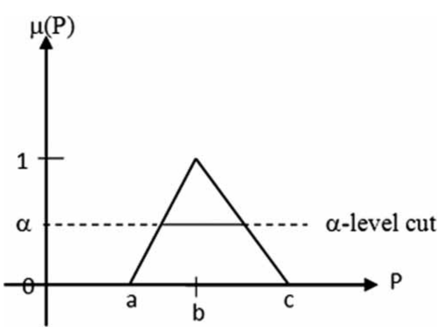

Figure 4 | Triangular fuzzy number.

parameter/s, the process is rather simple since only two simulations will be enough for each α-level (one for each boundary). Otherwise, optimization routines have to be carried out to determine the minimum and maximum values of the output for each α-level. Constrained nonlinear optimization algorithms have been used in this regard (Dou *et al.* 1995; Schulz & Huwe 1997; Reveli & Ridolfi 2002).

In this paper, the uncertainty analysis was conducted for the purpose of hydraulic performance analysis of separate domestic sewer systems. The following procedure was used to achieve this:

1. It was assumed that the basic parameters P, W and the Manning roughness coefficient are symmetric triangular fuzzy numbers. The construction of such fuzzy numbers begins by assigning a degree of possibility of $\alpha = 1$ to the most possible or recommended crisp values for the parameters. The supports of the fuzzy input parameters at $\alpha = 0$ were then selected based on the ranges recommended by the design standards, any available data, and expert knowledge.
2. The alpha-cut technique was used to find the membership functions for the different flow rates, Q_{ave}, $Q_{\max}$, $Q_{\min}$, and Q_{sc}, defined by Equations (1)–(4), at any particular year during the design period. In doing so, the dependency of $k_{\max}$ and $k_{\min}$ on P was considered. Although Q_{ave} is a triangular fuzzy number, the other calculated membership functions are not necessarily linear, considering the nonlinear structure of Equations (5) and (6).
3. In order to find the fuzzy membership functions for velocity and relative depth, the discharge fuzzy numbers were then used as input to the hydraulic models, Equations (7) and (8). This issue and the methodology for finding the performance values for each pipe are discussed in the following section.

Fuzzy formulation of hydraulic model and method for determination of performance values

The Manning equation (Equation (8)) is the hydraulic model used in this study. Equation (12) is the fuzzy expression of Manning equation for a circular section where it is assumed that the pipe diameters and longitudinal slope are specified crisp values and the Manning roughness coefficient and the discharge are fuzzy numbers.

$$\tilde{Q} = \frac{1}{\tilde{n}} S_0^{1/2} \left[\frac{D}{4}\left(1 - \frac{\sin\tilde{\theta}}{\tilde{\theta}}\right)\right]^{2/3} \left[\frac{D^2}{8}(\tilde{\theta} - \sin\tilde{\theta})\right] \tag{12}$$

where $\tilde{Q}$ and $\tilde{n}$ are the fuzzy discharge and fuzzy Manning roughness coefficient, respectively and $\tilde{\theta}$ is the output. The membership function of $\tilde{\theta}$ can be calculated using the alpha-cut technique, described in the fuzzy logic and fuzzy input parameters section. Briefly, this approach consists of writing several systems of interval equations corresponding to α-level cuts of fuzzy input variables and transforming the interval algebraic system into a problem of nonlinear constrained optimization in order to search for the corresponding maximum and minimum values of $\tilde{\theta}$.

The optimization problem searches for the maximum and minimum values of θ for any α-level cut and takes on the simple form of:

$$\text{Min} \quad \text{or} \quad \text{Max} \quad \theta_{\alpha=\alpha^*} \alpha \in [0,1] \tag{13}$$

Subject to the constraints:

$$\frac{Qn}{S_0^{1/2}\left(\frac{D}{4}\right)^{2/3}\left(\frac{D^2}{8}\right)} - \left(1 - \frac{\sin\theta_{\alpha=\alpha^*}}{\theta_{\alpha=\alpha^*}}\right)^{2/3} \times (\theta_{\alpha=\alpha^*} - \sin\theta_{\alpha=\alpha^*}) = 0 \tag{14}$$

$$Q_{\underset{\min}{\alpha=\alpha^*}} \le Q \le Q_{\underset{\max}{\alpha=\alpha^*}} \tag{15}$$

$$n_{\underset{\min}{\alpha=\alpha^*}} \le n \le n_{\underset{\max}{\alpha=\alpha^*}} \tag{16}$$

A mathematical analysis of Equation (14) shows that maximum θ corresponds to maximum Q and n values. Similarly, minimum θ relates to minimum Q and n values. These concepts are also in agreement with the physics of normal flow. Design charts available in hydraulic textbooks and a direct search method near the optimum points were used to find the maximum and minimum values of θ.

Knowing the interval $[\theta_{\min}, \theta_{\max}]_{\alpha=\alpha^*}$, the corresponding lower and upper bound values for flow velocity and depth can be calculated using the following equations:

$$[V_{\min}, V_{\max}]_{\alpha=\alpha^*} = \frac{1}{[n_{\max}, n_{\min}]_{\alpha=\alpha^*}} S_0^{1/2} \times \left[\frac{D}{4}\left(1 - \frac{\sin[\theta_{\min}, \theta_{\max}]_{\alpha=\alpha^*}}{[\theta_{\min}, \theta_{\max}]_{\alpha=\alpha^*}}\right)\right]^{2/3} \quad (17)$$

$$[y_{\min}, y_{\max}]_{\alpha=\alpha^*} = \frac{D}{2}\left(1 - \cos\left[\frac{\theta_{\min}}{2}, \frac{\theta_{\max}}{2}\right]_{\alpha=\alpha^*}\right) \quad (18)$$

Such information can be used to find the fuzzy membership functions of the flow velocity and depth for any particular discharge defined by Equations (2)–(4). This information can be used for two purposes: the uncertainty analysis and the hydraulic performance analysis. Hydraulic performance analysis was the main objective of this study.

The developed fuzzy membership functions for flow velocity and depth can be used to conduct the hydraulic performance analysis when they are combined with performance functions such as those proposed in Figure 3. One way of doing this is to defuzzify the fuzzy numbers first, and then use the calculated crisp numbers to find the performance values. The method used in this study is different and is explained here for finding the velocity performance value of a pipe under a particular discharge. The method is applicable to flow depth in a similar way. First, for any velocity fuzzy number, the upper and lower bounds of the velocity at different α-level cuts of 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 and 1 were found. Then, the corresponding performance values for these 19 velocity values were extracted from the velocity performance curve shown in Figure 3. Finally, the following averaging equation was used to find the average velocity performance value of the pipe.

$$\mathrm{PV} = \frac{\sum_1^{19} \alpha_j \mathrm{PV}_j}{10} \quad (19)$$

The difference between the two methods depends on the shape of the performance curves and fuzzy numbers. However, the second method involves a wider spectrum of the parameters' space and their interaction with the performance curves in the analysis.

APPLICATION, RESULTS AND DISCUSSION

A hydraulic performance analysis using the suggested procedure was applied to a separate domestic sewer system. Figure 5 shows the selected system which is part of the separate wastewater collection system of Amole, a city in the northern part of Iran. The system consists of 31 manholes and 30 polyethylene pipes of different sizes ranging from 200 to 400 mm. Table 1 refers to the different dimensions of the system as they were originally identified in the design phase of the project. The selected design period of the network is 25 years which begins in 2010 and ends in 2035.

Table 2 shows the characteristics of the triangular fuzzy numbers selected for different design and analysis parameters, at the beginning and end of the design period. The criteria for selecting the fuzzy numbers were discussed in the design parameters, discharges and uncertainties section.

An organized presentation of the potential model outputs and their application in a hydraulic performance analysis of sewer systems is now demonstrated.

1. Figure 6 shows how the uncertainty in basic parameters propagates to $Q_{\max}$ (in 2035), $Q_{\min}$ and Q_{sc} (in 2010) with the corresponding velocity and relative depth values for pipe 3, as an example. As the figure shows, an approximate triangular fuzzy number is obtained for these variables despite the nonlinearity in the equations.

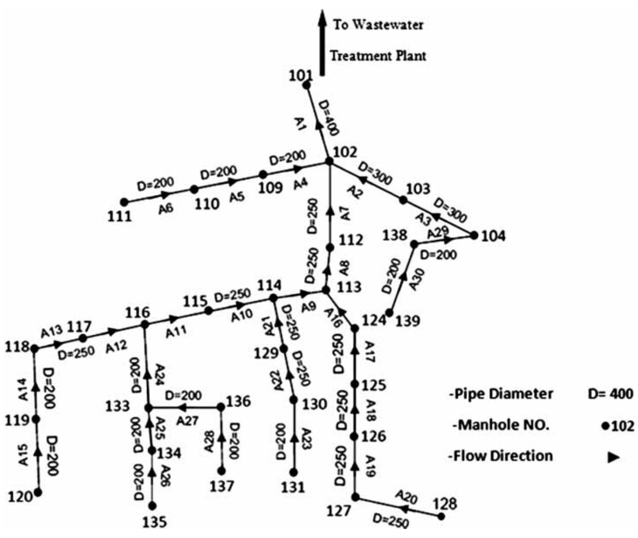

Figure 5 | Plan view of sewer system (not to scale).

Table 1 | Network characteristics and design dimensions

Pipe name (number)	Length (m)	Diameter (m)	Slope (m/m)	Area serviced (ha)
A1	60	0.40	0.0025	92.588
A2	76	0.30	0.005	32.879
A3	54	0.30	0.005	32.191
A4	50	0.20	0.035	1.791
A5	50	0.20	0.035	1.194
A6	50	0.20	0.008	0.597
A7	53	0.25	0.035	57.210
A8	32	0.25	0.035	54.640
A9	46	0.25	0.008	38.717
A10	59	0.25	0.008	18.057
A11	51	0.25	0.008	17.500
A12	48	0.25	0.004	14.174
A13	44	0.25	0.004	13.624
A14	52	0.20	0.005	13.207
A15	51	0.20	0.02	6.604
A16	45	0.25	0.008	15.563
A17	46	0.25	0.008	15.137
A18	46	0.25	0.008	14.666
A19	47	0.25	0.018	14.194
A20	71	0.25	0.004	13.710
A21	45	0.25	0.008	20.183
A22	44	0.25	0.008	19.732
A23	50	0.20	0.008	19.282
A24	59	0.20	0.005	2.766
A25	33	0.20	0.008	0.875
A26	44	0.20	0.008	0.444
A27	54	0.20	0.005	1.192
A28	50	0.20	0.005	0.571
A29	48	0.20	0.005	1.166
A30	52	0.20	0.055	0.656

2. Figures 7 and 8 show the variation of the self-cleaning velocity and corresponding relative depth during the design period for pipe 3. The figures show how these variables and their associated uncertainties increase over time due to the increase in both value and uncertainty in fundamental parameters. As discussed, these variables play a significant role in the hydraulic performance analysis of sewer systems.

3. Figures 9–12 are very important figures that demonstrate the performance of each pipe in terms of the self-cleaning velocity and relative depth at the beginning and end of the design period within the range of considered uncertainties and selected possibility degrees. Such figures which can be developed for any degree of possibility of input parameters are the most significant outputs of the model for performance analysis. Figure 9 shows that there is little possibility for pipes 6, 12–14, 20, and 24–29 to perform acceptably in terms of the self-cleaning velocity in the early stages of the design period (2010). Similarly, Figure 10 demonstrates a poor performance for pipes 4–6 and 25–30 in terms of the relative depth under Q_{sc} in the early stages. Also, Figure 10 shows a narrow band for performance in terms of relative depth. Observations shown in Figures 9 and 10 indicate a definite need for manual cleaning in pipes 6, 12–14, and 24–29 and a possible need for manual cleaning of pipes 4, 5, and 30 at the beginning of the design period. Improper selection of pipe sizes and dimensions have caused this problem. The correct adjustment of the slope and diameters of the pipes is a challenging issue for design engineers because in addition to other design requirements: (1) they are forced by standards to use a minimum diameter of 200 mm to control clogging and (2) they are limited to use pipe sizes which are available on the market. Pipes which have poor performance in terms of both criteria are in the far upstream locations of the sewer system with a low discharge and a minimum designed diameter of 200 mm.
   (a) Comparison between Figures 9–12 shows that the performance of the pipes improves over time due to increase in the discharge values. Despite the improvement in performance, Figures 9 and 11 show that pipes 6 and 24–29 have less possibility of performing well in terms of self-cleaning velocity up to the end of the design period. As Figure 5 and Table 1 show, pipes 24–29 are located in a particular zone of the sewer system which has a low slope and a small coverage area. Figures 10 and 12 show that pipe 30 performs poorly in terms of relative depth up to the end of the design period. It is interesting that although pipe 30 performs well in terms of self-cleaning velocity, it performs poorly in terms of relative depth.

Table 2 | Triangular fuzzy numbers for different design parameters

	Beginning of the design period			End of the design period		
Parameter	***a***	***b***	***c***	***a***	***b***	***c***
P[a]	2,917	3,241	3,565	8,796	9,259	9,722
W (LPCD)[b]	62.22	79.87	97.52	65.76	116.48	167.20
Infiltration for Q_{max}	10% of Q_{ave}[c]	15% of Q_{ave}	20% of Q_{ave}	10% of Q_{ave}	15% of Q_{ave}	20% of Q_{ave}
Infiltration for Q_{min} and Q_{sc}	−5% of Q_{ave}	0	+5% of Q_{ave}	−5% of Q_{ave}	0	+5% of Q_{ave}
Inflow	10% of Q_{ave}	15% of Q_{ave}	20% of Q_{ave}	10% of Q_{ave}	15% of Q_{ave}	20% of Q_{ave}
n	0.011	0.013	0.015	0.011	0.013	0.015

[a] *P* in the table refers to the total population serviced by pipe A1 at the end point of the network.
[b] *LPCD* stands for liters per capita per day.
[c] Q_{ave} is defined by Equation (1) at any particular year during the design period and specified possibility degree.

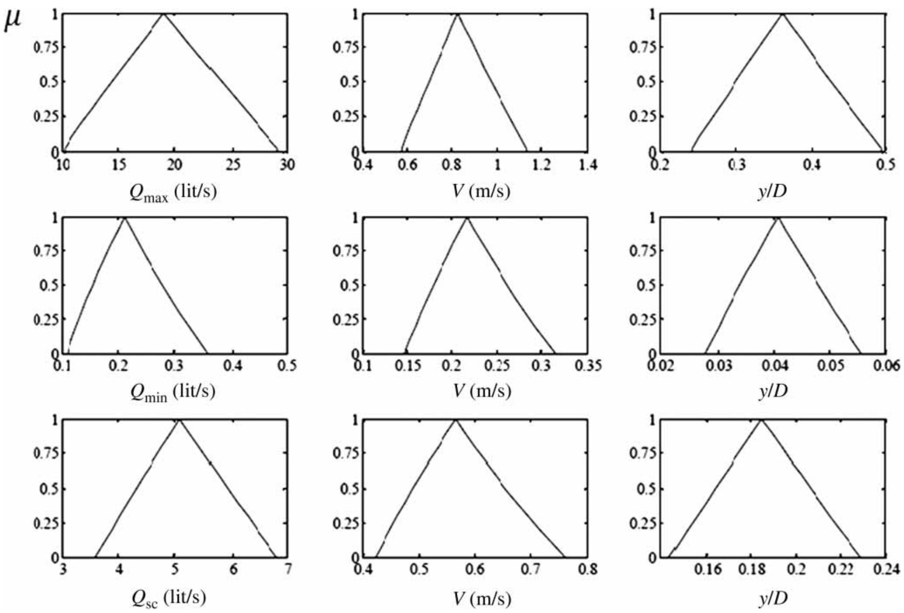

Figure 6 | Q_{max} (in 2035), Q_{min} (in 2010), and Q_{sc} (in 2010) and their corresponding velocity and relative depth fuzzy numbers for pipe 3.

This is due to the high longitudinal slope of this pipe that adjusts the velocity but not the required relative depth. An overall result of these interpretations is that the designer either can justify and accept the designed dimensions for pipes 6 and 24–30 together with a manual cleaning of the pipes during the design period, or revise the dimensions. Similar information and figures with different degrees of possibility of fundamental design parameters, resulting from updated data or expert knowledge, can be produced, providing the analyst with a plausible performance analysis under uncertainties.

4. Figures 13 and 14 present the performance of the pipes in terms of velocity and relative depth under maximum discharge (Q_{max} in 2035), respectively. These figures are not significantly different from Figures 11 and 12 because according to Equations (2) and (4), Q_{max} differs from Q_{sc} only in including infiltration and inflow which, as the results show, does not have a significant effect on performance values under the imposed uncertainties. Figures 15 and 16 are similar to Figures 13 and 14 but present performance values for α-levels of 0.5 and 1. As expected, a narrower band for performance values is obtained in this case, which is a result of considering a higher possibility

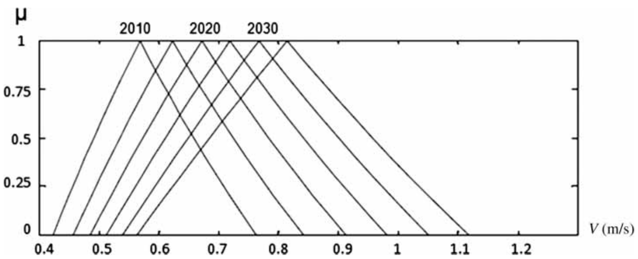

Figure 7 | Self-cleaning velocity fuzzy numbers for pipe 3 at 5-year intervals during design period.

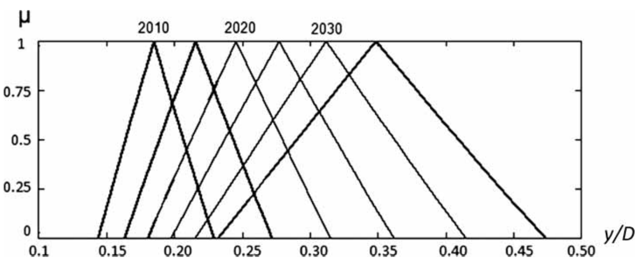

Figure 8 | Relative depth fuzzy number under self-cleaning discharge (Q_{sc}) for pipe 3 at 5-year intervals during design period.

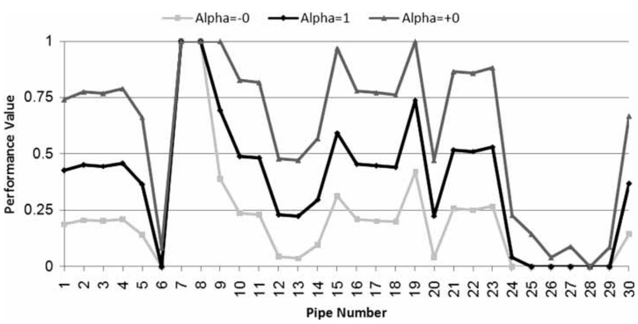

Figure 9 | Performance value of different pipes in terms of self-cleaning velocity at beginning of design period (2010); α-levels of 1 and 0 (− and + refer to lower and upper bounds, respectively).

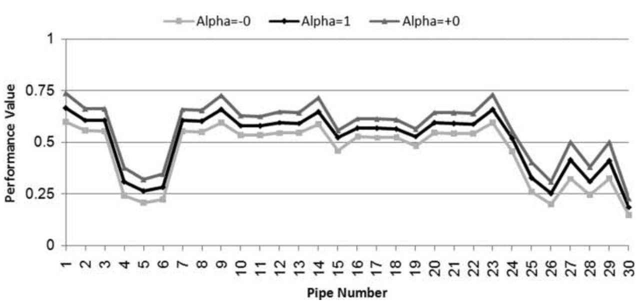

Figure 10 | Performance value of different pipes in terms of relative depth under self-cleaning discharge (Q_{sc}) at beginning of design period (2010); α-levels of 1 and 0 (− and + refer to lower and upper bounds, respectively).

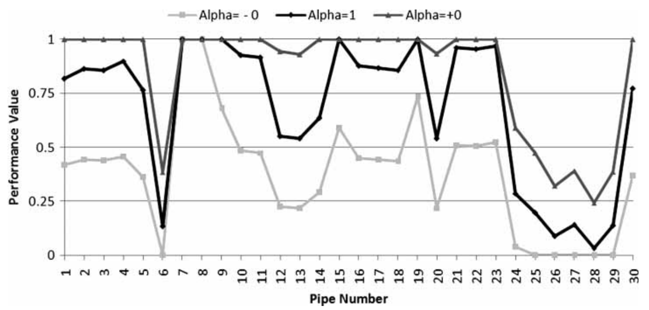

Figure 11 | Performance value of different pipes in terms of self-cleaning velocity at end of design period (2035); α-levels of 1 and 0 (− and + refer to lower and upper bounds, respectively).

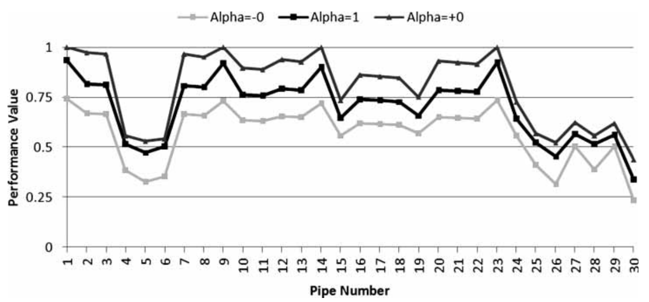

Figure 12 | Performance value of different pipes in terms of relative depth under self-cleaning discharge (Q_{sc}) at end of design period (2035); α-levels of 1 and 0 (− and + refer to lower and upper bounds, respectively).

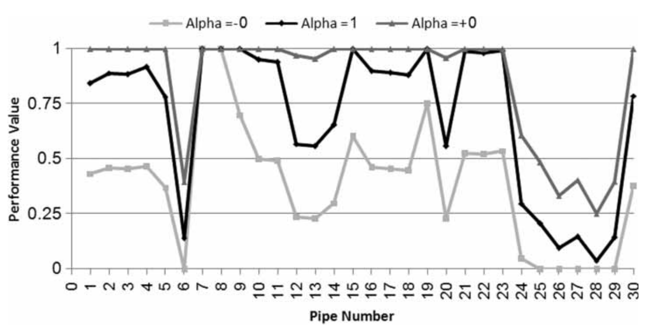

Figure 13 | Performance value of different pipes in terms of velocity under maximum discharge (Q_{max} in 2035); α-levels of 1 and 0 (− and + refer to lower and upper bounds, respectively).

for the input parameters, which consequently results in a smaller interval and a more reliable interpretation of performance values. As an example, in this case the performance value in terms of velocity for pipe 13 is between 0.388 and 0.744 with the most possible value of 0.557, while in Figure 13 this performance value has a wider range between 0.229 and 0.956.

5. Figure 17 presents the total or overall performance value of the network based on Equation (11) in terms

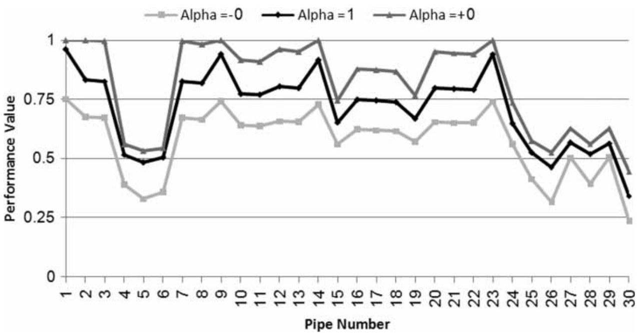

Figure 14 | Performance value of different pipes in terms of relative depth under maximum discharge (Q_{max} in 2035); α-levels of 1 and 0 ($-$ and $+$ refer to lower and upper bounds, respectively).

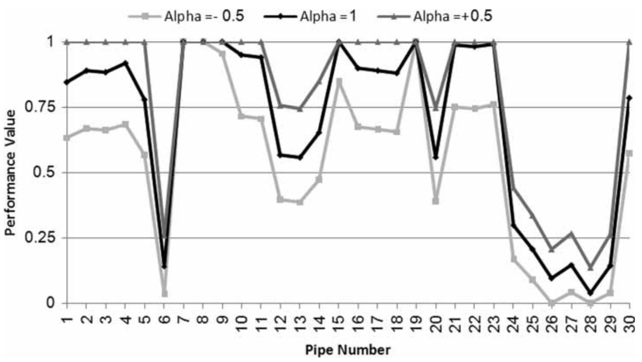

Figure 15 | Performance value of different pipes in terms of velocity under maximum discharge (Q_{max} in 2035); α-levels of 1 and 0.5 ($-$ and $+$ refer to lower and upper bounds, respectively).

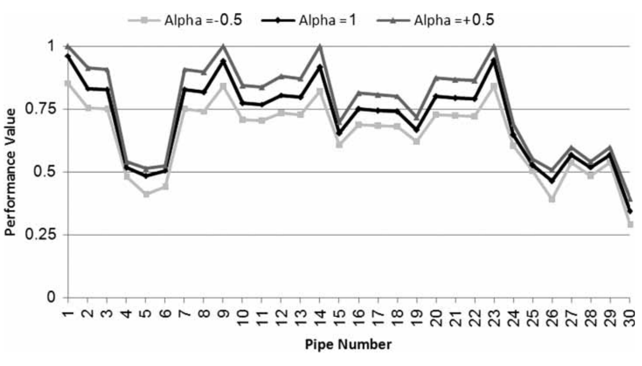

Figure 16 | Performance value of different pipes in terms of relative depth under maximum discharge (Q_{max} in 2035); α-levels of 1 and 0.5 ($-$ and $+$ refer to lower and upper bounds, respectively).

of self-cleaning velocity and relative depth every 5 years during the design period. As the figure shows, the global performance values are not significantly different but the performance in terms of velocity improves at a higher rate. Such figures can provide decision makers

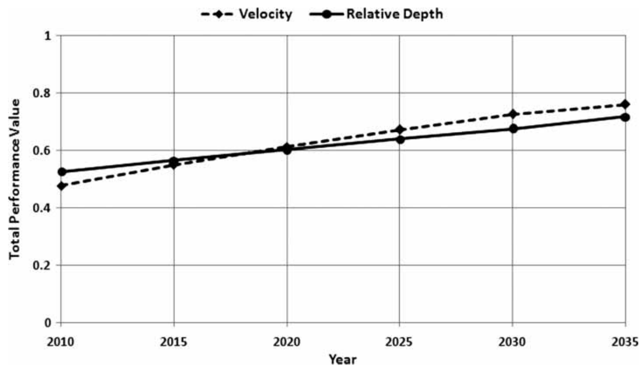

Figure 17 | Total performance values of sewer system in terms of self-cleaning velocity and corresponding relative depth at five-year intervals during design period.

with additional information for comparing the performance of different systems or different parts of an existing system for identifying priorities in the development of rehabilitation strategies.

CONCLUSIONS

In this research, a fuzzy model was developed for the hydraulic performance analysis of separate domestic sewer systems under uncertain parameters. The model was then applied to a case study to demonstrate its applicability. The results were shown in the form of element and overall system performance graphs for different hydraulic variables of interest, such as velocity and relative depth. The variation of performance values over time was highlighted. The following conclusions were drawn using the proposed model:

1. It is flexible and can include information available from different sources for quantifying the uncertainty in the basic analysis parameters.
2. It is conceptually understandable and computationally simple.
3. It can be used to calculate the performance value of each pipe in terms of any hydraulic variable of interest and for any possibility degree of significant parameters. For each pipe, an average performance value can also be calculated, which includes a wide spectrum of significant parameters in the performance analysis of the pipe. Although this kind of calculation can be computationally time-consuming, it can represent a more accurate

interaction of the uncertain variables with the performance assessment curves, especially in the rapidly varying zones of the curves.

Design engineers, operation managers and decision makers are among those who can use the results of the proposed model to improve the system performance. Design engineers can use the results for improving the quality of the design in the early stages. Operation managers can use the results for employing effective operation programs for an operational sewer. The results can also be useful for decision makers to identify priorities in the development of rehabilitation strategies.

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First received 15 October 2010; accepted in revised form 5 September 2011. Available online 29 November 2011