

**Finite Elements, An Introduction.** By E. B. Becker, G. F. Carey, and J. T. Oden. Prentice-Hall, Englewood Cliffs, N.J., 1981. 258 Pages. Price /24.95.

**REVIEWED BY T. BELYTSCHKO<sup>1</sup>**

The finite element method has been the topic of approximately 30 books and monographs that have been published over the last 15 years. Nevertheless, many instructors still have difficulties in finding a text for advanced undergraduate or beginning graduate courses which will develop a sound, fundamental understanding of the method. This book presents a significant advance in that direction for those who wish a more rigorous, mathematical development.

The book consists of six chapters. The first two are devoted to one-dimensional problems, emphasizing the development of a symmetric variational formulation for second-order, two-point boundary value problems and the smoothness required in the space of approximating functions. In the third chapter, a finite element program for one-dimensional problems is described, including the FORTRAN statements. Chapters 4 and 5 repeat the same material for two-dimensional problems, including shape functions for triangles and quadrilaterals and numerical quadrature. Chapter 6 presents an introduction to three-dimensional problems, fourth-order problems, and time-dependent problems.

A notable feature of this book is that it develops the weak, or variational form, from the partial differential equations, rather than simply presenting the variational form as given; the latter approach bothers many of the better students who usually wonder where the variational form comes from. The concepts in this book are all developed with rigor, clarity, and conciseness. Once a student has mastered this book, he will certainly have a broader understanding of the mathematics of the finite element method than would be obtained from more conventional treatments.

In using this book in my class, I found two types of response. Engineering students with a modest mathematical background found the book a little difficult as an introduction; it requires simultaneously tackling the concepts of the weak form, finite element approximations, and notation and concepts to which they are unaccustomed. On the other hand, mathematically inclined students tend to find this book delightful. In addition to its value as a text, it is also recommended to finite element specialists who wish to familiarize themselves with the more recent developments in the mathematical aspects of the method. Even recently I have received papers submitted to the ASME JOURNAL OF APPLIED MECHANICS that deal with the continuity requirements and natural boundary conditions in the Galerkin method; this book presents an unambiguous, consistent development at an introductory level.

<sup>1</sup>Professor, Department of Civil Engineering, The Technological Institute, Northwestern University, Evanston, Ill. 60201.

This book is the first volume of a series of six on finite elements. If the quality of this volume is maintained in the forthcoming volumes, it should prove a valuable contribution to the finite element literature.

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**Seismic Migration—Imaging of Acoustic Energy by Wave Field Extrapolation.** By A. J. Berkhout. Elsevier, Amsterdam, 1980. pp. xii-339. Price \$51.00.

**REVIEWED BY Y.-H. PAO<sup>2</sup>**

The purpose of this review is not to criticize Berkhout's book. Instead, it is intended to acquaint readers of the JOURNAL OF APPLIED MECHANICS with this seemingly mysterious topic.

Seismic migration is the construction of a vertical cross section of the ground from the time traces of signals recorded along a line of receivers. The signals are generated by either a single source, or a distribution of sources along the line of the receivers. Mathematically, the problem is formulated as the determination of the wave speed  $c(x, y, z)$  and mass density  $\rho(x, y, z)$  of an inhomogeneous half-space  $z \geq 0$ ,  $-\infty < x, y < \infty$ , from the known input at the surface,  $P_0(x_0, y_0, 0, t)$ , and the output  $P(x, y, z, t)$ . The  $P(x, y, z, t)$  satisfies a linear wave equation with a variable coefficient  $c^2 \nabla^2 P = \partial^2 P / \partial t^2$ .

The complexity of the problem apparently is far beyond the mathematical and computational tools currently available. In fact, this mathematical inverse problem may be ill-posed, for which the solutions are not stable, nonunique, or even nonexistent. Nevertheless, oil companies have to find oil, and do find them underground by seismic prospecting. Geophysicists specialized in this area have developed various approximate methods to map geological cross sections from records of map-generated seismic waves. The Migration is one of these methods.

A crude model for the cross section is a half space composed of many parallel layers, each having a constant wave speed  $c(z_i)$ , and density  $\rho(z_i)$ . A more refined model is to have nonparallel layers, and to allow  $c$  and  $\rho$  to vary laterally in  $x, y$  directions. Methods of seismic migration are developed to improve the lateral resolution of the data gathering and processing.

In this book, which is the first one devoted to the topic of seismic migration, the theory of migration is derived from first principles. Therefore, it contains some basic mathematics (Chapters 2-4) which are familiar to readers of the JOURNAL

<sup>2</sup>Professor, Department of Theoretical and Applied Mechanics, Cornell University, Ithaca, N.Y. 14853.