Computation of Rigid-Body Rotation in Three-Dimensional Space From Body-Fixed Linear Acceleration Measurements

L. E. Goodman and A. R. Robinson. The subject paper by Mital and King represents a valuable contribution to the practical problem of determining rotation histories from dynamical measurements. The main algorithm, the experimental validation of the computation and the discussion of error minimization are all interesting and useful. In describing the basis of the computation procedure, however, the authors are guilty of a slight verbal inaccuracy that may mislead readers unfamiliar with the literature of the subject. The statements in question occur in the first paragraph of the section entitled "Computation of Rotation From Accelerometer Data." The following discussion is offered in an effort to clarify matters.

The computational method presented in the subject paper is based on what the authors term the "Goodman-Robinson" theorem (authors' reference [8]). The correct bibliographic reference to this theorem is given below [8]. Briefly, the theorem concerns a rigid body constrained to turn about a fixed point, 0, which is also the origin of a set of body-fixed axes, x, y, z (not necessarily orthogonal). Consider a fixed sphere of unit radius centered on 0. As the body moves, the intersection with the sphere of any one of the body-fixed axes, say x, describes a curve on the sphere. From any starting point at t = 0 the body will, at time t, have reached a new position that could have been reached by a single rotation Φ. The Goodman-Robinson theorem asserts that the component of Φ about the x-axis, φx, is given by the expression (equation (4) in [8]):

\[ \phi_x = \int_0^t \omega_x(t) \, dt + A_x \pm 2m\pi \]

Here \( \omega_x(t) \) is the x-component of the angular velocity; \( A_x \) is the area on the unit sphere bounded by the curve traced by the intersection of the x-axis with the sphere plus a closure arc corresponding to the negative of Φ; and m is an integer or zero. Similar expressions hold for \( \phi_y \) and \( \phi_z \). The theorem is exact. No "correction term" has been omitted. In fact, no additional term is possible.

In technological applications of the Goodman-Robinson theorem it is often desirable to express \( A_x \) in terms of other geometric parameters that are of interest. After it is proven in [8], the theorem is first illustrated by a simple example in which the area \( A_x \) is computed exactly. Then the theorem is applied to solve what Dr. Bortz in the authors' reference [6] terms "the famous coning problem." In this application the area \( A_x \) is small compared with 4\pi. In this circumstance it is permissible to replace \( A_x \) by the corresponding area on the tangent plane—what cartographers would call the central or gonmonic projection of \( A_x \). In the technological problem to which Dr. Bortz applies the Goodman-Robinson theorem, on the other hand, \( A_x \) is not necessarily small and he, quite correctly, employs an exact expression for \( A_x \). So also in the authors' paper under discussion. This expression for \( A_x \) entails adding a term to the area of the central projection and it is this addition that the authors appear to have in mind when they write of a correction term. This term, however, has nothing to do with the theorem itself. That theorem is completely and correctly stated and proved in [8].

Authors' Closure

The authors appreciate the comments made by Professors Goodman and Robinson relative to the usefulness of the method proposed in the paper. We agree that the Goodman-Robinson theorem does contain a correction term \( A_x \) and that the word "omit" in our paper was a poor choice. What we meant to say was that we needed an exact expression which was not available in reference [8]. It should be noted that Bortz [6] derived the expression for the orientation vector independently of the Goodman-Robinson theorem and that his expression was more suitable and accurate for the computation of rigid-body rotation from linear accelerometer measurements. We apologize for the error in reference [8] which was rectified by the discussants.

Stability of a Rotor Partially Filled With a Viscous Incompressible Fluid

F. G. Kollmann. This discusser would like to congratulate the authors on their very valuable contribution to this interesting field and also would like to draw their attention to the following:

1. The first comprehensive analytical solution for the inviscid case...
was given by Kuipers [1] where he compared his theoretically predicted stability charts [2] with the discusser's experimental results and excellent agreement was found.

2 The authors' remark, that the discusser attempted no analysis is a little bit misleading. The discusser used a lumped mass model for the trapped liquid and could predict the independence of the critical spin frequency from the mass of the contained fluid but by the discusser's model, he was not able to analyze the interaction of the motions of the rotor and the fluid waves.

3 The discusser is interested to learn, whether the authors have performed experimental investigations to confirm their theoretical predictions.

References


Authors' Closure

The author is indebted to Dr. Kollmann for bringing the two references to his and the reader's attention. The article by Kuipers supports the author's assertion that both external rotor damping and damping due to the viscosity of the entrapped fluid must be considered in order to have a consistent theory.

The author apologizes for any misconceptions that may have arisen concerning Dr. Kollmann's article.

The authors have not performed any comprehensive experiments to date. A few unreported experiments have been conducted using a rotor which was clamped and free at the other. The experiments confirm the dramatic rise in the upper stability boundary when rotor damping was increased (Fig. 6). Since the experimental rotor allowed the cup to tilt (creating gyroscopic stiffening effects in the rotor and exciting axial dependence in the fluid waves), a direct comparison with the current theory was not attempted. A more comprehensive theory incorporating axial dependence in the fluid motion has now been completed and will be reported in another paper. In the meantime the author would encourage Dr. Kollmann or anyone else to undertake a comprehensive experimental investigation.

Dynamic Response of a Cylindrical Shell in a Potential Fluid

R. L. Citerley. The authors have touched upon several problem areas that can be encountered in the analysis of fluid-structure systems. Three points in particular should be addressed.

1 In solving a transient response problem for an incompressible fluid, the authors attributed the observed numerical instability to the addition of the fluid, arguing that the Houbolt difference operator is unconditionally stable. The proof of stability for this operator has been given only for symmetric systems [1]. Using a finite-difference formulation of the Sander shell equations, with displacements and moment results as fundamental variables, results in system equations which are nonsymmetric. Thus a formal proof of unconditional stability is lacking for the problem under consideration. The same basic shell equations, coupled with a compressible fluid, produced a similar instability phenomenon in a recent study [2]. The instability was removed by using a spatial Euler difference operator rather than central difference at the fluid-shell interface. It can be shown that the eigenvalues, ω, of the system [K][d] = ω[M][d] will provide the insight with respect to numerical stability. Using an incompressible fluid, either the "stiffness" matrix, K, or the "mass" matrix, M, can be modified to account for the fluid. These matrices are generally full and nonsymmetric for the particular geometries encountered in the nuclear energy field [3]. Further, the eigenvalue may even be complex, but will appear as conjugate pairs. Using a diagonal mass matrix to represent the fluid for all harmonic responses, although attractive for its economy, is only correct for rigid body motions and must be applied only to the mass terms corresponding to the normal displacement.

2 When representing a fully coupled fluid-structure system with an incompressible fluid, responses are instantaneous felt throughout the domain, and are solely determined by the accelerations of the wetted interface and imposed pressures. Therefore, when performing an eigenvalue analysis, the fundamental variables are those of the shell. With a compressible fluid, the pressures at points within the fluid must be added as fundamental variables in the vector [d], in the foregoing. Again, an eigenvalue analysis can be performed, but the physical interpretation of results becomes a little more involved. In either case, for containment shells some eigenvalues will have the same, or nearly the same values as predicted for the empty shell, but the corresponding eigenvectors will be considerably different. Whether or not these modes should still be classified as shell modes is simply a matter of semantics. As the fluid height approaches the shell height, the magnitude of the eigenvalues associated with shell responses will dramatically change under the incompressible assumption. Little reduction in the "shell natural frequency" is observed by the introduction of compressibility, but additional "acoustic modes" will now be present. The eigenvalues corresponding to these acoustic modes can greatly differ from the rigid wall modes. Of course, the acoustic modes are, by definition, affected by compressibility. For a narrow annular fluid configuration with a 12.91 m fluid height, several acoustic modes would exist within the frequency range of interest (<500 Hz); whereas, for a 0.7 m fluid height, the first acoustic mode is at 525 Hz.

Generally, when a fluid-structure system is excited by an imposed pressure or volume source within the fluid, and one is concerned with fluid responses (i.e., the acoustic modes), the dynamic characteristics of the fluid take on a greater importance than do the structure modes. Conversely, when the structure is excited by an externally applied surface or body force, and one is concerned about containment responses only, the dynamic characteristics of the structure begin to take precedence. If one can perform an eigenvalue analysis for a reasonable frequency bandwidth, then periodic, random or transient analyses become straightforward tasks. Wave propagation problems are more readily solved by direct time integration since they are equivalent to a modal response involving a very large number of contributing modes.

3 Experimental verification of a numerical procedure for fluid-structure analysis is not a trivial task. As noted by the authors, specific boundary conditions may be difficult to achieve. Numerical procedures are available for combining structural modes of a dry structure with either incompressible or compressible fluids [4]. In this way, measured data of nonideal boundary conditions of the dry structure can be directly coupled with a fluid. However, it has been this writer's experience that minute volumes of air, either entrapped in pockets or in the form of bubbles at the fluid-structure interface, have a far greater effect on acoustic modes than does structural compliance [5]. Only after careful treatment of the water, such as raising the fluid temperature to near boiling and the addition of wetting agents, can consistent results over a wide frequency band be obtained.