Quantifying the economy of flow distribution in water supply looped networks

Jose Bienvenido Martínez

ABSTRACT

Optimization of water supply looped networks has attracted a great deal of attention from researchers for more than 30 years. As the classical water supply looped network optimization problem is mathematically non-convex and multimodal, the resulting solution of most approaches is uncertain in the sense of how close it is to the ''best'' solution. In many cases, this ''best''' or ''global'' solution is invoked and pursued only intuitively without a clear understanding of its meaning. This paper discusses what is involved in ''global'' solutions and the role that pipe flow distribution can play to deal with non-convexity and multimodality in a new context. The author has introduced this new context recently after formulating a new objective function capable of finding a looped network that can be economically more attractive than its related branched one. Therefore, the convenience of an approach dealing with flows and heads, as relevant decision variables, is encouraged in this paper and its advantages enumerated under the new concepts. The entropy approach is studied critically and an example is provided for comparison with the proposed approach.

Key words | flow distribution, looped networks, network design, network reliability, optimization, water supply

Jose Bienvenido Martínez (corresponding author)
Centro de Investigaciones Hidráulicas (CIH),
Instituto Superior Politécnico ''José Antonio Echeverría'' (CUJAE),
Calle 114 No. 11901 e/119 y 127 CP -19390,
Marianao,
La Habana,
Cuba
E-mail: *bienvenido@cih.cujae.edu.cu*; *jbmr27@yahoo.es*

NOTATION

a	coefficient of pipe failure frequency formula
B	parameter in Equation (11)
C	constant defined in Equation (1)
C_{n1}, C_{n2}	number of pipes connected to nodes n_1 and n_2 of broken pipe
c_1	annualizing factor
c_a	average cost of supplying water to affected consumers in dollars per unit volume.
c_f	average cost of repair in dollars per day.
d_k	diameter of pipe k
e^{xs}	excess pressure in node s
fQ_1	fraction of flow Q_1 of Figure 1
hf	head loss in pipe
H	total head in source node
i	pipe initial node; node counter
j	pipe final node
K	parameter in Equation (7)
k	pipe counter
L_k	length of pipe k
m	exponent of pipe cost formula
NN	total number of nodes
NP	number of pipes
NS	number of source nodes
n	exponent of flow in friction formula
pm	required minimum pressure
Q_k	pipe flow
Q_{break}	flow to be supplied to affected consumers.
Q_{n1}, Q_{n2}	demand flow in nodes n_1 and n_2 of broken pipe.
q	exterior flow at node
r	exponent of diameter in friction formula

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s	source node counter
t_f	average number of days for repair of pipe failure.
u	exponent of pipe failure frequency formula
V_f	volume per day that must be supplied to affected consumers
z	ground elevation
β	parameter defined in Equation (10)
η	coefficient of pipe cost formula
λ	constant defined in Equation (4)
μ	parameter defined in Equation (6)
ω	parameter defined in Equation (8)
ψ	objective function
ζ	set of pipes connected to given node

INTRODUCTION

Most of the many different approaches that have been presented for the optimization of water supply looped networks use, as relevant decision variables, either flows and heads (or friction losses) or pipe diameters. As for mathematical techniques, linear (LP) and non-linear programming (NLP) can be found as well as evolutionary and integer programming algorithms. In many cases the optimization procedure involves two or more stages. This diversity is a consequence of the complexity of the problem.

Uncertainty is a major drawback inherent in the field of water-supply looped-network design optimization. Uncertainty is present when estimating current and future demands, diurnal demand variation, pipe friction coefficient, network and system reliability, etc. Adding to this assertion is the uncertainty arising from the results of the optimization procedure itself. As the classical water supply looped network optimization problem is mathematically non-convex and multimodal, despite the existence of numerous approaches to deal with it, no single one can claim achievement of global optimality in the general case. Therefore, the final solution is uncertain because it is not known how far it can be from the actual global optimal solution.

This uncertainty is typical (although not exclusive) for some approaches whose solutions are dependent on initial trial solutions (Morgan & Goulter 1985; Park & Leibman 1993; Gupta & Bhave 1996; Xu & Goulter 1997). Even more, for the classical formulation (minimize capital cost objective under nodal and loop constraints), the achievement of actual global optimality would be useless in practice because it would lead to purely branched networks. The meaning of global optimality will be further discussed below.

Apart from the uncertainty regarding optimality, some approaches produce continuous pipe diameter solutions (Varma *et al.* 1997; Tanyimboh & Templeman 2000) or the so-called split pipe solutions (Loganathan *et al.* 1995) which further deviate them from the practical engineering solution.

Procedures dealing directly with discrete pipe diameters have been introduced. A group of these techniques apply evolutionary algorithms (Savic & Walters 1997; Cunha & Ribeiro 2004) that, despite their capacity to evaluate tens of thousands of solutions, are not free from the above-mentioned drawbacks. Also, they seem to be limited by the computer time load for actual large networks. Just recently, an integer-programming algorithm (IPA), handling discrete diameters, has been proposed (Samani & Mottaghi 2006). Although the IPA example solution provided is a quasi-branched (pseudo-looped) network, the technique might be promising if it proves to be consistent and robust (Martínez 2008).

In some cases a reliability constraint has been added (Xu & Goulter 1999; Afshar *et al.* 2005) but most reliability definitions existing so far are not effective in assuring the necessary redundancy. Recent developments on multiobjective techniques are worth mentioning (Devi Prasad & Park 2004; Farmani *et al.* 2005) although they might be limited in problem size as well as the need for further research regarding the type of reliability parameter and to enhance comparability of solutions with other techniques.

After the previous discussion, it follows that a procedure would be quite advantageous if it could be devised with the following characteristics:

- an adequately redundant, looped network is obtained,
- the solution is a unique, reproducible solution,
- the solution is a global optimum under certain circumstances,
- it is suitable for large networks.

This is the purpose of the present paper.

CLASSICAL OBJECTIVE FUNCTION

Although the basic decision variables are pipe diameters, perhaps due to the high non-linearity associated with diameters, some researchers have formulated the problem in terms of separated sets of flows and heads (Alperovitz & Shamir 1977; Chiong 1985; Sarbu & Kalmar 2002; Martínez 2007, 2010).

The objective function (OBF) in Chiong (1985) cited by Martínez (2007, 2010) is restricted to account for costs only in the pipe network. The sum ψ of annualized capital costs and annual energy costs which is to be minimized is then

$$\psi = \mathrm{c}_1\eta\sum_{\mathrm{k}=1}^{\mathrm{NP}} L_k d_k^m + \mathrm{C}\sum_{\mathrm{s}=1}^{\mathrm{NS}} q_s(pm_s + \exp(x_s) + z_s) \qquad (1)$$

where k,s: subscript for pipes and source nodes; L,d: pipe length and diameter; NP: number of pipes; NS: number of source nodes; η,m: coefficient and exponent, respectively, of pipe cost formula; c_1: annualizing factor; C: constant including annual pumping time, unit price of energy and units conversion; q_s: known inflow to source node s; pm_s: known minimum pressure requirement in node s; z_s: ground elevation in node s; $\exp(x_s)$: excess pressure in node s, the use of this type of variable is because it improves convergence of the solution algorithm;
subject to:

$$hf_k = (pm_i - pm_j) + (\exp(x_i) - \exp(x_j)) + (z_i - z_j)$$
$$\text{for } k = 1,\ldots, NP \qquad (2)$$

$$\sum_{k\in\zeta} \mathrm{Q}_k + \mathrm{q_i} = 0 \quad \text{for } i = 1, \ldots, NN - 1 \qquad (3)$$

$$hf_k = \lambda_\mathrm{k} L_k \frac{Q_k^n}{d_k^r} \quad \text{for } k = 1, \ldots, NP \qquad (4)$$

where i,j: subscripts in Equation (2) for nodes belonging to pipe k; Q_k: flow in pipe k (positive if leaving the node); hf_k: head loss in pipe k; q_i: exterior flow in node i (outflow $+$); ζ: set of pipes k connected to node i; NN: total number of nodes in network; λ_k: constant including the friction coefficient; n,r: exponents of the friction formula.

Equation (2) is an expression of Bernoulli's law for each pipe, Equation (3) is the node flow continuity and Equation (4) is a generic friction formula. Substitution of Equations (4) into the OBF leads to

$$\psi = c_1\eta\sum_{\mathrm{k}=1}^{\mathrm{NP}} \mu_k \frac{Q_k^{mn/r}}{hf_k^{m/r}} + C\sum_{\mathrm{s}=1}^{NS} q_s(pm_s + \exp(x_s) + z_s) \qquad (5)$$

where

$$\mu_k = \mathrm{L_k}(\lambda_\mathrm{k} \quad \mathrm{L_k})^{\mathrm{m/r}}. \qquad (6)$$

In this model the decision variables (unknowns) are the x values in nodes (all but one) and one Q value for each loop. This formulation is the same as the classical one except for the energy term.

In order to obtain a convenient expression for the OBF, substitute Equation (3) into Equation (5) and assume that nodal heads at sources are given so the energy term in Equation (5) is constant. Then a specific equation can be written for the simple one-loop network of Figure 1 as follows:

$$\psi' = c_1\eta\left[\left(\frac{\mu_1}{hf_1^{m/r}}\right)Q_1^{mn/r} + \left(\frac{\mu_2}{hf_2^{m/r}}\right)(Q_1 - q_C)^{mn/r} + \left(\frac{\mu_3}{hf_3^{m/r}}\right)(q_A - Q_1)^{mn/r} + \left(\frac{\mu_4}{hf_4^{m/r}}\right)(q_B + q_C - Q_1)^{mn/r}\right]$$

where ψ' equals the former ψ by removing the constant energy term, flow Q_1 has been chosen as the independent flow value in the loop and then all Q_k values are expressed in terms of Q_1 through Equation (3).

A mathematical study of this equation would show that if Q_1 is given then the variation of ψ' with nodal heads would be convex with one single minimum. On the other hand if all hf_k were known then a plot of ψ' versus Q_1 could be drawn from the following equation:

$$\psi' = K_1 Q_1^{mn/r} + K_2(Q_1 - q_C)^{mn/r} + K_3(q_A - Q_1)^{mn/r} + K_4(q_B + q_C - Q_1)^{mn/r} \qquad (7)$$

where now all K values are constant. By taking equal values for them, the plot will not be biased by differences in the hf_k values.

Assuming reasonable values for the parameters ($K_1 = K_2 = K_3 = K_4 = 3790$; $m = 1.5$; $n = 2$; $r = 5$; $q_A = q_B + q_C + q_D$; $q_B = 80$; $q_C = 60$; $q_D = 60$ L/s) a plot is made of Equation (7) as the case (a) thick line in Figure 1. The plot is presented with a non-dimensional abscissa scale ($fQ_1 = Q_1/q_A$) and the

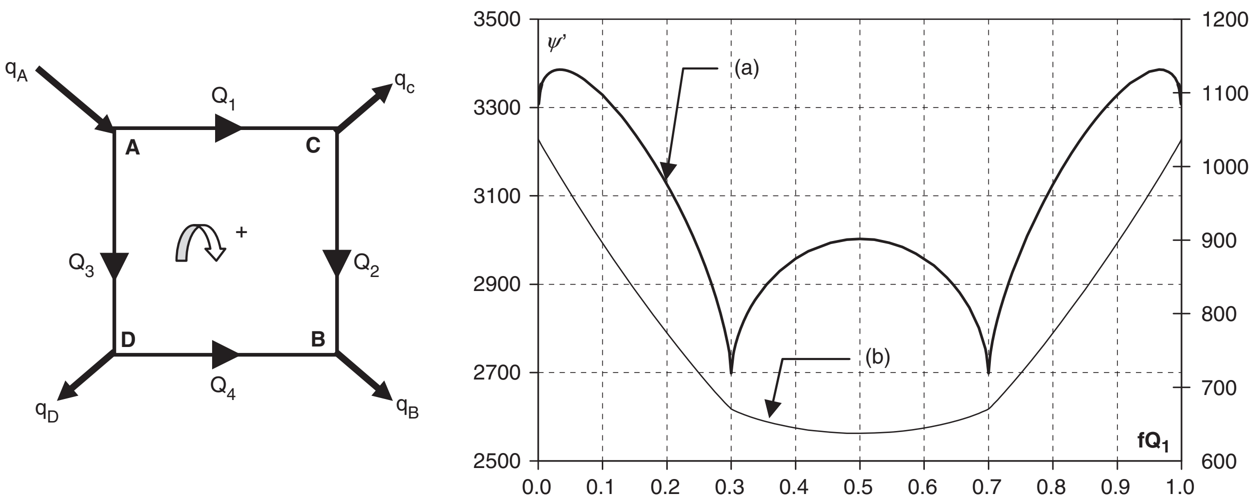

Figure 1 | Plot of Equation (7) for one-loop network and cases (a) and (b).

ordinate scale is $/year but its numerical values are irrelevant. Most interesting here is the shape of the curve and the following characteristics: (i) the curve has three maxima as stationary points; (ii) there are also four non-stationary minima located precisely where each one of the four pipe flows are zero: $Q_1 = 0$ for $fQ_1 = 0$; $Q_2 = 0$ for $fQ_1 = 0.3$; $Q_4 = 0$ for $fQ_1 = 0.7$; $Q_3 = 0$ for $fQ_1 = 1.0$.

Thus the concavity and multimodality here can be clearly noticed. It is also apparent from Figure 1 that in all minima there is a discontinuity in the derivative. This can also be obtained by calculating the derivative of Equation (7) with respect to Q_1 and noticing that it goes to infinity at those points. It has been shown (Chiong 1985) that this function would become *convex* and *unimodal* for $mn/r > 1$, which could only happen for $m > 2.5$ and this would deny the economy of scale in pipe costs; that is why it cannot be achieved under common values of the exponents.

Where are, in Figure 1, the global optima? In this case there are four local minima and because of the symmetry of this simple network there are two global minima. Not even the global optimum is unique. When minimum diameter or minimum flow restrictions are included, the search for minimum cost would lead to, say, in Figure 1 with four possible solutions. Generalization of this reasoning to a large network is obvious: the restricted global optimum can be multimodal!

Although it is true that Figure 1 has been drawn for constant hf_k for the sake of simplicity, it is believed that this analysis gives enough insight into the relationships involved. As an extension, similar conclusions can be derived from more general cost-flow drawings for two loops given by Loganathan *et al.* (1995) and (even better) from Kessler & Shamir (1989).

For varying parameters the plot of ψ' can show many different shapes, for instance, after a large increment in the length of one pipe the graph will show a very well defined global minimum which implies the removal of that pipe. This effect is stronger as pipe size increases.

Another interesting feature arises for the hypothetical case of $m > 2.5$. The case (b) thin-line curve in Figure 1 is the plot of ψ' for $m = 3$ (scale on the right). Now there is only one global minimum at $fQ_1 = 0.5$ because of symmetry and the derivative is discontinuous again at $fQ_1 = 0.3$ and 0.7. It can be shown that changing the parameters (for instance by increasing one pipe length) to eliminate symmetry the minimum could be located at or near one of these two values where one of the flows is zero. This has a very instructive conclusion: even when there would be no economy of scale and the classical looped-network problem had a unique absolute global minimum, there is no point in searching for this global optimum as the curse of the branched result would still be there.

Why does this happen? This happens because mathematics responds strictly to what is written in the OBF and constraints. The main reason for looping a network has not been expressed mathematically.

NEW OBJECTIVE FUNCTION

A new OBF has been proposed by Martínez (2007, 2010). The new OBF is obtained by adding a new term to Equation (1), which accounts for the expected annual cost involved in a pipe breakage. This expected cost includes the cost of failure repair and the cost of supplying affected consumers by other means. In such a formulation, explicit use is made of an empirical formula to express the frequency of failures:

$$\psi = c_1\eta\sum_{k=1}^{NP}L_k d_k^m + \sum_{k=1}^{NP}\omega_k L_k d_k^{-u} + C\sum_{s=1}^{NS}q_s(pm_s + \exp(x_s) + z_s) \quad (8)$$

where $\omega = a\cdot t_f(c_f + c_a\cdot V_f)$: coefficient associated with each pipe; $a\cdot L\cdot d^{-u}$: empirical formula giving the expected number of failures per year in terms of pipe diameter and length (a and u are known constants); t_f: average number of days for complete repair of each pipe failure; c_f: average cost of repair in dollars per day; c_a: average cost of supplying water to affected consumers in dollars per unit volume; $V_f = 86400\cdot Q_{break}$: volume per day that must be supplied to affected consumers; $Q_{break} = (Q_{n1}/C_{n1} + Q_{n2}/C_{n2})$ for broken pipes in loops; Q_{n1}, Q_{n2}: demand flow as volume per second in nodes n_1 and n_2 of the broken pipe; C_{n1},C_{n2}: number of pipes respectively connected to nodes n_1 and n_2; $Q_{break} = Q_k$ for pipes not in loops (the whole pipe flow). Further details can be found in the original paper of Martínez (2007). Substitution of Equation (4) into Equation (8) leads to

$$\psi = c_1\eta\sum_{k=1}^{NP}\mu_k\frac{Q_k^{mn/r}}{hf_k^{m/r}} + \sum_{k=1}^{NP}\beta_k\frac{Q_k^{-un/r}}{hf_k^{-u/r}} + C\sum_{s=1}^{NS}q_s(pm_s + \exp(x_s) + z_s) \quad (9)$$

where

$$\beta_k = \omega_k L_k(\lambda_k \quad L_k)^{-u/r}. \quad (10)$$

By substitution of Equation (3) into (9) and making similar assumptions to those leading to Equation (7) it can be obtained that

$$\psi'' = \psi' + B_1Q_1^{-un/r} + B_2(Q_1 - q_C)^{-un/r} + B_3(q_A - Q_1)^{-un/r} + B_4(q_B + q_C - Q_1)^{-un/r} \quad (11)$$

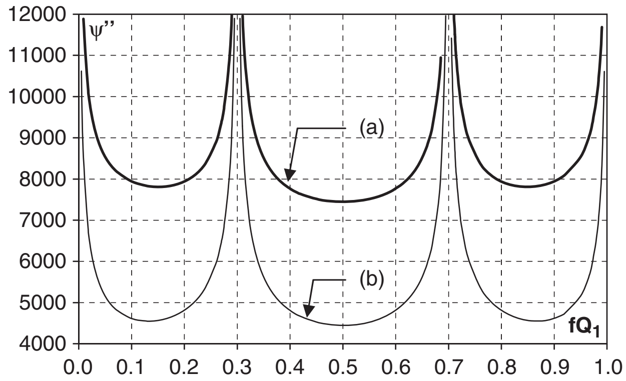

Figure 2 | Plot of Equation (11) for one-loop network and cases (a) and (b).

where ψ' is the same as in Equation (7) and the B values are constant. For a plot of Equation (11) another set of parameters are assumed ($10^5a = 3.50$; $u = 1.27$; $t_f = 2.0$ day; $c_f = 500$ \$/day; $c_a = 2.0$ \$/m^3). The thick line, case (a), in Figure 2 represents ψ'' while the thin line, case (b), is the sum of terms to the right of ψ' or $(\psi'' - \psi')$. The latter $(\psi'' - \psi')$ accounts alone for the new term in the new OBF mentioned above. Let it be called $New = (\psi'' - \psi')$.

It can be seen that the shape of the new term *New*, case (b), is convex although multimodal. The function goes to infinity where any one flow is null. The complete plot ψ'' of Equation (11), case (a), is very similar to case (b). This means that under the chosen parameter values, the addition of ψ' to *New*, despite its concave nature, cannot reverse the convexity of the new term *New*. But if the parameters are so changed that the relative weight of ψ' increases, a situation might be reached in which the minima will be located near the zero flow points and concavity will show up in the shape of the ψ'' curve. This situation is illustrated in Figure 3. This figure was obtained by multiplying the pipe cost coefficient by a factor of 10. Exactly the same drawing can be obtained by dividing the two cost indices of pipe breakage and the vertical scale by 10.

The effect of enlarging a pipe here also tends to enhance one of the minima with a similar implication of greatly reducing the flow of that pipe. This effect and the one shown in Figure 3 are considerably less dramatic with the new OBF – notice that now it is impossible to reach a zero flow and it is very likely that this only happens for parameters well away from common values – but they might still be present in a large real network due to the spatial diversity of its lengths and demands. The new OBF then, despite being a clear economic vindication of the looped network (Martínez

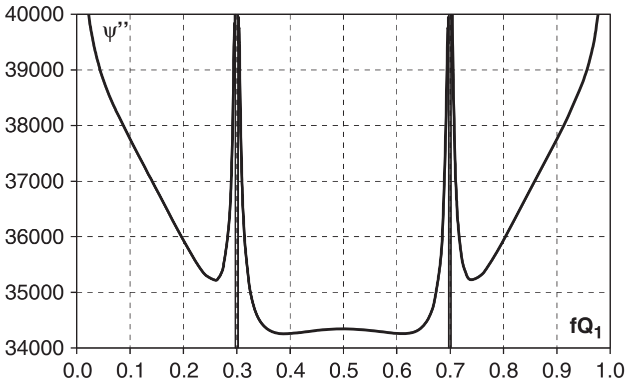

Figure 3 | Alternative plot of Equation (11).

2007, 2010), is not free from certain drawbacks when the intention is to find the global optimum.

TWO STAGES

The main purpose of the foregoing discussion was to demonstrate the inconvenience of the search for the global optimum in the classical sense. Particularly when the optimization procedure uses diameters as decision variables it is perhaps not possible to avoid that kind of search. The alternative of including reliability as an additional constraint in the diameters search has not been fully successful because reliability under most existing definitions cannot make a distinction between looped and branched networks. But even if it could make the distinction that approach would keep the uncertainty as to the quality (optimality) of the solution.

In order to practically eliminate the uncertainty about the optimality of the solution and, at the same time, to get both non-convexity and multimodality out of the way, a two-stage approach was proposed by Chiong (1985) formulating the problem in terms of separated sets of flows and heads.

The stages are: (1) calculate flows in pipes only as a function of demands looking for the maximum flow uniformity; (2) given all pipe flows calculate nodal heads to obtain minimum cost using Equation (1) as the OBF and the usual constraints. Chiong (1985) introduced the principle of minimum variance to calculate the flow distribution in the first stage. This principle has a unique global minimum for any network. Flow uniformity not only prevents the "opening" of the loops but also is relevant for the sake of reliability; for this reason researchers have frequently dealt with pipe flow distribution, other arguments and citations can be found in Martínez (2007) and Gupta *et al.* (2008).

The variance V_Q of a set of N values of Q flows can be calculated as

$$V_Q = N^{-1}\left(\sum_{j=1}^{N} Q_j^2\right) - \left(N^{-1}\sum_{j=1}^{N} Q_j\right)^2 \tag{12}$$

where N is number of pipes in the network and Q_j is flow in any pipe j.

Recall that on each loop any one of its pipe flows can be selected as independent variable. The other pipe flows belonging to the loop are then a function of the selected one. After substitution of nodal flow Equations (3) for all loops, and calculating derivatives with respect to each selected-as-independent loop pipe flow, the result is Equation (13) for each loop. Calculation of second derivatives shows that this is a global minimum:

$$\sum_{k \in \mathrm{Loop}} Q_k = 0 \quad \textit{for each loop}. \tag{13}$$

As stated in Equation (13) the flows involved are of course those in pipes belonging to the loop. These flows have a sign according to the usual loop convention: clockwise is positive.

The second stage produces also a unique global minimum because, if the flows are given, the problem is convex and unimodal as has been shown elsewhere. Therefore, the two-stage approach is a straightforward procedure that can produce a unique, global, reproducible solution.

In spite of these advantages, the described approach was not able to realize the economic advantages of the looped network. This was introduced just recently as Equation (8) (Martínez 2007) where not only the advantages of the looped network are brought into light but also a methodology is proposed to obtain optimal reliability level as well as the optimal design demand based on cost analysis (Martínez 2010).

Then the proposed two-stage approach fulfills the characteristics stated before as follows:

- *an adequately redundant, looped network is obtained*; this is accomplished by the first stage with the flow distribution and completed by the second stage where the choice of diameters takes into account the failure frequency.

- *the solution is a unique, reproducible solution*; the first stage gives a unique solution and so does the second stage.
- *the solution is a global optimum under certain circumstances*; the result of each stage is a global optimum, the first is a sort of statistical optimum while the second is an economic one.
- *it is suitable for large networks*; the example in Martínez (2007) suggested this possibility and recently the author tested a 184-node, 273-pipe, 90-loop network for one design demand in 4.5 minutes of a Pentium 4 type personal computer.

There is one drawback though and it is the need for diameter rounding to discrete commercial values. This is a minor drawback if the available commercial diameter set is "tight''. It is perhaps only natural if one might like to add a third stage: find the discrete diameters by applying an evolutionary or integer-programming algorithm, now with only two diameters per pipe reach. This of course could be the subject of future research.

THE ENTROPY APPROACH

The idea of uniform flow distribution has also been analyzed by other researchers in association with the maximum entropy principle (Awumah *et al.* 1991; Tanyimboh & Templeman 2000).

Goulter & Bouchart (1990) used an approach with a fixed flow distribution but they did not say how it was obtained. Awumah *et al.* (1991) use an entropy formulation based on inflows to nodes in an optimization model and apply it to the same example of Morgan & Goulter (1985) with a constraint enforcing a minimum entropy level and giving a split-pipe type solution. This approach should be considered at least doubtful because the results are very close to the original example, which was found to be highly infeasible by Afshar *et al.* (2005). Nevertheless, the entropy inflow formulation, from a practical viewpoint, would seem to be more redundancy-related than the outflow formulation.

A two-stage approach can be applied if maximum entropy is used as the uniform flow distribution generator. This was done by Tanyimboh & Templeman (2000) together with a NLP routine as their second stage which gives continuous diameters as the final solution. They also have proposed a different two-stage approach where the second stage would be the Alperovits & Shamir (1977) LP split-pipe solution. Their entropy formulation is based on outflows from nodes.

The main disadvantage here is that if the first stage is based on entropy, the result is multimodal (Ang & Jowitt 2005) because entropy can only be calculated after the flow directions are given. Outflow-based entropy drawings are shown in Figure 4, one for the network of Figure 1 and the others for each of its nodes. In the network drawing three maxima can be observed corresponding to the three possible flow directions, the one shown in Figure 1 and the others when either Q2 or Q4 flows are reversed. It is interesting to notice that entropy is not zero at the points where any one flow is zero. This is because, even if some flow is zero, there is always one node that has two outflows. These zero-flow points correspond to branched versions of the network showing their large entropy values. Calculating the proportion between these branched versions entropy points and the maximum in this figure, it gives about 80%. A similar value is obtained from an example in Ang & Jowitt (2005) as well as from other tests made by the author. So this entropy is not only not zero in a branched network but it seems to be a large part (about 75–80%) of the total entropy of the associated looped network.

The drawing for source node A shows a maximum and two zeros when either Q1 or Q3 flows are zero. The drawings for the other nodes show segments of zero entropy corresponding to the interval in which the node only receives water, i.e. has no outflow pipe. Except for node A, which is the source, maximum nodal entropy occurs at points of null flow. Another shortcoming is that, for large networks, several nodes might have only one outflow, which means a zero entropy value, even when those nodes may have two or more inflowing pipes.

The multimodality of network entropy is considerable, notice that in most of its vertical range there are six flow distributions with the same entropy value. For two or more loops they are certainly infinite. For any classical optimization with an entropy constraint this adds another dimension to the already cumbersome multimodality. So far no method has been proposed to find the global maximum entropy.

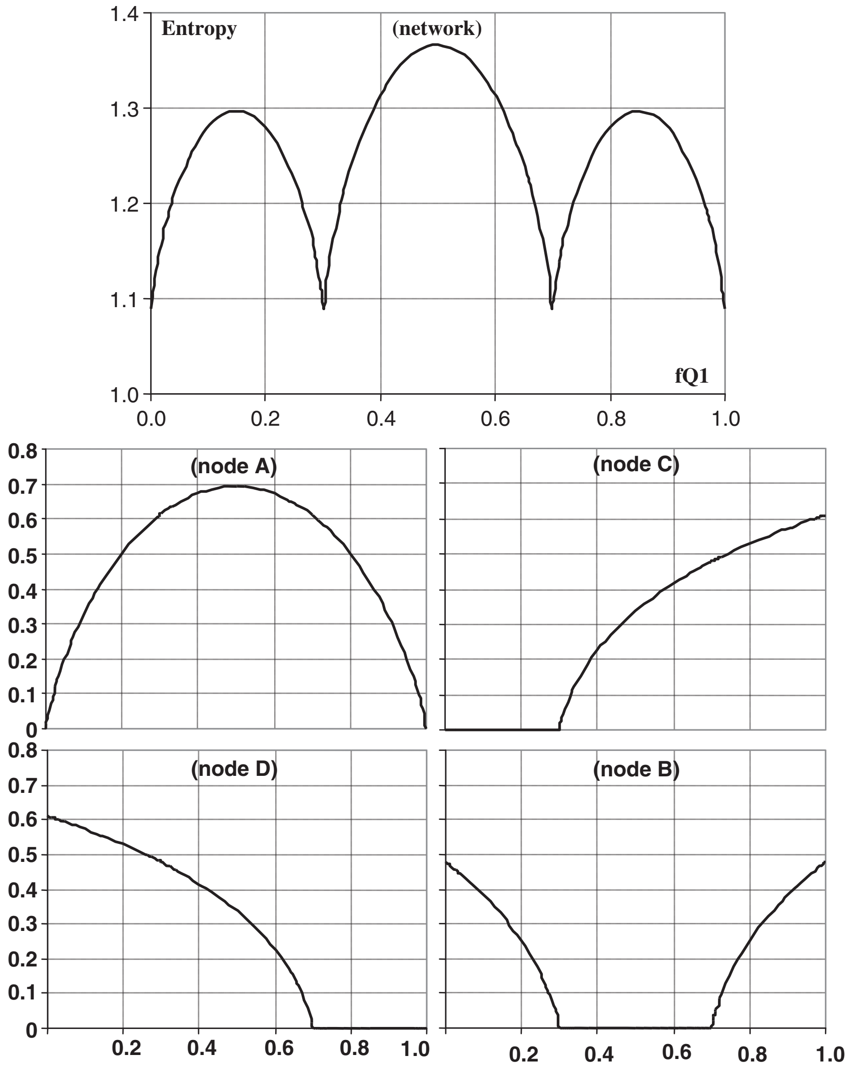

Figure 4 | Entropy for one-loop network and nodes.

Given all flow directions the calculation of flow values for its local maximum entropy is very easy for one-source networks. But for two or more sources, contrary to the optimistic statements of Yassin-Kassab *et al*. (1999), the same is not true. Despite the well debated and clearly explained procedure for finding the flow distribution of multi-source networks for *given flow directions*, the examples presented are so small that disadvantages could not be foreseen.

Let *NS* be the number of sources. The procedure involves solving a set of *NS*-1 polynomial equations with *NS*-1 unknowns. The degree of each polynomial is equal to the number of demand nodes that are reachable from its corresponding source. A small network may have, for instance, 15 demand nodes reachable from all sources. The more sources in the network, the more complex the solution of polynomials of degree 15 which indeed is not a trivial pocket calculator solution.

Nevertheless, there might be another problem: the multiplicity of solutions because such a polynomial set may have up to 15 (roots) sets of solutions. Some of them may be real while others may be complex. If the real ones are multiple then there is another multiplicity drawback, which originates within the specifically given flow directions. The problem of maximizing entropy for multi-source networks with given

Table 1 | Statistical comparison of flow distribution

Network	**1**	**1**	**2**	**2**
Method	Maximum entropy	Minimum variance	Maximum entropy	Minimum variance
Sum(Q^2)	1334.40	943.75	67309.00	62853.40
Mean(Q)	11.146	9.375	80.429	80.429
Variance	42.572	30.078	3146.816	2510.302
Std. deviation	6.525	5.484	56.096	50.103
Coeff. of variation	0.5854	0.5850	0.6975	0.6229

flow directions may have a unique solution but this short-cut procedure may not represent a much easier approach to obtain it in a large network with several sources.

As compared to entropy based criteria, the principle of minimum variance has several advantages (Martínez 2007) to achieve maximum flow uniformity: (a) there is an explicit comprehensive statistical measure of uniformity; (b) the flow solution is extremely easy to obtain; (c) the solution is unique; (d) there is no need to assign flow directions (e) the solution process is independent of the number of source nodes.

As an illustration, Table 1 shows a statistical comparison of flow distribution calculated with maximum entropy and minimum variance for two small networks. Network 1 is the 8-pipe example given as case A in Tanyimboh & Templeman (1993*a*) and Network 2 is the 7-pipe example of Tanyimboh & Templeman (1993*b*). In Table 1, except the mean value, the other parameters are measures of dispersion. The original Q values are expressed in liters per second (L/s). It is seen that the comparison favors the minimum variance method.

EXAMPLE

In order to compare optimization results between flow distributions obtained from maximum entropy and from minimum variance, an example is introduced. The example is taken from Tanyimboh & Templeman (2000) where, for the complete network layout shown in Figure 5, a solution is given for maximum entropy flows and minimum capital cost. The fixed flow direction for each pipe corresponds to the numbering order of its two nodes.

This network has only one source at node 1 with total head H = 100 m. In all other nodes ground elevation is zero and minimum required pressure is 30 m. Pipes are all 1000 m long and Hazen-Williams friction coefficient is C = 130. Demands in liters per second for all other nodes are, orderly: 27.8; 41.7; 41.7; 41.7; 27.8; 55.5; 55.5; 55.5; 27.8; 41.7; 27.8. Pipe cost per unit length is formulated with $c_1 = 0.10$, $\eta = 800$ and $m = 1.50$. The original results of this example are given as continuous diameters.

Optimization is now performed under Equation (8) as OBF and constraint Equations (2)–(4). As stated before, the flow distribution is calculated as a first stage by the principle of minimum variance and then the second stage calculates optimal heads and continuous diameters. The diameters are then rounded to commercial values. Available commercial values exist in multiples of 25 mm. Rounding assigns the nearest upper commercial value unless the calculated diameter is very near the nearest lower. Other parameters for the new OBF are the same as the ones used in plotting Equation (11).

Three alternatives are devised for comparing results. The first is the original optimization (denoted here as T & T), the

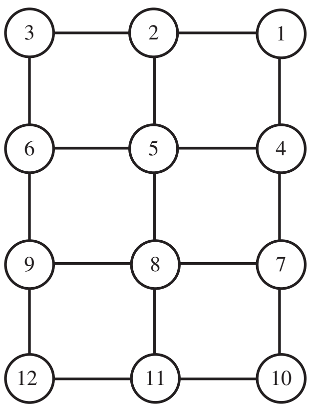

Figure 5 | Example network.

Table 2 | Comparison of dispersion in flows and diameters

Pipe # (1)	Node 1 # (2)	node 2 # (3)	T & T Q(L/s) (4)	JB-cont Q(L/s) (5)	JB-discr Q(L/s) (6)	T & T D(mm) (7)	JB-cont D(mm) (8)	JB-discr D(mm) (9)
1	1	2	175.65	209.71	204.98	302	313	325
2	1	4	268.85	234.79	239.52	361	321	325
3	2	3	61.57	87.96	91.10	192	224	225
4	2	5	86.28	93.96	86.08	228	231	225
5	4	5	87.61	68.89	72.19	226	209	225
6	4	7	139.54	124.20	125.63	275	247	250
7	3	6	19.87	46.26	49.40	138	192	200
8	5	6	42.94	40.26	37.92	175	181	175
9	5	8	89.26	80.89	78.65	239	224	225
10	7	8	44.23	25.57	25.69	179	158	175
11	7	10	39.81	43.13	44.44	169	182	200
12	6	9	35.00	58.71	59.52	182	221	225
13	8	9	37.49	18.08	18.17	178	154	150
14	8	11	40.49	32.88	30.66	184	181	175
15	10	11	12.01	15.33	16.64	119	150	150
16	9	12	16.99	21.30	22.20	162	182	200
17	11	12	10.81	6.50	5.60	135	145	150
Mean			71.08	71.08	71.08	202.59	206.76	211.76
Variance			4637	4272	4282	4003	2645	2744
Std. deviation			68.09	65.36	65.44	63.27	51.43	52.38
Coeff. of variation			0.9579	0.9195	0.9206	0.3123	0.2487	0.2473

second the new optimization with continuous diameters (denoted JB-cont) and the third reflects rounding the latter diameters to discrete ones (denoted JB-discr). A comparison of dispersion in resulting flows and diameters is shown in Table 2 for the three alternatives. The unique flow distribution resulting from minimum variance has exactly the same flow directions as the original example. No surplus head in the critical node is allowed under the new optimization process.

Column (4) is for maximum entropy flows, column (5) is for minimum variance flows and column (6) is for flows obtained after diameter rounding. It is seen that the two alternatives of the new optimization are less disperse than

Table 3 | Cost comparison in $/year

Alternative (1)	Costs from Objective Function: Capital (2)	Failure (3)	Total (4)	Cost of Additional Shortfall (5)	Grand total (6)
T & T	128125	53687	181812	49412	231224
JB-cont	130577	50079	180656	47724	228380
JB-discr	135314	48412	183726	35698	219424

column (6) = column (4) + column (5)

Table 4 | Reliability comparison in percent

Alternative (1)	Time without failure (2)	Time with failure (3)	Geometric mean nodal reliability (4)	Network Volumetric reliability (5)	Time with Dem ≥90% (6)
T & T	97.199096	2.800904	99.631410	99.650490	99.094143
JB-cont	97.380157	2.619843	99.662241	99.668198	99.097657
JB-discr	97.458004	2.541996	99.714407	99.716534	99.238049

the original solution. Similar comments can be drawn from columns (7)–(9) which relate to dispersion of diameters.

The T&T alternative is evaluated for cost with the same OBF, Equation (8). To complete the cost and reliability comparison (Martínez 2007), each one of the three alternatives is then analyzed with a pressure-driven simulator as many times as there are pipes (17 in this case) considering one pipe broken at a time. Average network-wide additional shortfall is obtained and its cost computed with the same price for water in the OBF. Additional energy costs were found negligible. Table 3 shows the cost comparison. Columns (2)–(4) are costs from OBF. Capital cost increases and failure cost decreases with increasing average diameter. Total OBF cost is less for the second alternative and higher for the third. When the cost of additional shortfall is added the best alternative is the third.

Several reliability parameters are shown in Table 4. Columns (2) and (3) add up to 100% and are estimated from pipe failure frequency. Using shortfall calculations a nodal reliability is obtained as the expected fraction of satisfied demand in the node. Column (4) is the geometric mean of nodal reliabilities while column (5) is the volumetric reliability of the whole network. Volumetric reliability is obtained considering all nodes, it is the expected fraction of satisfied demand in the whole network. Column (6) shows the expected fraction of time that at least 90% of demand will be fulfilled.

Although the results of the three alternatives are very close to one another, the two alternatives from the proposed approach are slightly better.

CONCLUSIONS

A discussion about the inconvenience of the search for the global optimum in the classical sense has been introduced. When diameters are used as decision variables it is perhaps not possible to avoid that kind of search. The alternative of including reliability as an additional constraint in the diameters search has not been fully successful and in any case keeps the uncertainty as to the quality (optimality) of the solution.

A two-stage approach was presented formulating the problem in terms of separated sets of flows and heads. This approach practically eliminates the uncertainty about the optimality of the solution and gets rid of non-convexity and multimodality. It is also a clear economic vindication of the looped network (Martínez 2007, 2010).

A critical study of the entropy approach shows its inherent drawbacks. A comparative example demonstrates that the proposed approach, besides the advantages in using the principle of minimum variance, is quite competitive as concerns uniformity of flow distribution, uniformity of diameters, cost and reliability.

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