A Modified Glauber Approximation and the Formation of Shock Waves in High-Energy Heavy Ion Collisions

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Coherent features in high energy heavy ion collisions are investigated from a microscopic viewpoint. A modified Glauber approximation which enables us to include the slowing down of the relative motion between the centers of masses of both nuclei is presented. The generation of the shock wave is shown to be suppressed by this slowing down effect. As an application inclusive nucleon spectra and the two-particle correlation are calculated in the case of Ne+U collision at 400 MeV/A.

§ 1. Introduction

In previous papers\(^1\) (which are referred to as I hereafter) we investigated a possible microscopic mechanism for shock wave generation in high energy heavy ion collisions. The nuclear excitation induced by a nuclear collision was described in terms of phonons. It was shown that as the collision proceeds phonons are coherently excited and show a characteristic feature of the shock wave. In this treatment the action of the projectile on the target was replaced by that of a potential moving with a constant velocity. We may construct this potential by superposing the nucleonic potential between projectile nucleons and the target ones with amplitudes of nucleonic states in both nuclei. To treat both systems equally from a microscopic viewpoint, one of the authors (M.I) proposed a formalism which is essentially an application of the Glauber approximation to the process.\(^2\) The assembly of nucleons in the projectile nucleus was assumed to move in a group during the collision and the Glauber approximation was applied to the relative motion between the centers of masses of the projectile and the target nucleus. In this treatment the deceleration of the relative motion due to the nuclear excitation was not taken into account. In this paper we extend the Glauber approximation so as to take into account the slowing down of the relative motion. In § 2 the formulation of the modified Glauber approximation is presented.

As an application of this formalism we come back to the shock wave problem in high energy heavy ion collisions, which was treated in I. We are anxious to know how the coherency of phonon production is modified when the slowing down
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of the relative motion is taken into account. First we briefly recapitulate in § 3 the phonon formalism introduced in I. The velocity of the relative motion between the projectile and the target is a quantum mechanical operator. In high energy collisions, however, the momentum transferred in each individual step of the collision is small compared with the momentum of the relative motion. Thus the variation of the relative velocity is rather small and we can safely approximate the velocity operator by a classical quantity. Moreover, since the transverse momentum transfer in high energy nuclear collisions is small, we assume that the relative motion is on a straight line. Under these approximations the scattering amplitudes are explicitly written in terms of phonon amplitudes in § 4. Phonons are generated coherently by a moving source which is gradually decelerated as the collision proceeds. The resulting nucleon spectra are discussed in § 5 and the typical example is shown in § 6. As discussed in I, the angular distribution of nucleons displays a peak corresponding to the Mach angle, if the slowing down of the relative motion is neglected. The deceleration of the relative motion works so as to suppress the peak as is expected. Angular correlations between emitted particles are also discussed.

§ 2. Extension of the Glauber approximation

Our dynamical system is composed of a projectile nucleus and a target nucleus. The energy-momentum of each nucleus is governed by the relativistic relation:

\[ \begin{align*}
E_{T(P)}^2 & = (M_{T(P)}^* + K_{T(P)})^2, \\
M_{T(P)}^* & = M_{T(P)} + e_{T(P)}. 
\end{align*} \]

Here \(K_{T(P)}, M_{T(P)}\) and \(e_{T(P)}\) denote the momentum, the ground-state mass and the excitation energy of the target (projectile) respectively. The suffix \(T(P)\) means the quantity associated with the target (projectile) nucleus. If we use a potential \(V\) for the interaction energy between the projectile and the target, the total energy becomes

\[ E = E_P + E_T + V(r), \]

where \(r\) is the position vector of the center of the projectile measured from the center of the target. It is convenient to take the center of mass system \((K_P = -K_T = K)\). In this reference system, we assume hereafter that the projectile energy satisfies the condition

\[ K \lesssim M_P \ll M_T. \]

This restricts us to the cases where the projectile is much lighter than the target,
and a nucleon within the projectile has the kinetic energy at most comparable with its rest mass (\( \sim 1 \text{ GeV} \)). Thus \( E_T \) may be expanded as \( E_T \approx M_T^* + K^2/2M_T^* \). Noting that \( K^2/2M_T^* \approx E_P(M_P/M_T) \ll E_P \), the recoil of the target may be neglected: \( E_T \approx M_T^* \). Moreover, for the sake of simplicity we assume that the excitation of the projectile is negligible compared with its kinetic energy, and we expand \( E_P \) as \( E_P = \sqrt{M_P^2 + K^2} (1 + O(e_P/M_P)) \approx \sqrt{M_P^2 + K^2} \). The total energy is thus expressed as

\[
E = (M_T + e_T) + \sqrt{M_P^2 + K^2} + V(r). 
\]

Next we go over to the quantum theory. The stationary Klein-Gordon equation corresponding to the classical Hamiltonian (2.5) is written as

\[
[M_P^2 - \mathbf{V}^2] \Phi = [E - M_T - H_T - V(r)]^2 \Phi. 
\]

\( H_T = e_T \) denotes the quantized intrinsic Hamiltonian of the target and \( \Phi \) is the total wave function of our system (stationary state). The potential \( V(r) \) contains the intrinsic degrees of freedom of target nucleus which are not written explicitly. If the interaction is relatively weak \((\mathbf{V}(r) \ll M_T)\), Eq. (2.6) is approximated as

\[
(\mathbf{V}^2 + H_T^2 - M_T^2) \Phi = [H_T, V(r)] \Phi, 
\]

where \( H_T \) is defined by the equation

\[
H_T = E - M_T - H_T. 
\]

The physical meaning of \( H_T \) is the "projectile energy" if the interaction with the target is neglected.

In order to solve Eq. (2.7) we expand \( \Phi \) as

\[
\Phi = \sum_s e^{i\mathbf{k}_s \cdot \mathbf{r}} \varphi_s(r) |s>, 
\]

where \( |s> \)'s are the eigenstates of \( H_T \): \( H_T |s> = \varepsilon_s |s> \). We also factorize the plane wave with the constant vector \( \mathbf{k}_s \) which will be determined below. Substituting Eq. (2.9) into Eq. (2.7), we obtain the simultaneous equations for \( \varphi_s \):

\[
[2i\mathbf{k}_s \cdot \mathbf{V} + \mathbf{V}^2 + (E_s^2 - M_T^2 - \varepsilon_s^2)] \varphi_s \\
= \sum_{s'} \langle s | [H_T, V] | s' \rangle e^{i(\mathbf{k}_s - \mathbf{k}_{s'}) \cdot \mathbf{r}} \varphi_{s'}. 
\]

where an eigenvalue of the "projectile energy" \( H_T \) is given by \( E_s = E - M_T - \varepsilon_s \). It will be shown later that the energy transfer \( \omega_s \) per one elementary process is small compared with the projectile energy. This leads us to the approximation

\[
\langle s | [H_T, V] | s' \rangle = \langle s | V | s' \rangle [2E_s + O(\omega_s)]. 
\]
Now we set up the fundamental assumption in our theory:

(i) The vector $k_s$ in Eq. (2·9) has the incident direction (z-axis) and its magnitude is given by

$$k_s^2 = E_s^2 - M_p^2,$$

which shows that the $k_s$ corresponds to the "projectile momentum".

(ii) The $\phi_s(r)$ varies slowly compared with the plane wave $e^{ik_s z}$ so that its second derivatives can be neglected. These assumptions reduce Eq. (2·10) to

$$\phi_s \approx \frac{1}{\omega} < s | V | s' > e^{i(k_s - k_s') z} \phi_s',$$

where $\omega \equiv k_s / E_s$. It should be noted that our approximation is different from the usual one (Glauber approximation) in which all the quantities $k_s$ are set to be equal to the initial momentum of the projectile. Therefore one may expect that the present model is capable of dealing with the slowing down of the projectile.

In order to rewrite Eq. (2·13) in a more compact form, we introduce

$$k \equiv \sqrt{H_p^2 - M_p^2},$$
$$\tilde{v} \equiv \frac{k}{H_p},$$

which represent the momentum and the velocity of the projectile respectively if the interaction is switched off. Equation (2·13) is then transformed into

$$\frac{\partial \phi_s}{\partial z} = \sum_{s'} < s | V | s' > \phi_s',$$

$$\tilde{V}(r) = (i \tilde{v})^{-1} e^{-ik_z} V(r) e^{ik_z}.$$  

(2·14)
(2·15)

(2·16)
(2·17)

It can easily be shown that these simultaneous equations have the following formal solution satisfying our initial condition:

$$\phi_s(b, z) = < s | Z \exp \int_{-\infty}^{z} \tilde{V}(b, z') dz' | 0 >.$$

(2·18)

The two-dimensional vector $b$ represents the $(x, y)$-component of $r$ and the symbol $Z$ means $z$-coordinate ordering operator analogous to the time ordering operator known as $T$-product.

Now we are in a stage to derive the scattering amplitude in terms of the approximate wave function obtained above. The scattering amplitude is defined by the asymptotic behavior of the wave function:

$$\Psi_{r \rightarrow \infty} \rightarrow e^{ik_s r} | 0 > + \sum_s F_s(k) \frac{e^{i k_s r}}{r} | s >.$$

(2·19)

The differential cross section with the excited state $| s >$ is expressed by...
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The Klein-Gordon equation (2·7) can be transformed into the integral equation

$$\varphi(r) = e^{ikr}|0\rangle - \frac{1}{4\pi} \int d\mathbf{r} \frac{e^{ik|\mathbf{r}|}}{|\mathbf{r}-\mathbf{r}'|} s\langle s|H_{ef}, V,|\varphi(r')\rangle.$$  

Comparing this equation with Eq. (2·19), we obtain

$$F_s(k) = \frac{F_s}{2\pi} \int d\mathbf{r} e^{-i\mathbf{k} \cdot \mathbf{r}} \langle s|V(r)|\varphi(r)\rangle,$$  

where the use is made of Eq. (2·11) and |k| = k_s. One may replace $\varphi(r)$ in this equation by its approximate solution (2·18). Carrying out the z-integration, the scattering amplitude is expressed as

$$F_s(k) = \frac{k_s}{2\pi} \int d\mathbf{r} e^{-i\mathbf{k} \cdot \mathbf{r}} \langle s|Z \exp \int_{-\infty}^{\infty} \tilde{V}(\mathbf{b}, z) dz |0\rangle - \delta_{s,0}. $$  

This form shows that the forward scattering is dominant because of the presence of the rapid oscillation function $e^{-i\mathbf{k} \cdot z}$ in the integral as expected in our basic assumption. The total and the reaction cross sections are given by the optical theorem:

$$\sigma_{tot} = 2 \int d\mathbf{b} \left[ 1 - \text{Re}\langle 0|Z \exp \int_{-\infty}^{\infty} \tilde{V}(\mathbf{b}, z) dz |0\rangle \right],$$

$$\sigma_r = \sum_{s \neq 0} k_s |F_s(k)|^2.$$  

§ 3. Intrinsic motion (phonons)

To proceed further we must know the nature of the intrinsic state $|s\rangle$. We assume that the intrinsic motion of the projectile is inert during the collision as discussed in the previous section. The intrinsic motion of the target nucleus is visualized by the nuclear density fluctuation

$$\rho'(x) = \rho(x) - \rho_0.$$  

Here $\rho(x)$ is the density operator and $\rho_0$ is its mean value with respect to the ground state. In the phonon approximation, the fluctuation is expanded in terms of phonon operators as follows:\textsuperscript{16}

$$\rho'(x) = \sum_{\mathbf{k}} \sqrt{\frac{\rho_0}{2m\omega_s\Omega}} (a_{\mathbf{k}} + a_{\mathbf{k}}^*) e^{i\mathbf{k} \cdot x},$$  

where $m$ denotes the nucleon mass and $\Omega$ the nuclear volume. The operator $a_{\mathbf{k}}^*$
represents the creation operator of a phonon with momentum $k$ and the energy

$$\omega_k = k \sqrt{\frac{k^2}{4m^2} + \frac{\rho_0 V_k}{m}},$$

(3.3)

where $V_k$ is the Fourier component of the nuclear interaction in the target. The intrinsic Hamiltonian is

$$H_I = \sum \omega_k a_k^\dagger a_k + H_I^\prime,$$

(3.4)

where $H_I^\prime$ means the Hamiltonian for the other mode which is independent of the phonon mode. Since our interaction is described by the density which is just the phonon mode we neglect $H_I^\prime$ hereafter.

If one denotes the creation operator of a particle (hole) with momentum $k$ by $a_k^\dagger (b_k^\dagger)$, the phonon creation operator is expressed in terms of particle-hole operators with the usual manner:

$$a_k^\dagger = \frac{1}{2\sqrt{\rho_0 \Omega}} \sum \left( \phi_k b_{k'} + \phi_{k'} b_k \right)$$

(3.5)

in which the forward amplitude and the backward one are defined by

$$\left\{ \begin{array}{c}
\phi_k = \sqrt{\frac{2m\omega_k}{k^2} + \sqrt{\frac{2m\omega_k}{k^2}}} \\
\phi_{k'} = \sqrt{\frac{2m\omega_{k'}}{k^2} - \sqrt{\frac{2m\omega_{k'}}{k^2}}} 
\end{array} \right.$$

(3.6)

This is the same as the plasma mode in the electron gas. Noting that $a_k$ and $a_k^\dagger$ satisfy the boson commutation relation, we obtain the approximate commutation relation between pair operators:

$$[b_{p'}, a_{p', k'}^\dagger, a_0^\dagger b_p^\dagger] \approx 4\delta_{kk'} \delta_{pp'}.$$  

(3.7)

The factor 4 comes from the spin-isospin variables.

The interaction between the projectile and the target is given in terms of the effective nucleon-nucleon potential $v(r)$ as

$$V(r) = \frac{1}{2} \int \int v(|r-x+x'|) \rho_0(x') \rho(x) dx dx'$$

$$= \int U(|r-x|) \rho(x) dx.$$  

(3.8)

$U(r)$ is regarded as the potential produced by the projectile having the density $\rho_0(x)$. Substituting Eq. (3.2) into Eq. (3.8), we get

$$V(r) = V_0(r) + V_1(r),$$  

(3.9)
where \( V_0 \) and \( V_i \) are given by

\[
V_0(r) = \int U(|r - x|) \rho_0(x) \, dx, \tag{3·10}
\]

\[
V_i(r) = \sum_k k \sqrt{\frac{\rho_0}{2m \omega_k \Omega}} U_k(\alpha_k + \alpha_k^*) e^{ik \cdot r}. \quad (|r| < R) \tag{3·11}
\]

The Fourier transform of \( U(r) \) is approximately replaced by

\[
U_k = \int U(r) e^{ik \cdot r} \, dr. \tag{3·12}
\]

To manipulate the potential operator \( \bar{V}(r) \) introduced in Eq. (2·17), it will be useful to note that for many phonon states

\[
[f(a' \alpha), \alpha] = [f(a' \alpha) - f(a' \alpha - 1)] \alpha = -\frac{\partial f}{\partial (a' \alpha)} \alpha, \tag{3·13}
\]

where \( f(a' \alpha) \) is any function of the phonon number operator.\(^*\) From Eq. (2·14) and this formula, we obtain

\[
e^{-iz \alpha} a^* \alpha e^{iz \alpha} = \exp(-iz \bar{V}^{-1} \omega_k) a^*_k, \tag{3·14}
\]

which enables us to rewrite Eq. (2·17) into

\[
\bar{V}(r) = (i \bar{\varepsilon})^{-1} \left[ V_0(r) + \sum_k k \sqrt{\frac{\rho_0}{2m \omega_k \Omega}} U_k(\alpha_k + \alpha_k^*) e^{(\bar{\varepsilon} - \omega_k \beta^*)} a^*_k \right] \tag{3·15}
\]

If we replace the operator \( \bar{V} \) by a c-number and substitute Eq. (3·15) into Eq. (2·18), our intrinsic wave function would become a phonon coherent state. However, Eq. (3·15) has the rather complicated structure than it looks. It is here worthwhile to notice that our intrinsic wave function depends not on the relative momentum but on the velocity only. In the high energy limit \( (v \rightarrow 1) \), our intrinsic state approaches the definite state (scaling),\(^5\) if particle productions were neglected.

§ 4. Quasi-classical approximation for the intrinsic motion

In this section we will calculate the scattering amplitude (2·22) in which the interaction is replaced by the boson expression (3·15). Our main task is to calculate the matrix element

\[^*\) For example, it can be shown that from Eq. (3·13)

\[
[H_0, V] = \left[ -\sum \omega_k \alpha_k a_k, V \right] \sim O(\omega_k V)
\]

which has been used in Eq. (2·11).
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\[ T_{s,0} = \langle s | \mathcal{Z} \exp \int_{-\infty}^{\infty} \tilde{V}(\tilde{v}; b, z) dz | 0 \rangle. \]  

(4.1)

Note that \( \tilde{V} \) contains the operator \( \tilde{v} \) in the complicated form as seen in Eq. (3.15). Now we take up the quasi-classical approximation which replaces the operator \( \tilde{v} \) by the classical variable \( v(r) \) (c-number). It may be justified by noting that the commutator

\[ [\alpha_k, \tilde{v}] \approx \frac{1}{\tilde{v}} \left( \frac{\omega_k}{E} \right) \alpha_k \]  

(4.2)

derived from Eq. (3.13) is small because of the relation \((\omega_k/E) \ll 1\) and \( \tilde{v} \ll 1 \). Then the \( Z \)-symbol in Eq. (4.1) can be easily computed as follows:

\[ Z \cdot \exp \int_{-\infty}^{\infty} \tilde{V}(v(r); b, z) dz \approx e^{ix} \prod_{s} \exp[f_{ks}e^{-ib_{ks}}a_{k's} - \text{h.c.}] \equiv U_{c}(b), \]  

(4.3)

where \( x(b) \) and \( f_{ks}(b) \) are defined by

\[ \begin{align*}
    x(b) &= -\int_{-\infty}^{\infty} \frac{V_{sk}(r)}{v} \, dz + \frac{i}{2} \int_{-\infty}^{\infty} dz \int_{-\infty}^{\infty} dz' \left[ \tilde{V}(b, z'), \tilde{V}(b, z) \right], \\
    f_{ks}(b) &= \frac{\hbar}{i} \sqrt{2m\omega_{ks}x} \int_{-\infty}^{\infty} v \, e^{i(a_{ks} - \omega_{ks}t)} \, dz.
\end{align*} \]  

(4.4)

(4.5)

Equation (4.3) shows that our intrinsic state is excited to form phonon coherent states through the collision. Therefore our scattering amplitude (2.22) is rewritten as

\[ F_{s}(k) = \frac{k_{s}}{2\pi i} \int d\mathbf{b} e^{-ik \cdot \mathbf{b}} \{ e^{ix} \prod_{s} \exp[f_{ks}e^{-ib_{ks}}a_{k's} - \text{h.c.}] | 0 \rangle - \delta_{s,0} \}. \]  

(4.6)

Finally let us determine the classical dynamical variable \( v(r) \). It should be derived from the original expression by some physical approximation. However, we will take the following phenomenological form for convenience. It is based on the nucleon-nucleon scattering data presented in Fig. 1. The horizontal axis represents the incident nucleon energy (GeV) in the laboratory frame of reference. The incident velocity is shown by the dashed line (c

\[ \begin{align*}
    \text{Fig. 1. Velocity curve (v) of the incident nucleon \((c=1)\) and the damping rate (\(\mu\)) of the longitudinal component defined by Eq. (4.7).}
\end{align*} \]
The solid line shows the damping rate of the longitudinal velocity \( v_L \) defined by

\[
\mu = \frac{v_L - v'_L}{v_L} = \frac{m}{m+(E\rho'/\rho')} \frac{p_L - p'_L}{p_L},
\]

where the values of the longitudinal momentum transfer \( (p_L - p'_L) \) is taken from Ref. 6).

If we denote the mean free path of a nucleon travelling through the nuclear matter as \( \lambda \), we get the relation

\[
\lambda \frac{\partial v}{\partial z} = -\mu v.
\]

According to Fig. 1, one may be allowed to assume that the damping rate \( \mu \) has a constant value. Then Eq. (4.8) has the solution

\[
v(b, z) = v_0 \exp\left[-\frac{\mu}{\lambda}(z + d)\right],
\]

where \( d \) means the \( z \)-coordinate of the projectile at which the interaction between the projectile and the target is switched on.

§ 5. Inclusive distributions

Since the exhaustive data on the high-energy heavy ion collision have not been obtained yet, our present knowledge on the reaction mechanism in these collisions is mainly derived from the inclusive cross section. The experimental results measured up to the present are one-particle and two-particle inclusive distributions. In this section we will calculate these quantities within our theoretical framework.

According to Eq. (2.20), the probability that the projectile is scattered in the direction \( \Omega \) together with the intrinsic state \( |s\rangle (s+0) \) is given by

\[
\frac{1}{\sigma_r} \left( \frac{d\sigma}{d\Omega} \right)_s.
\]

Subsequently the nucleons which have energies enough to escape from the excited nucleus are emitted. The one-particle (momentum) distribution in the target after the collision is expressed as

\[
\langle N_p \rangle = \sum \int d\Omega \frac{1}{\sigma_r} \left( \frac{d\sigma}{d\Omega} \right)_s \langle s| N_p |s \rangle,
\]

where \( N_p \) is the number operator for the nucleon in the target. With the use of Eq. (2.23), Eq. (5.1) is transformed into the following expression:
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\[ \langle N_p \rangle = \frac{1}{\sigma_f} \sum_s \int d\Omega \int d\mathbf{b} \int d\mathbf{b'} \frac{\kappa_0^3}{(2\pi)^3} e^{i\mathbf{b} \cdot \mathbf{b'}} \times \langle 0 \mid U(\mathbf{b'})^\dagger | s \rangle \langle s | N_p | s \rangle \langle s | U(\mathbf{b}) | 0 \rangle , \] (5.2)

where \( U(\mathbf{b}) \) is \( Z \exp \int_{-\infty}^{\infty} V(\mathbf{b}, z) \, dz \) in Eq. (2.23).

The single particle state \( \mathbf{p} \), which we are interested in, belongs to the unbound state so that \( \langle s | N_p | s' \rangle = \delta_{ss'} \langle s | N_p | s \rangle \). This relation, together with the completeness relation transforms Eq. (5.2) into a simple form

\[ \langle N_p \rangle = \frac{1}{\sigma_f} \int d\mathbf{b} \langle 0 \mid U(\mathbf{b})^\dagger \frac{\tilde{k}}{k_0} N_p U(\mathbf{b}) | 0 \rangle , \] (5.3)

where use is made of the forward scattering approximation \( d\Omega \approx dk_x dk_y / k_s^2 \). If the phonon and quasi-classical approximation (4.3) is taken, Eq. (5.3) becomes

\[ \langle N_p \rangle \approx \frac{1}{\sigma_f} \int d\mathbf{b} \frac{k(\mathbf{b})}{k_0} \langle 0 \mid U_c(\mathbf{b})^\dagger N_p U_c(\mathbf{b}) | 0 \rangle , \] (5.4)

where the operator \( \tilde{k} \) which is a function of the phonon number \( \alpha_s^a \alpha_s^a \) is approximated by

\[ U_c(\mathbf{b})^\dagger \tilde{k}(\alpha_s^a \alpha_s^a) U_c(\mathbf{b}) \approx k(|f_s|^2) = k(\mathbf{b}). \] (5.5)

In order to proceed further calculations, we note that

\[ U_c(\mathbf{b})^\dagger a_{\mathbf{p}}^\dagger U_c(\mathbf{b}) \approx a_{\mathbf{p}}^\dagger - \frac{1}{2\sqrt{\rho_0 \Omega}} \sum_k e^{-ik \mathbf{b} \cdot \mathbf{p}} (f_k \phi_k - f^*_k \phi_k^*) b_{\mathbf{p} - \mathbf{k}} + O\left( \frac{1}{\rho_0 \Omega} \right) \] (5.6)

with the help of Eq. (3.5). Substituting this equation into Eq. (5.4), we get

\[ \langle N_p \rangle \approx \frac{1}{\sigma_f} \int d\mathbf{b} \frac{k(\mathbf{b})}{k_0} S(\mathbf{p}, \mathbf{b}) , \] (5.7)

\[ S(\mathbf{p}, \mathbf{b}) \equiv - \frac{1}{\rho_0 \Omega} \sum_k (\phi_k \phi_k^* - \phi_k^* \phi_k) |\mathbf{p} - \mathbf{k}|^2 . \] (5.8)

As for the decay process from the excited state, we set up a following simple picture: (i) A nucleon with the momentum \( \mathbf{p} \) satisfying \( \mathbf{p}^2 / 2m > E_0 \) (\( E_0 \): binding energy) decays outwards with the observed momentum \( \mathbf{p'} = \sqrt{1 - (2mE_0 / p^2)} \mathbf{p} \). (ii) \( E_0 = (p_F^2 / 2m) + 8 \text{ MeV} \) (\( p_F \): Fermi momentum). Two-particle inclusive distribution is given by

\[ \langle N(\mathbf{p}, \mathbf{p'}) \rangle = \frac{1}{\sigma_f} \int d\mathbf{b} \frac{k(\mathbf{b})}{k_0} \langle 0 \mid U_c(\mathbf{b})^\dagger a_{\mathbf{p}}^\dagger a_{\mathbf{p}} a_{\mathbf{p'}} a_{\mathbf{p'}} U_c(\mathbf{b}) | 0 \rangle . \] (5.9)

The expectation value of the right-hand side of the above equation can be calculated in the same manner as in the case of \( \langle N(\mathbf{p}) \rangle \) and is given by
where the function \( S \) is given by Eq. (5-8) and \( \Delta S \) is defined as

\[
\Delta S(p_1, p_2) = \frac{1}{k_0} \sum_{k,h} \left( \phi_{k-h, f_k-p_1} - \phi_{k-h, f_k-p_1} \right) \left( \phi_{k-h, f_k-p_2} - \phi_{k-h, f_k-p_2} \right) \right) \right)^2.
\]

(5-11)

Our expression (5-10) for the two-particle inclusive distribution consists of two parts. The first term means the product of each one-particle inclusive distribution (5-7). The second corresponds to the two-particle correlation within each individual collision event. From Eq. (5-11), it has always minus sign mainly due to the Pauli principle. Experimentally, however, the following correlation function is often used instead of Eq. (5-10):

\[
R(p_1, p_2) = \frac{\langle N(p_1, p_2) \rangle - \langle N(p_1) \rangle \langle N(p_2) \rangle}{\langle N(p_1) \rangle \langle N(p_2) \rangle}.
\]

(5-12)

It should be noted that the above correlation function does not vanish even if \( \Delta S = 0 \). This is because the single-particle spectrum depends on the impact parameter of the collision. Then the numerator of Eq. (5-12) gives

\[
\frac{1}{2} \left( \frac{1}{\sigma_r} \right)^2 \int d^2k \frac{k(b)}{k_0} \int d^2b \frac{k(b)}{k_0} (\bar{N}_p(b) - \bar{N}_p(b')) (\bar{N}_p(b) - \bar{N}_p(b')) ,
\]

(5-13)

where \( \bar{N}_p(b) = \langle 0 | U_c(b) N_p U_c(b) | 0 \rangle \) and use is made of \( \sigma_r \approx \int d^2b \). It is almost evident that the integrand of Eq. (5-13) is positive so that

\[
R(p_1, p_2) > 0 \quad \text{(for } \Delta S = 0)\]

which is numerically confirmed (§ 6).

§ 6. Application to the Ne+U collision

As a typical example, we apply our model to the case of Ne+U collision (\( E = 400 \text{ MeV} / \text{A} \)).? It is our aim to investigate the influence of the slowing down effect on the inclusive distributions. Our calculations are preliminary so that the extensive study on this model including the comparison with the experiments will be made in a subsequent paper.

To begin with, let us consider the potential \( U(r) \) produced by the projectile. Equation (3-8) gives

\[
U(r) = \frac{1}{2} \int u(|r + x|) \rho_0(x) dx.
\]

(6-1)

It must be noted that this expression is written in the reference system belonging...
to the target. Therefore the projectile density $\rho_0(x)$ in Eq. (6.1) has a Lorentz-contracted shape. If the density in the projectile frame has the form of $\rho_0 \exp[-(r/2a)^2]$ the Fourier transform of $U(r)$ becomes

$$U_k = 4\pi^{3/2}a^3 \rho_0 V_k \exp\left[ -a^2 \left( k_x^2 + k_y^2 + \frac{k_z^2}{\gamma^2} \right) \right], \quad (6.2)$$

where $V_k$ is the Fourier transform of the effective nucleon-nucleon potential and $\gamma = (1 - v^2)^{-1/2}$. The Fourier components (6.2) of our potential is obviously enhanced in the forward and backward direction. As for $V_k$, we may take a simple soft core potential $V_k = V_0 \exp(-\lambda^2 k^2) \ (\lambda = 0.25 \text{ fm})$ considering the relevant high-energy phenomena. The value of $V_0$ is left as a parameter.

In the first place, the total and the reaction cross sections are computed and the results are as follows:

$$\sigma_{tot} = 245 \ (\text{fm}^2), \quad \sigma_r = 147 \ (\text{fm}^2) \quad \text{for } \mu = 0,$$

$$\sigma_{tot} = 271 \ (\text{fm}^2), \quad \sigma_r = 150 \ (\text{fm}^2) \quad \text{for } \mu = 0.15.$$  

Noting that we take $\pi R^2 = 147 \ (\text{fm}^2)$, our results support the geometrical nature of the reaction (shadow scattering). The slowing down effect appears in the decrease of transparency of the target.

Next, the one-particle inclusive distributions are calculated and are shown in Figs. 2~4. We study first the case of no slowing down effect ($\mu = 0.0$). The

![Fig. 2](https://academic.oup.com/ptp/article-abstract/69/1/142/1834252)
angular distributions for 20 MeV and 100 MeV\textsuperscript{*}) nucleons are presented in Figs. 2(a) and 3(a) respectively. Figure 4(a) shows the energy spectra for nucleons emitted in the directions of \( \theta = 18^\circ \) and \( 90^\circ \). These results show the following features:

(i) The Mach peak\textsuperscript{9}) \( \theta_M \approx \cos^{-1}(0.2/\nu) = 74^\circ \) is seen at lower energies but disappears at higher energies.

(ii) Backward emission is enhanced irrespective of the energies.

(iii) The energy spectrum has an exponential form and its slope is more steep in the case \( \theta = 90^\circ \).

Taking into account the slowing down effect \( \mu = 0.15 \), Figs. 2(a)\textasciitilde4(a) turn into Figs. 2(b)\textasciitilde4(b) respectively. Generally the emitted particle number is very enhanced and the above-mentioned features are altered as follows:

(i) The Mach peak disappears in two cases.

(ii) The enhancement of the backward emission is unchanged.

\textsuperscript{9}) It will become important to take account of the simultaneous excitation of the projectile as for the emission of the high energy nucleons. Therefore the values calculated in the case of 100 MeV nucleons is not so significant for us.
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Fig. 4. (a) Energy spectra of nucleons at $\theta = 18^\circ$ (filled circles) and $\theta = 90^\circ$ (open circles). The slowing down effect is not included.
(b) Energy spectra of nucleons at $\theta = 18^\circ$ (filled circles) and $\theta = 90^\circ$ (open circles). The slowing down effect is included.

Fig. 5. Zenith angle correlations between two nucleons of 20 MeV. The notation is explained in the text.

Fig. 6. Azimuthal angle correlations between two nucleons of 20 MeV. The notation is explained in the text.
(iiiib) The slope of the energy spectrum becomes independent of the emitting direction.

Next we consider the two-particle correlation. For convenience, our interest is limited to the correlation between two nucleons with 20 MeV energy. The polar angles of $\mathbf{p}_1$ and $\mathbf{p}_2$ are denoted by $(\theta_1, \varphi_1=0)$ and $(\theta_2, \varphi_2)$ respectively. The zenith angle correlations are shown in Fig. 5 where $\mathbf{p}_1$ is fixed and $\varphi_2=0$. On the other hand, Fig. 6 represents the azimuthal angle correlations where $\mathbf{p}_1$ is fixed and $\theta_2=\theta_1$. These figures show that:

(iv) The minus correlation due to the Pauli principle is seen in all cases.
(v) The peak of positive correlation appears near the region of Pauli correlation in the cases of zenith angle correlation. The slowing down effect ($\mu=0.15$) raises these curves in the positive direction uniformly.

§ 7. Summary and discussion

The center-of-mass motion of the colliding nuclei can be described in the framework of the Glauber approximation. We modify it so as to take into account the excitation of the target nucleus and the resulting slowing down of the center-of-mass motion. During the collision the target nucleus is excited due to the action exerted by the projectile and the phonons are generated. If the velocity of the projectile is approximately constant throughout the collision, phonons are generated coherently. A characteristic feature of the shock wave appears when the projectile velocity exceeds the velocity of the phonon propagation. In our case the projectile is decelerated transferring its energy to the target and the coherency of phonons diminishes. The velocity of the projectile is an operator in our treatment and its eigenvalue is determined by the state of the target. Since the mean momentum transfer in the individual nucleonic collision is small compared with the momentum of the center-of-mass motion, we approximate the velocity operator of the projectile as a c-number. The deceleration of the projectile originates from its energy transfer to the target. These quantities should be treated selfconsistently. Since our main interest here is, however, to see the effect of deceleration on the coherency of phonon generation, we have treated the rate of deceleration as a parameter. As the damping rate we have taken the value which is determined from the nucleon-nucleon collision. This may too much overestimate the actual value for the center-of-mass motion.

Our model mentioned above is based on the basic assumption that the nucleons belonging to the projectile can be always distinguished from the ones belonging to the target. This leads us to the requirement that the final longitudinal momentum of each nucleon in the projectile is larger than the Fermi momentum, which restricts the value of $\mu$.

The single particle spectra were calculated for the case of Ne+U collision at
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400 MeV/A. The Mach peak in the one-particle inclusive spectra disappears in the strong damping case. We see that the backward emission is enhanced. This is mainly due to the effect of the Lorentz contraction of the potential between colliding nuclei. However this tendency is not seen in the experimental data, which may be caused by the neglect of the simultaneous excitation of the projectile and the oversimplified phonon mode ((3·5) and (3·6)). Two particle correlations were also calculated. The negative correlation due to the Pauli principle is seen as is expected. Positive peak which appears in the correlation function is considered to originate from a sort of collective phenomenon.

Throughout this paper the excitation of the projectile is not considered. To include it formally is a straightforward matter. The production of mesons as well as the formation of nucleonic resonance states should be taken into account. These will be done in subsequent papers.

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