Seismic moment tensors and kinematic source parameters

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Summary. Parameters pertaining to the kinematics of a finite source are usually estimated by fitting specific fault models to the data. On the other hand, these parameters, including source location, are also contained in the moment tensors of higher degree. In this paper, the seismic response is represented in terms of 20 source parameters which are related to components of the moment tensors; they are also related to the parameters of fault models, as will be demonstrated for a number of ‘classical’ models. A linearized inversion for the moment tensor shows that with real data, or with realistic synthetic data, the results are not necessarily physically meaningful, unless constraints are imposed. The constraints are precisely those appearing as a priori assumptions in the conventional methods of source analysis; it is thus possible to investigate the impact of these assumptions. We will discuss in particular the assumption of a general deviatoric point source (not necessarily a double couple) versus that of a plane fault in finite sources. Although at this stage experience with practical performance of the new method is limited, it is suggested that in the appropriate circumstances constrained inversion for the seismic moment tensors offers a viable alternative to estimate kinematic source parameters.

1 Introduction

The moment tensor representation of a seismic point source has now become widely appreciated, both for the purpose of modelling the seismic wavefield (the forward approach), and for obtaining source parameters (the inverse approach). A review of the development of this concept is to appear elsewhere (Doornbos 1981). Here we investigate a problem that arises in that the moment tensor applies to a point source whereas seismic sources are, in general, of finite extent. It may not be a serious problem in the forward approach, because the representation is linear and the fields from a spatial distribution of point sources can be summed or integrated to simulate the effect of a finite source; the source is in fact represented by a moment tensor density (Backus & Mulcahy 1976a,b). But in the inverse problem this is not a practical representation because it implies an infinite number of unknowns. In contrast, ‘classical’ source models, like those of Haskell (1964) and Savage (1966), are described by only a limited number of parameters; because they pertain to the kinematics of seismic
sources, these parameters have been called kinematic source parameters. A good description of these models is contained in Aki & Richards (1980). If the source region is ‘reasonably’ small (compared to the wavelengths), it is possible to approximate the effect of a moment tensor density by a limited number of moment tensors of degree zero and higher. This representation was discussed in some detail by Backus (1977a, b), but no attempts to do the inverse problem were reported, previous studies dealing almost exclusively with the moment tensor of degree zero (referred to above as the moment tensor). The zero degree tensor does not contain spatial information (including source location), and parameters like fault geometry and rupture velocity are still obtained by fitting observations to specific models like those of Haskell and Savage (e.g. Somerville, Wiggins & Ellis 1976; Chung & Kanamori 1980). These source parameters represent averages over the fault, so it is the long-wavelength part of the seismograms (relative to fault dimensions) that is used to estimate them. Clearly, these parameters must be related to the moment tensors of higher degree (Backus 1977a), and it thus appears worthwhile to explore the feasibility of estimating these moment tensors. In doing so it is important to keep the number of unknown parameters small, and this consideration suggests to approximate not only the spatial extent, but also the source time function, by a few parameters. Such a procedure is not only practical, but also meaningful in the sense that, ideally, spatial and temporal source functions should be approximated in the same way, as demonstrated by Backus (1977a). We will give the necessary formulation to estimate the 20 time and space parameters which are adequate to represent the seismic source (in the long-wavelength approximation) and show how they are related to the parameters of particular ‘classical’ source models. Some examples with synthetic and real data (from the SRO network) demonstrate that the results of such an inversion are not necessarily physically meaningful unless further constraints are imposed, and the reasons for this are explored.

2 Moment tensor representations

The moment tensor was originally introduced in connection with the excitation of the normal modes of the Earth (Gilbert 1970). Along these lines it was extended by Woodhouse (1980). It can also be introduced with a Green’s function representation, which is more adapted to our present purposes. Partly as a means to introduce the various quantities which are needed later in the paper, we first give some established results (e.g. Aki & Richards 1980; Stump & Johnson 1977). The representation of displacement at position \( \mathbf{x} \) and at time \( t \), due to a source within a volume \( V \), is

\[
U_t(\mathbf{x}, t) = \int_{-\infty}^{t} \int_V \partial \xi_k G_j^i(\xi, \mathbf{x}, t-\tau) m_{jk}(\xi, \tau) \, dV \, d\tau
\]

where \( G_j^i \) is Green's function which is supposed to satisfy the reciprocity theorem, and \( m_{jk} \) is a moment tensor density. In an explosion it represents an imposed stress pulse, but in an earthquake it has been called the stress glut (Backus & Mulcahy 1976a), representing the difference between the true physical stress \( \tau_{ij} \) and the model stress \( \sigma_{ij} \) which satisfies Hooke's law:

\[
m_{ij} = \sigma_{ij} - \tau_{ij}
\]

and the equivalent force is then

\[
f_j = -\partial_t m_{ij}.
\]
Equation (1) can be replaced by a point source representation with the aid of moments, by expanding $G_j^i$ about a suitably chosen reference source point $\xi^0$:

$$u_i(x, t) = \partial \xi_k G_j^i(\xi^0, x, t) * M_{jk}(t) + \partial \xi_l \partial \xi_k G_j^i(\xi^0, x, t) * M_{jk,l}(\xi^0, t) + \ldots$$

where $M_{jk,l} \ldots$ are spatial moment tensors of the stress glut.

$$M_{jk}(\xi, \tau) = \int_V m_{jk}(\xi, \tau) \, dV$$

$$M_{jk,l}(\xi^0, \tau) = \int_V (\xi_l - \xi^0) \, m_{jk}(\xi, \tau) \, dV.$$  

For asymptotic wave functions, the spatial derivatives in equation (4) are simply evaluated from an integral representation (Doornbos 1981):

$$G_j^i(\xi^0, x, t - \tau) = \int_\Gamma g_j^i(\xi^0, x, t - \tau) \, dp$$

where $p$ is the ray parameter or slowness, and

$$\partial \xi_k G_j^i(\xi^0, x, t - \tau) = -\int_\Gamma \frac{c_k}{\beta} \frac{\partial}{\partial \tau} g_j^i(\xi^0, x, t - \tau) \, dp = \hat{G}_{j,k}^i(\xi^0, x, t - \tau)$$

where $\cdot$ denotes a temporal derivative of $g$, $\beta$ is the wave velocity ($S$ or $P$) and $c_k$ the direction cosine of the wave in $\xi^0$ (more precisely, a generalized cosine as defined in Richards 1976). $G_{j,k}$ is defined in this equation merely for notational convenience in the sequel.

Green's function may consist of a sum of wave functions; in that case, this sum is to be taken over the right side of (6). The response (4) may now be written

$$u_i(x, t) = G_{j,k}^i(\xi^0, x, t) * \hat{M}_{jk}(t) + \hat{G}_{j,k,l}^i(\xi^0, x, t) * \hat{M}_{jk,l}(\xi^0, t) + \ldots$$

and the convolutions replaced by summations, with the aid of temporal moments

$$u_i(x, t) = G_{j,k}^i M_{jk} - \hat{G}_{j,k}^{(1)}(\tau^0) + \hat{G}_{j,k,l}^i M_{jk,l}(\xi^0)$$

$$+ \frac{1}{2} \hat{G}_{j,k}^{(2)}(\tau^0) - \hat{G}_{j,k,l}^i M_{jk,l,t}(\xi^0, \tau^0) + \frac{1}{2} \hat{G}_{j,k,l,m}^i M_{jk,l,m}(\xi^0) + \ldots$$

where

$$G_{j,k}^i \equiv G_{j,k}^i(\xi^0, x, t - \tau^0)$$

$$M_{jk} = \int_{-\infty}^{+\infty} \hat{M}_{jk}(\tau) \, d\tau, \quad M_{jk}^{(n)}(\tau^0) = \int_{-\infty}^{+\infty} (\tau - \tau^0)^n \hat{M}_{jk}(\tau) \, d\tau$$

$$M_{jk,l}^{(1)}(\xi^0, \tau^0) = \int_{-\infty}^{+\infty} \int_V (\tau - \tau^0) (\xi_l - \xi^0) \hat{m}_{jk}(\xi, \tau) \, dV \, d\tau.$$  

Equation (8) contains 90 source parameters (moment tensors up to degree 2), but if the six components of the moment tensor density have a common space and time history (a 'smoothing' assumption), the number of parameters may be reduced:

$$M_{jk}^{(1)}(\tau^0) = M_{jk} \Delta \tau, \quad M_{jk,l}(\xi^0) = M_{jk} \Delta \xi_l, \quad \text{etc.}$$
where
\[ \Delta \tau = \int_{-\infty}^{+\infty} (\tau - \tau_0) f(\tau) d\tau, \quad \Delta \xi_l = \int_V (\xi_l - \xi_l^0) g(\xi) \, dV, \quad \text{etc.} \quad (11) \]
and \( g(\xi), f(\tau) \) are normalized source space and time functions:
\[
\int_V g(\xi) \, dV = 1, \quad \int_{-\infty}^{+\infty} f(\tau) \, d\tau = 1.
\quad (12)
\]

The representation (8) then becomes
\[
u_i(x, t) = M_{jk} \left( G_{j,k}^i - \dot{G}_{j,k}^i \Delta \tau + \ddot{G}_{j,k}^i \Delta \xi_l \right.
+ \frac{1}{2} \dot{\ddot{G}}_{j,k}^i \Delta (\tau^2) - \dot{\ddot{G}}_{j,k}^i \Delta (\tau \xi_l) + \frac{1}{2} \dot{\ddot{G}}_{j,k,l}^i \Delta (\xi_l \xi_m) + \ldots \right)
\quad (13)
\]
with the 20 source parameters
\[
M_{jk}, \Delta \tau, \Delta \xi_l, \Delta (\tau^2), \Delta (\tau \xi_l), \Delta (\xi_l \xi_m).
\]

### 3 Particular source models

An interpretation of moment tensors of degree one and two in terms of kinematic source parameters, together with some quantitative estimates based in part on a statistical analogy, was given by Backus (1977a). The interpretation of quantities in equation (13) would be similar (Doornbos 1981). In summary, the reference source location and time can be chosen such that
\[
\Delta \tau = \Delta \xi_l = 0
\]
and this provides a means to find the 'optimum' location and time. Recently, Dziewonski, Chou & Woodhouse (1981) used this criterion to relocate earthquake sources by moment tensor analysis. Referred to the optimum coordinates \( \tau^0, \xi^0 \), the source 'rise time' is determined by \( \{\Delta (\tau^2)\}^{1/2} \), an estimate of rupture velocity averaged over the source region is \( \bar{v}_r = \Delta (\tau^2)/\Delta (\tau^2) \), and the size, shape and orientation of the final static source region is described by \( \Delta (\xi_l \xi_m) \). The quantitative relationships will of course vary with varying models of faulting. The parameters introduced above are commonly estimated by a priori assuming a specific model, and it is of interest to have the moment tensors of these commonly used source models; some results for Haskell and Savage type of models will be given in this section. These models have a plane fault, and in an isotropic medium (as assumed here) the zero degree moment tensor represents a double couple. The Haskell model is specified by fault length \( L \), width \( W \), rupture velocity \( u \), \( T = L/u \) in unidirectional rupture propagation, \( T = L/2u \) in bidirectional rupturing, the final dislocation \( [u] \) is uniform and the slip velocity is \( c, \theta = [u]^t/c \). We take the local z-axis to coincide with the normal to the fault plane, rupture initiating at the origin and propagating in the x-direction. The stress glut is, for unidirectional propagation,
\[
m_{jk}(\xi, \tau) = \frac{\mu c (n_j s_k + n_k s_j)}{2} \left[ H\left(\frac{\tau - x}{u}\right) - H\left(\tau - \theta - \frac{x}{u}\right) \right] \left( H(x) - H(x - L) \right) \left( H\left(y + \frac{W}{2}\right) - H\left(y - \frac{W}{2}\right) \right) \delta(z) \quad (14)
\]
for bidirectional propagation,

\[ \dot{m}_{jk}(\xi, \tau) = \mu c (n_j s_k + n_k s_j) \left\{ H \left( \tau - \frac{|x|}{v} \right) - H \left( \tau - \theta - \frac{|x|}{v} \right) \right\} \left\{ H \left( x + \frac{L}{2} \right) - H \left( x - \frac{L}{2} \right) \right\} \delta(z) \]

where \( \mu \) is the shear modulus and \( n, s \) are the unit normal and slip vector, respectively. We will consider a circular Savage type of model, fault radius \( R \), rupture velocity \( v \), \( T = R/v \). If the slip is uniform and occurs instantaneously in each point of the fault, the stress glut rate is

\[ \dot{m}_{jk}(\xi, \tau) = \mu [u]_s^2 (n_j s_k + n_k s_j) \delta \left( \tau - \frac{R}{v} \right) \left\{ H(r) - H(r - R) \right\} \delta(z) \]

where \( r = (x^2 + y^2)^{1/2} \). Modification of this model to get a more realistic slip function in time and space have also been considered. As an example we give the model proposed by Molnar, Tucker & Brune (1973). The stress glut rate for this model is

\[ \dot{m}_{jk}(\xi, \tau) = \mu [u]_s^2 \left( n_j s_k + n_k s_j \right) \left\{ H \left( \tau - \frac{r}{v} \right) - H \left( \tau - \frac{2R - r}{v} \right) \right\} \left\{ H(r) - H(r - R) \right\} \delta(z). \]

The moment tensor can now be obtained with equations (5) and (9) and performing some straightforward algebra. Results are summarized in Table 1 and Fig. 1. This shows that rupture time \( T \) and slip time \( \theta \) cannot be directly distinguished in interpreting the temporal moment \( \Delta(r^2) \). In principle, \( \theta \) could be estimated using the results for \([u]_s^2\), and an estimate of slip velocity (e.g. Abe 1975). The results for second degree moments are referred to the ‘optimum’ source coordinates \( \xi_0, \xi_0 \). More general (cf. Backus 1977a):

\[ \Delta(r^2) = \Delta(r^2) + (r^2 - r_0^2) \]

\[ \Delta(r \xi_j) = \Delta(r \xi_j) + (r^2 - r_0^2) (\xi_0^2 - \xi_j^2) \]

\[ \Delta(\xi_i \xi_m) = \Delta(\xi_i \xi_m) + (\xi_0^2 - \xi_j^2) (\xi_0^2 - \xi_m^2). \]

To assess the relative contribution of second-degree moments to the seismogram, we use equation (8), noting that

\[ \ddot{G}_{j,k} \sim \omega^2 G_{j,k}, \quad \ddot{G}_{j,k} \sim \omega^2 \frac{c_j}{\beta} G_{j,k}, \quad \ddot{G}_{j,k,m} \sim \omega^2 \frac{c_j c_m}{\beta^2} G_{j,k} \]

Table 1. Moment tensors of fault models. \( M = \text{scalar moment}; S = \text{fault surface area}; \Delta(\tau^2), \Delta(x^2), \Delta(y^2) \) referred to ‘optimum’ reference point \( \xi_0, \xi_0^* \), \( \xi_0^* \) such that \( \Delta \tau = \Delta x = \Delta y = 0 \). For further details see text.

<table>
<thead>
<tr>
<th>Haskell (mdfr.)</th>
<th>Haskell (mdfr.)</th>
<th>Savage (circular)</th>
<th>Molnar et al (circular)</th>
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</thead>
<tbody>
<tr>
<td>( M )</td>
<td>( u[u]_s^3 )</td>
<td>( [u]_s^3 )</td>
<td>( 2ucTS/3 + 2ucRS + 3v )</td>
</tr>
<tr>
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<td>( r + 1 ) + 1</td>
<td>( r + 1 ) + 1</td>
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<td>( \Delta \xi )</td>
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<td>( r + 1 ) + 1</td>
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<tr>
<td>( \xi (x^2) )</td>
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<td>( r^2 + 1 ) + 1</td>
<td>( r^2 + 1 ) + 1</td>
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<td>( \xi (y^2) )</td>
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<td>( \xi (x^2) )</td>
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</tr>
</tbody>
</table>
Figure 1. (a) Far field source time functions for several source models. $\tilde{M}(\tau)$ is the scalar moment rate function:

$$M = \int \tilde{M}(\tau) \, d\tau.$$ 

Models are (1): Haskell (unidir.), (2): Haskell (bidir.), (3): Savage (circular), (4): Molnar et al. (circular). For further details, see text. (b) Spatial dependence of the stress glut, for the same source models as in (a).

$$M = \int m(x) \, dx dy = \int m(\tau) \, r dr \phi.$$

where the notation of equation (6) has been used. From Table 1 we then infer relative contributions of the order

$$\frac{3}{2} \frac{T^2}{T_0^2}, \frac{3}{2} \frac{L}{T_0}, \frac{3}{2} \frac{L^2}{T_0^2}$$

where $T_0$ is a wave period in the seismogram. From estimates of source rise time and fault surface area (e.g. Kanamori & Anderson 1975; Chung & Kanamori 1980) one may then infer that for earthquakes with $M_s > 6$, the relative contribution may be of the order of 10 per cent or more, even in long-period seismograms.

We can also assess the significance of moment tensors of degree 3 and higher. Most source time and space functions in our examples are symmetric with respect to the optimum source location. In these cases, moment tensors of degree 3 will then vanish, and they should be small in other cases. The relative excitation of moment tensors of degree 4 involves factors of the order $T^4/T_0^8, \ldots, L^4/(\beta^4 T_0^8)$. In practice it means that these contributions are insignificant as long as source rise time and spatial extent are smaller than wave period and wavelength. In most cases, these higher degree moments may then be deleted in the analysis of long-period seismograms.

4 Inversion

For the inversion experiments we took long-period SRO and ASRO records of a deep event in the Bali Sea; event information is contained in Table 2. The event was selected mainly because coverage of the focal sphere was reasonable with eight stations in the 90° epicentral distance range, and because of the apparent simplicity of the source. In these circumstances, some of the practical problems with inversion for the source parameters should stand out more clearly. One of the practical problems which has been addressed by several authors is concerned with anomalous or unexpected path effects. To minimize this problem we used
Table 2. Conditions for source inversion. Event with PDE origin time 1978 June 10, 17 hr, 38 m, 19.9 s, location 6.09°S, 114.237°E, depth 520 km, m_p 5.5. For further details see text.

<table>
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<th>Moments Degree 1</th>
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only a limited number of bodywave phases. Records with P, SH and SV phases are shown in Fig. 2. In a standard earth model (PREM of Dziewonski & Anderson 1981) and with the given hypocentral depth, the S and SP phase are expected to interfere on the radial component. Although this is no problem in seismic modelling, the use of surface reflected phases like SP would necessarily imply rather simplifying assumptions about upper mantle and near-surface structure. To avoid this, in keeping with our desiderata outlined above, we decided to use here only P (vertical and radial component) and SH (transverse component). Fig. 2 shows that azimuthal coverage of the focal sphere is far from ideal, but this is partly compensated by the fact that P and S 'sample', the source under mutually perpendicular angles. Source parameters are to be obtained on the basis of equation (13). This is a non-linear equation, but it can be linearized:

\[
\bar{u}_i - \bar{u}_i = H_i, \delta M_{jk} + M_{jk} \left\{ - \bar{G}_{i, k} \delta (\Delta \tau) + \bar{G}_{i, kl} \delta (\Delta \xi_i) + \frac{1}{2} \bar{G}_{i, k} \delta (\Delta \tau^2) \right\}
\]

(19)

\[
- \bar{G}_{i, kl} \delta (\Delta \tau \xi_i) + \frac{1}{2} \bar{G}_{i, klm} \delta (\Delta \xi_i \xi_m)
\]

Figure 2. SRO and ASRO records from Bali Sea event (see Table 2 for details). Vertical component with P, horizontal components rotated into transverse and radial, with SH and SV. Record length is 4 min. Different amplitude scale for different components. The P vertical component of TATO was not used because of presumed non-linearity effects.
where
\[ u_i = H_{i,k}^T M_{jk} \]
\[ H_{i,k}^T = C_{i,k}^T - D_{i,k} \Delta \tau + \tilde{G}_{i,k,m} \Delta \xi_l \]
\[ + \frac{1}{2} \tilde{G}_{i,k}^T \Delta (\tau^2) - \tilde{G}_{i,k} \Delta (\tau \tau_l) + \frac{1}{2} \tilde{G}_{i,k,c} \Delta (\xi_l \xi_m) \]
and \((M_{jk} + \delta M_{jk})\) is taken to be an updated version of \(M_{jk}\), and similarly for the other parameters.

Each equation of the set (19) is for one component, one time, and one station. Thus, with eight stations recording 40s with two components of \(P\), one component of \(SH\), and sampling interval \(4s\), there will be 240 equations. Although these will not all be independent, there is reason to expect that the problem is sufficiently overdetermined for the source inversion to be feasible. This is one of the principal advantages of inversion in the time domain.

Before discussing some of the results, a few other preliminary remarks should be made. For practical purposes it is important to use a fast and efficient procedure of calculating Green’s functions and their derivatives in equation (19). In the present work we applied a version of the so-called WKBJ method of computing synthetic seismograms (Chapman 1978), while including the effects of anelastic damping and system response. Calculations usually do not account for station anomalies, but these are often significant. The SRO station anomalies have not yet been evaluated, so an ad hoc procedure has been followed by adding time corrections to Green’s function so as to ‘line up’ synthetic seismograms with the observations; a similar procedure was used by Ward (1980). However, these corrections will obscure information on source location, so they should not be applied if equation (19) is used to estimate \(\Delta \tau\) and \(\Delta \xi\). In a least-squares inversion of the set (19), each equation is weighted according to the wave amplitude. Ideally, the weight should be in accord with the standard deviation of the observed amplitude. Usually, amplitudes of \(S\)-waves are roughly a factor 4 stronger than amplitudes of \(P\)-waves (mainly due to a factor \((\alpha_\omega^2/\beta_\omega^2)\) difference in the Green’s functions \(G_{i,k}\)). On the other hand, observations of first arriving \(P\)-waves are often as reliable as those of later arriving \(S\). This fact can be taken into account by giving equations with \(P\) and \(S\) data different weight. In cases 1–4 of the subsequent experiments, we compare unweighted with weighted inversion, where the weight of \(P\) observations was 4 times that of \(S\). We will not present the comparison in detail, but the root-mean-squared (rms) error of fit confirms that weighted inversion is the preferred procedure.

In Table 2 we summarize the conditions and options applied in nine different inversions, and the resultant rms error of fit. Some of the results are compared in Figs 3–6. Fig. 3 shows that the inferred source reference time is about 3s late with respect to PDE time.

**Figure 3.** Results of source relocation in space and time. Case numbers refer to those in Table 2.
Consistently late times were also inferred by Dziewonski et al. (1981). In the present case the time shift seems to be too large to be explained by the finite rise time of the source alone; the use of an earth model different from that used in PDE determinations might be a factor. Another feature in Fig. 3 is the relatively large relocation, which again is unlikely to be associated with finite fault surface area alone. This result is essentially unchanged by the inclusion of depth phases $pP$ and $sS$ (cases 6 and 8).

In some cases we allowed an isotropic component to develop (tr $M \neq 0$). That possibility was investigated previously by Randall (1972), and more recent investigations (e.g. Backus & Mulcahy 1976a; Doornbos 1977; Okal & Geller 1979; Patton & Aki 1979) were provoked by some results of Gilbert & Dziewonski (1975). Fig. 4 shows that for the event in hand, the inferred isotropic component can be made to essentially disappear by relocating the source. The decomposition of the deviatoric part of the moment tensor into a major and minor double couple, though non-unique, serves to illustrate the deviation from a representation by one double couple only, and the major double couple can be used to construct conventional fault plane solutions (Fig. 5). It appears that differences among the various solutions are

Figure 4. Scalar moments of decomposed source. Decomposition into isotropic part and major and minor double couple. Case numbers refer to those in Table 2.

Figure 5. Fault plane solutions in equal area projections, for case numbers 2, 4, 5, 7, 9 of Table 2, with observed polarities and amplitudes of $P(\uparrow)$ and $SH(\rightarrow)$. 
small, and the solutions are consistent with observed polarities and amplitudes (corrected for path effects).

Of particular interest is case number 9 where moments of degree two are estimated. Fig. 6 illustrates the convergence toward the final solution, and the rms error of fit is smaller than in all other cases, as expected with the larger number of degrees of freedom. However, the solution is unacceptable on physical grounds. In general, the spatial moment tensor should describe a source ellipsoid or ellipse, rupture should be in the fault plane, and numerical values for moments of degree two are expected within certain bounds based, in part, on the solution for the moment tensor of degree zero. The solution in case 9 does not satisfy these criteria (we omit a detailed presentation of the results, but remark that similar features were obtained with some other events involving a different coverage of the focal sphere). In the next section we discuss how to take these criteria into account. Then we will investigate, by means of a few numerical experiments, the effect of errors in the data and of a priori assumptions about the source.

5 Linearized constraints

Results in the previous section suggest that a solution for the moment tensors should be constrained, for it to be physically meaningful and consistent. Here we obtain the constraints, in a linearized form. For notational convenience, we denote by $\mathbf{M}$ the tensor with components $M_{ij}$. Also, $\mathbf{W} \equiv \Delta(\xi_1, \xi_2)$, $\mathbf{w} \equiv \Delta(\tau_1, \tau_2)$. Variations of these quantities are denoted by $\delta \mathbf{M}$ etc. The linear constraint for a deviatoric moment tensor is

$$\sum_i M_{ii} = 0 \quad \text{or} \quad \sum_i \delta M_{ii} = - \sum_i M_{ii} \quad (20)$$

which is also a sufficient condition to uniquely obtain stress glut moments of spatial degree two from the seismic response, at least in principle (Backus 1977a; Doornbos 1981).

---

**Figure 6.** Convergence of moment tensors during iteration, case number 9 of Table 2, arbitrary units. ——: moments of degree zero, ----: moments of degree two, ——: rms error of fit.
A moment tensor satisfying this constraint can be decomposed in two double couples, with scalar moments $M$ and $N$ respectively:

$$M = M(ff^T - gg^T) + N(ff^T - hh^T)$$

(21)

where $f, g, h$ are the unit eigenvectors of $M$, and usually we have $|M| > |N|$. The decomposition (21) is non-unique, and the final results do not depend on it. More commonly, the decomposition has been into a double couple and a compensated linear vector dipole (Knopoff & Randall 1970). However, the form (21) does simplify derivations and some of the expressions in this section.

Equations (20) and (21) provide 11 relations among the variations $\delta M, \delta M, \delta N, \delta f, \delta g, \delta h$. We will need, in particular, the relation between variations in the moment tensor $M$ and in the 'normal' $n$ defined by

$$n = (f \pm g)/\sqrt{2}, \text{and } s = (f \mp g)/\sqrt{2}.$$  

By operating directly on (21), the linearized relation is found to be

$$\delta n = \left( \frac{ss^T}{2M + N} + \frac{(2M + N)hh^T}{2(M + 2N)(M - N)} \right) \delta M s + \frac{sh^T \delta M h}{2(2M + N)} - \frac{3Nh^T \delta Mn}{2(M + 2N)(M - N)}.$$  

(22)

Generally, the tensor describing the source geometry defines an ellipsoid, and it is reasonable to have the minor axis aligned with the normal $n$:

$$W = a^2 pp^T + b^2 qq^T + c^2 nn^T$$

(23)

where we may choose

$$a^2 > b^2 > c^2 > 0.$$  

(24)

Then we can use the tensor invariants

$$\sum_i W_{ii}, \sum_i |W_{ii}|, |W|$$

...
Also, the temporal moment of degree two describes the time history of the source and should be positive:

\[ \Delta(\tau^2) = T^2 > 0 \text{ or } \delta(\Delta\tau^2) = T^2 - \Delta(\tau^2). \]  

(27)

The form of equations (26) and (27) suggests that more than just inequality constraints can be inserted, if the appropriate information concerning the presumed value and the standard deviation, is available or assumed. Such use of a priori information in inverse problems has been advocated by Jackson (1979). Rupture velocity \( v \) and its average over the fault \( \bar{v} \) could similarly be bounded:

\[ \frac{\delta a^2}{\Delta(\tau^2)} = \gamma v^2 \text{ or } \delta a^2 = \gamma v^2 \delta(\Delta\tau^2) \approx \gamma v^2 \Delta(\tau^2) - a^2 \]  

with \( \delta a^2 \) given by equation (25), and the constant \( \gamma \) depends on the source type (e.g. unidirectional versus bidirectional).

\[ \mathbf{w}^T \mathbf{w} = \bar{v}^2 \{ \Delta(\tau^2) \}^2 \text{ or } 2 \mathbf{w}^T \delta \mathbf{w} - 2\bar{v}^2 \Delta(\tau^2) \delta(\Delta\tau^2) \approx \bar{v}^2 \{ \Delta(\tau^2) \}^2 - \mathbf{w}^T \mathbf{w}. \]  

(29)

If \( \bar{v} = 0 \) then, of course, this equation is replaced by

\[ \delta \mathbf{w} = 0 \]

with prescribed standard deviation.

For solutions of \( \mathbf{M}, \mathbf{W}, \mathbf{w} \) and \( \Delta(\tau^2) \) to be mutually consistent, the following may be required:

the ‘normal’ to the fault coincides with the minor axis of source ellipsoid:

\[ \mathbf{w} \mathbf{n} = c^2 \mathbf{n} \text{ or } (\delta \mathbf{w} - \delta c^2 \mathbf{I}) \mathbf{n} + (\mathbf{W} - c^2 \mathbf{I}) \delta \mathbf{n} \approx -(\mathbf{W} - c^2 \mathbf{I}) \mathbf{n} \]  

(30)

with \( \delta \mathbf{n} \) and \( \delta c^2 \) given by equations (22) and (25).

The average rupture vector is normal to \( \mathbf{n} \):

\[ \mathbf{w}^T \mathbf{n} = 0 \text{ or } \mathbf{w}^T \delta \mathbf{n} + \mathbf{n}^T \delta \mathbf{w} \approx -\mathbf{w}^T \mathbf{n}. \]  

(31)

More specifically, if \( \mathbf{w} \neq 0 \) it would be expected that the average rupture vector is aligned with the major axis of the source ellipsoid:

\[ \mathbf{w} \mathbf{w} = a^2 \mathbf{w} \text{ or } (\delta \mathbf{w} - \delta a^2 \mathbf{I}) \mathbf{w} + (\mathbf{W} - a^2 \mathbf{I}) \delta \mathbf{w} \approx -(\mathbf{W} - a^2 \mathbf{I}) \mathbf{w}. \]  

(32)

Equations (20)–(32), together with (19), provide an iterative scheme for finding a solution consistent with physical criteria and a priori information. Not all of the constraint equations may be necessary at any given time, and not all of them are independent. For working purposes, the usual conception of a source region has been that of a plane fault. Then

\[ \mathbf{N} = 0, \mathbf{h}^T \delta \mathbf{M} \mathbf{h} = -\delta \mathbf{N} = 0, c^2 = 0 \]

and equations (22), (25), (30) are simplified in an obvious way. In practice a problem may arise since by linearizing equations we have tacitly assumed that the variations are relatively small. This assumption is usually justified for \( \delta \mathbf{M} \), but not always for some of the other parameters, in particular \( \delta \mathbf{W} \) in equation (26). One way to satisfy this requirement is by introducing constraints on the variations, e.g.

\[ \delta \mathbf{W} = 0 \]
Seismic moment tensors

with prescribed standard error. Alternatively, the problem can be avoided by supplying more information about \( W \) itself. Thus, we may assume an orientation of axes \( p \) and \( q \) in equation (23):

\[
p = s \cos \phi + h \sin \phi, \quad q = s \sin \phi - h \cos \phi, \quad 0 < \phi < \pi
\]

with \( s \) and \( h \) defined in equation (21). The problem of solving for \( W \) subject to non-linear constraints, is then replaced by the problem of solving for \( a^2, b^2, c^2 \), subject to the linear constraints

\[
\delta a^2 = A^2 - a^2, \quad \delta b^2 = B^2 - b^2, \quad \delta c^2 = C^2 - c^2.
\]

In this way, the number of unknowns may be reduced and the inversion simplified but of course, the simplification rests on the assumed value of \( \phi \), and solutions must be compared for \( \phi \) varying between 0 and \( \pi \).

6 Constrained inversion

Inversion of the data used before (Fig. 2) is now repeated, where the (finite) fault is constrained to be plane, and all the other constraints discussed in the previous section are included. The constraints (30), (31) assume the normal \( n \) to the fault plane to be constructed from the moment tensor \( M \) (equation 21), but this involves the well-known ‘fault plane ambiguity’, i.e. the choice between \( n \) and \( s \). We try both possibilities and choose the solution with smallest rms error; this does provide a means to resolve the fault plane ambiguity. The result is summarized as case number 10 in Table 3 and in Fig. 8 (together with results of experiments to be discussed next). Standard deviations of moment tensor components can be computed as a usual byproduct of the least-squares procedure. Since most of the parameters in Table 4 are non-linear functions of the moment-tensor components, standard deviations of these parameters have been approximated by linearizing the functional relationship (cf. Ward 1980). The usefulness of the values thus obtained is somewhat limited not only due to the approximations involved, but also due to the role of ‘soft’ constraints

Table 3. Results of inversion with constraints. \( M \) and \( N \) are scalar moments of major and minor double couple, \( \bar{v} \) is the average rupture velocity, \( a^2, b^2, c^2 \) are eigenvalues of source ellipsoid. Standard deviations in parentheses. Fault constraints are those in equations (26)–(32). For event information, see Table 2. For further details, see text.

<table>
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<tr>
<th>Nr</th>
<th>Data</th>
<th>Fault Constrained</th>
<th>M (10^25 dyne cm)</th>
<th>N (10^25 dyne cm)</th>
<th>( \Delta (t^2) )</th>
<th>( v ) (km/s)</th>
<th>( a^2 ) (km²)</th>
<th>( b^2 ) (km²)</th>
<th>( c^2 ) (km²)</th>
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*In case number 13, rupture velocity is unconstrained.
**In case number 14 and 15, random errors have been introduced into synthetics.
with corresponding a priori standard errors (Jackson 1979). The possibility of formal standard errors to differ from real uncertainties was mentioned in a similar context, by Dziewonski et al. (1981).

Although the solution for case number 10 seems to be 'reasonable' (partly because of the constraints so imposed), it should be noted that the rms error is about the same as for the point source solution, case number 4 of Table 2. Since the latter involves less degrees of freedom, it would be preferred from a statistical point of view. The point source solution does not completely specify a single double couple, hence the corresponding fault is not necessarily plane. This raises a question about the effect of the plane fault assumption in source analysis. Another question concerns the effect of errors (or anomalies) in the data.

To investigate these problems we computed synthetic seismograms for the point source solution (displayed in Fig. 7), and these synthetics formed the basis of a number of inversion experiments. Both unconstrained and constrained inversion gave small non-zero values for second-degree moments (cases 11, 12). In case number 12, it appeared that the small values for spatial extent of the fault were primarily due to the constraint of 'reasonable' rupture velocity. Without that constraint (case 13), the spatial source parameters attain values which are clearly inconsistent with the point source model. In this case, the original component along the minor principal axis of the moment tensor (the B-axis in Fig. 8) is carried, in part, by solutions for the spatial source parameters, which explains why the major axis of inferred fault ellipse is near the B-axis.

In the next experiments we introduced random time (±1 s) and amplitude (±2 dB) anomalies in the synthetics. Unconstrained inversion (case 14) now gives unphysical results.

![Figure 7. Synthetic records with P and SH at SRO and ASRO stations, for a source corresponding to the solution of case number 4 in Table 2. Record length is 2.5 min. Different amplitude scale for different components.](https://academic.oup.com/gji/article-abstract/69/1/235/561999)
Figure 8. Fault plane solutions in equal area projections for the cases in Table 3. •: principal axes of moment tensor of degree zero; X: major axis of moment tensor of degree two, in cases where fault constraints were imposed.

reminiscent of case 9 with real data. Imposing the constraints improves the solutions (case 15); some parameter values are now comparable to those of case 10. It is concluded that unjustified assumptions (in the present examples, a plane fault) and data errors, can significantly effect estimates of source parameters related to second-degree moments. How significant the effect really is depends on wave period and wavelength, and on the actual source rise time and spatial extent, because they determine the relative excitation factors. Relatively large effects are expected for small faults with large curvature (or large deviation from a single double couple).

From equations (21)–(30) it is clear that in the inversion method proposed here, the plane fault assumption need not be made (although it does simplify the procedure). Attempts to reduce data errors or anomalies are rather common. Here we have followed an ad hoc procedure to correct for travel-time anomalies. Although it has not been applied here, we suggest that amplitude anomalies could be treated similarly, and source parameters related to second-degree moments would then be estimated solely from variations in pulse width. This type of observation has been used before for purposes of source analysis (e.g. Chung & Kanamori 1980; Douglas, Hudson & Marshall 1981).

7 Conclusions

In this paper, the seismic response is represented in terms of 20 source parameters which are related to components of the moment tensors; they are also related to the parameters of finite fault models, as demonstrated for Haskell and Savage type of models. The usual representation in terms of six components of the zero-degree moment tensor is adequate for point sources at given location. For point sources at unknown location, a 10 parameter representation (including first-degree moments) is necessary and adequate, and the location may be determined (Dziewonski et al. 1981). For sufficiently extended sources, the 20 parameter representation (including second degree moments) is necessary; it is estimated that for sources with M, > 6, the relative contribution of second-degree moments may be of the order of 10 per cent or more, even in long-period seismograms. The representation is
adequate as long as source rise time and spatial extent are smaller than seismic wave period and wavelength. Contrary to the zero-degree moment tensor representation, the extended source representation is non-linear, but for inversion purposes it can be efficiently linearized. The inversion procedure was applied to SRO data from a deep event in the Bali Sea. Inferred principal axes of the zero-degree moment tensor were consistent with observed polarities and amplitudes, and a decomposition into minor and major double couple showed that their ratio was about 10 per cent.

Parameters related to second-degree moments can be interpreted in terms of source rise time, orientation and spatial extent, and average rupture velocity. Orientation of the source region can also be inferred, in part, from the zero-degree moments. Thus, solutions for the different source parameters should be mutually consistent. Furthermore, values for some of the parameters should be positive or, more specifically, be in a range of 'acceptable' values dictated by our conception of the mechanism of faulting. These considerations lead to (generally non-linear) constraints on the solution, which in this paper are given in a linearized form, to be included in the inversion procedure. The constraints are precisely those appearing as a priori assumptions in the conventional methods of source analysis; it is thus possible to investigate the impact of these assumptions. From experiments with inversion of synthetic data from a general deviatoric point source (e.g. a sum of major and minor double couple), it is concluded that unjustified assumption of a plane fault may lead to overestimate the fault surface area. The same is true if the data are corrupted by errors (or anomalies). The significance of the effect depends on the relative excitation factors of the source parameters. It should be realized that this effect is not a consequence of the moment tensor representation; it is to be expected in any method of source analysis. In the moment tensor representation however, the plane fault assumption can be avoided.

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Seismic moment tensors


