A method for calculating synthetic seismograms in laterally varying media

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Summary. An effective algorithm for computing synthetic seismograms in laterally inhomogeneous media has been developed. The method, based on zero-order asymptotic ray theory, is primarily intended for use in refraction and reflection studies and provides an economical means of seismic modelling.

A given smoothed velocity-depth-distance model is divided into small squares with constant seismic parameters and first-order interfaces are represented by an arbitrary number of dipping linear segments. The computation of ray propagation and amplitudes through such a model does not involve complicated analytic expressions and therefore minimizes computer time.

Amplitudes are determined by geometrical spreading of spherical wavefronts and energy partitioning at interfaces. Synthetic seismograms calculated for laterally homogeneous models are in good agreement with those obtained by the Reflectivity Method.

Introduction

In the past decade it has become standard practice to interpret explosion seismic record sections in terms of amplitudes as well as travel times. Amongst a variety of techniques to compute synthetic seismograms (Hron & Kanasewich 1971; Helmberger 1968; Chapman 1978), one of the most popular is the Reflectivity Method (Fuchs & Müller 1971), which is capable of generating the complete wavefield within a medium composed of a large number of homogeneous layers. Unfortunately Reflectivity is cumbersome because it consumes a large amount of computer time depending on the proportions of the model. Methods like Disk Ray Theory (Wiggins & Madrid 1974; Wiggins 1976) have alleviated this problem and reduced computer costs associated with trial and error seismic modelling. The Earth, however, is not composed of a stack of plane layers but of more complicated media with both seismic parameters and interfaces varying not only vertically but also horizontally, and at present there is need for methods able to cope with such models. Hong & Helmberger (1978) have presented a method with which to calculate wave propagation in non-planar structures. Other techniques, such as finite difference and finite element methods (Kelly et
al. 1976; Smith 1975) are slow and expensive to use. Theoretical seismograms from complicated media can be calculated using zero-order asymptotic ray theory (ART), which is a high-frequency approximation and corresponds to the geometrical optics solution. Although ART is only approximate and not as exact as full wave methods, it can be used effectively if inhomogeneities are smooth and medium properties change little within a wavelength. Significant advances in this field have been achieved by Červený, Langer & Šenčík (1974) and Červený, Molotkov & Šenčík (1977).

To synthesize the propagation of elastic waves through laterally inhomogeneous media, the wavefield must be decomposed into families of elementary waves, the rays of which must be generated at given take-off angles from the source. If a ray with a predetermined number of peg-leg multiples and P-SV conversions does not arrive at a given receiver location on the Earth's surface, additional rays must be computed until the prescribed one is found. This involves the calculation of a large number of rays. In the method described by Červený et al. (1974), ray paths, which are perpendicular to wavefronts, are computed by solving a set of first-order differential equations. Since the numerical solution of these is excessively time consuming, especially where the routine interpretation of record sections is concerned, a more rapid method with which to generate large numbers of trial models is desirable. The computer program of the method presented here is capable of calculating ray paths and amplitudes in realistic earth models, and since only simple analytical expressions must be evaluated it is relatively fast and therefore convenient for application in seismic prospecting. Once all the prescribed ray types arriving at given receiver points along the surface have been found using a search procedure, the resulting set of displacement amplitudes and travel times are combined so as to form a set of impulse seismograms. These, in turn, need only be convolved with a suitable source wavelet and with the response of the receiving equipment to obtain the synthetic seismogram. As a check on the working of the program synthetic seismograms from laterally homogeneous models are compared with those obtained by the Reflectivity Method (Fuchs & Müller 1971).

Parameterization of the model

Before a specific ray tracing system is adopted, the main objectives must be brought to mind: the theoretical model must approximate any realistic medium sufficiently; it should be possible to define it by a minimal amount of input parameters; it must allow for a rapid and accurate computation of any type of elementary ray path and the corresponding amplitude.

These points are essential in that many trial models must be calculated and repeatedly modified. It is convenient to confine oneself to two-dimensional structures. Since ray paths in homogeneous media are linear and thus simple to compute, it is advantageous to divide a given model into a system of small squares, referred to here as boxes, each containing constant seismic parameters. In a two-dimensional structure these represent the square ends of infinite prisms (Fig. 1). Any ray crossing a box boundary changes its direction according to Snell's Law. Since such a box structure is meant to simulate an isotropic medium with velocity gradients in any direction, box boundaries transmit the total energy and therefore do not cause reflections of rays impinging below the critical angle. A ray reaching a box boundary above the critical angle corresponds to a diving ray with a turning point at the box boundary depth. The approximation of such turning points depends entirely on the side length of a box. In order to refine turning point locations it is possible to calculate ray segments as arcs of circles by using velocity gradients determined by velocity values of neighbouring boxes. As will be shown later on, side lengths of 1 km are a good approximation for lithospheric models. An effective computational method is to designate P-velocity values α
The velocity model is set up by dividing the medium into squares (boxes) containing constant seismic parameters. It is further divided into geological layers by laterally varying interfaces, represented by heavy lines, which cut across the boxes. Within a layer the velocity distribution is smoothed vertically and horizontally, distributing the variation in discrete steps from box to box. The interfaces between the layers are approximated by linked linear segments. The thin lines are rays propagating from the source S in the upper left corner. Inset: each square can be subdivided into arbitrary numbers of thin horizontal slices. The velocities of the slices are interpolated by using box velocity values from above and below.

Figure 1. The velocity model is set up by dividing the medium into squares (boxes) containing constant seismic parameters. It is further divided into geological layers by laterally varying interfaces, represented by heavy lines, which cut across the boxes. Within a layer the velocity distribution is smoothed vertically and horizontally, distributing the variation in discrete steps from box to box. The interfaces between the layers are approximated by linked linear segments. The thin lines are rays propagating from the source S in the upper left corner. Inset: each square can be subdivided into arbitrary numbers of thin horizontal slices. The velocities of the slices are interpolated by using box velocity values from above and below.

to each box within the limits of the model. S-wave velocities $\beta$ can then be established by using $\beta = \alpha / \sqrt{3}$ or any other predetermined relation. The density $\rho(\alpha)$ can be evaluated by

$$\rho(\alpha) = 0.252 + 0.3788 \alpha$$

in the absence of more reliable information on the distribution of these parameters. This roughly approximates the Nafe & Drake relationship between compressional P-wave velocity and density (Talwani, Sutton & Worzel 1959). In order to reduce the number of input parameters necessary to construct a model it is sufficient to specify a minimal number of velocity-depth-distance points along vertical mesh lines. These points can then be connected in vertical and horizontal directions by cubic splines to create intermediate values for interlying boxes. Thus, for a given velocity gradient within a medium, velocity contrasts at box boundaries decrease with decreasing side lengths. Once all the P-velocity values have been stored on a disc file, they need only be recalled when their respective boxes are crossed by a ray. The degree of change in the ray angle upon crossing a box boundary is minimized by a smooth velocity $(v(x, z))$ distribution.

The method described above is suitable for models without geological boundaries. To avoid the complexities of the interaction of spherical wavefronts with curvilinear boundaries, interfaces consisting of arbitrary numbers of linked linear segments with individual dips are introduced. These can also include boundaries which reach the Earth's surface. The velocity values are then smoothed within regions of continuous seismic parameters, i.e. areas between interfaces. In this case special attention must be given to boxes intersected by interfaces. Such boxes contain two velocity values, one on either side of the interface, the bottom of which is taken from the box directly below. In order to make these boxes conspicuous for later identification a notation scheme for interfaces and related boxes must be set up. When a ray impinges on such a box, special steps are taken to calculate the intersection point and the energy partitioning and propagation of reflected or transmitted components. In the system presented here, the velocity values for the complete model are set up before the ray tracing procedure comes into effect.
Ray tracing procedure

As a first step towards calculating synthetic seismograms to compare with observations, one must trace rays through the model from source to successive receiver locations to produce the travel-time curves. This involves generating a large number of trial models and so the ray tracing system should not consist of lengthy algorithms. As mentioned in the previous section, models are composed of many homogeneous boxes within which ray paths are linear. The seismic source can be located on any box boundary in the model or at the surface. In the former case, rays leaving the source upwards as well as downwards are included. From a given point source a ray path is defined by its box velocity and take-off angle by which the emergence point from the box is given implicitly. Box corners are not normally encountered due to numerical inaccuracy. Successive take-off angles are given by Snell's Law

\[ \phi_{i+1} = \arcsin \left( \sin \phi_i \frac{u_{i+1}}{u_i} \right) \]

and the travel time is obtained by the distance between the points \((x_0, z_0), (x_1, z_1)\) and the box velocity:

\[ t_0 = \sqrt{\left(x_1 - x_0\right)^2 + \left(z_1 - z_0\right)^2} / u_0. \]

Large models, in relation to a given box side length, require a large array of velocity values. In extreme cases the size of the array could be limited for storage reasons. As a result a combination of zones of strong velocity gradients and relatively large boxes would decrease the accuracy with which curved trajectories are to be approximated by linear ray path segments. Here individual boxes can be divided into an arbitrary number of horizontally stratified homogeneous segments having an overall velocity gradient determined by the velocities above and below (Fig. 1). A ray may only penetrate a neighbouring box if it reaches the box boundary at an angle below the critical angle, taken to the normal to the boundary. When a ray reaches a box boundary at an angle greater than the critical angle it will change its direction by reflection which implies that it has passed through a turning point. Since there can be no turning point in a box with a constant velocity it is necessary to modify the calculation of ray segments which begin to propagate under large angles taken to the vector of the local velocity gradient. A ray path within a constant velocity gradient is a circular arc with a radius of curvature given by

\[ R = \frac{1}{\sqrt{\text{grad} u}} \sin \epsilon \]

(Gebrande 1976) where \( \epsilon \) is the angle between the tangent to the circle and the velocity gradient \( \text{grad} u \) at the point where the calculation of the arc begins. The velocity gradient is specified by the velocity values of the surrounding boxes. This calculation can be repeated each time a ray impinges upon a box until it has passed the turning point and begins to propagate under a predetermined 'safe' angle.

Geological boundaries, which consist of linked linear segments of varying dip represent interfaces with significantly larger velocity contrasts compared to those at box boundaries. Should a ray enter a box crossed by an interface which lies in the path of the ray within that box, simple geometry determines the intersection point. Here the coordinate system is rotated according to the dip of the interface and rays are either reflected, refracted or terminated depending on the current ray code. The numerical ray code is given by an array of switches which are called up successively whenever an interface is encountered. These switches determine which action is to be taken at that point and the type of elementary ray to leave the interface. If the switch is set for transmission where a ray impinges on the interface at an angle greater than the critical angle, calculation of the ray path will terminate...
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Figure 2. Comparison of ray paths and travel-time curves for primary reflections from a model by Christie & Sclater (1980). The box method produces a shadow zone for first arrivals between 100 and 120 km, caused by an interface corner. For all calculations shown in this paper, a box side length of 1 km was used. Left: the method described in Červený et al. (1974). P-velocities are given.
since an elementary ray of that type and take-off angle does not exist. In this way it is possible to generate multiples of any ray type which can include surface reflections and head waves. It would be simple to extend the algorithm to take into account P to S conversions.

Head waves, which correspond to higher-order terms of the ray series are simulated by shooting rays off interfaces at the critical angle. The critical angle and point (inner point of total P reflection) is found by a trial and error search routine wherein rays are emitted from the source, the initial angles of which are automatically corrected until the critical point on a predeterminet interface is found. Head waves are then computed for that particular interface segment. Although pure head waves rarely exist (Červený et al. 1977), this approximation adequately describes the effect caused by diving rays which have their turning points directly beneath interfaces. McMechan & Mooney (1980) calculate such rays by introducing small gradient zones beneath layer boundaries. The interaction of rays with interface corners is ignored. Should a ray impinge near a corner, local continuity of an interface is assumed. As can be seen in Fig. 2 ray paths and travel-time curves compared with those calculated by the method by Červený et al. (1974) show good agreement and differ only where interface corners produce shadow zones. Since the method here is directed at the problem of providing an approximate solution in order to produce synthetic seismograms, these effects are not disturbing. The model used to compute Fig. 2 has been taken from Christie & Sclater (1980) and for this particular job the box method required half as much computer time as the method by Červený et al. (1974).

Amplitudes

The computation of amplitudes plays a major role and adheres closely to the theory described in Červený & Ravindra (1971). The amplitude calculation, which corresponds to the zero-order approximation of the ray series, consists of two steps: the evaluation of the geometrical or ray path spreading factor and the calculation of the energy partitioning at interfaces, both of which must be accumulated along ray paths (Fig. 3).

![Figure 3](https://academic.oup.com/gji/article-abstract/69/2/339/764058/fig3)

Figure 3. Symbols used in the amplitude calculation along a ray with a take-off angle φ, between points N₀ and Nₖ₊₁. The N_j represent successive ray-boundary and ray-interface intersection points with j = 1, 2, ..., k +1. Angles δ are taken to the normals to boundaries. The ray path spreading factors and interface reflection and transmission coefficients are accumulated along the ray path. In the program boxes are distinguished by integers (IX, IZ), the shaded box is (2, 4). The lower portions of boxes intersected by interfaces are assigned the velocity values of boxes directly below.
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The ray path spreading effect can be calculated by tracing a ray tube consisting of several neighbouring rays (Marks & Hron 1979). The ratio of cross-sections \( \frac{d\sigma_j}{d\sigma_{j+1}} \) of the ray tube between points \( j \) and \( j+1 \) is inversely proportional to the square of the amplitude ratio, i.e.

\[
\frac{d\sigma_j}{d\sigma_{j+1}} \sim \left( \frac{\text{amp}_{j+1}}{\text{amp}_j} \right)^2.
\]

Since this would imply that several rays must be traced per arrival, a technique is used here whereby displacements may be calculated along single ray paths. This can be done by expressing the ratio of cross-sections in terms of the principal radii of curvature of the wavefront along the boundary. A two-dimensional model is assumed which requires that the angle between the vertical plane parallel to the ray path, and the normal to the interface is zero. Three additional assumptions must be made: the wave generated by the source propagates as a spherical wave in the first box; the principal radii of curvature of interfaces are continuous in the neighbourhood of points \( N_j (j = 1, 2, \ldots, k) \) (Fig. 3) which are points of reflection or refraction on any boundary; these radii are large in comparison with the wavelength.

Ray theory can be used except in the vicinity of singularities (caustics, critical points) where certain corrections must be made. Since all boundaries are plane, their radii of curvature are infinite and need not be taken into account and Červený & Ravindra’s (1971) expressions for the spreading function \( L \) between points \( N_0 \) and \( N_{k+1} \) become:

\[
L(N_0, N_{k+1}) = \left[ \left( \sum_{j=1}^{k+1} \frac{l_j u_j}{v_1} \right) \left( \sum_{j=1}^{k+1} \frac{l_j u_j}{v_1} \prod_{\lambda=1}^{j-1} \cos^2 \delta(N_\lambda) \prod_{\lambda=1}^{j} \cos' \delta'(N_\lambda) \right) \right]^{1/2}
\]  

(1)

defining

\[
\prod_{\lambda=1}^{j-1} = 1 \quad \text{for } j = 1.
\]

\( j \) is the ray segment and intersection index along the ray; 
\( l_j \) is the distance between successive points \( N_j \) and \( N_{j+1} \); 
\( u_j \) is the velocity along each ray segment \( j \); 
\( N_j \) is the point of incidence of the ray upon the \( j \)th interface it encounters; 
\( k + 1 \) is the total number of ray segments in the ray; 
\( \delta, \delta' \) are the angles of incidence and reflection or refraction which the ray makes with the boundary. For horizontal box boundaries angles are taken to the normal to the boundary. The primed quantities are to be evaluated on the side of the interface from which energy travels away.

The displacement \( U(N_{k+1}) \) can be calculated:

\[
U(N_{k+1}) = \frac{1}{L} \prod_{j=1}^{k} C_j
\]

and \( j \) is as above where \( C_j \), the complex displacement amplitude coefficient along the ray path can represent either the reflection or the transmission coefficient, depending on what action is to be taken when a ray encounters an interface. At box boundaries the transmission coefficient \( C_T \) is 1. \( C_f \) at an interface is calculated by applying Zoeppritz’s amplitude equations, which take into account \( P - SV \) interaction. Numerous authors have developed various forms of these amplitude equations and there has been some confusion as to the validity of some of the versions published. Young & Braile (1976) have found the expressions given in Červený & Ravindra (1971, p. 63) to be correct and have developed a FORTRAN computer.
program to calculate them which has here been adapted to suit the requirements for the ray
tracing procedure. The frequency dependence of reflection coefficients in an inhomogeneous
medium can be ignored since asymptotic ray theory is an infinite frequency approximation.
The complex value C-coefficients in terms of displacements, which are valid for plane
harmonic elastic waves (P and SV) at an interface between two isotropic, homogeneous
elastic solids, are (for P-waves):

\[ C_{11} = -1 + 2P_1D^{-1}(\alpha_2\beta_2 P_2 X^2 + \beta_1 \alpha_2 \rho_1 \rho_2. P_4 + q^2 \Theta^2 P_2 P_3 P_4) \]
\[ C_{13} = 2\alpha_1 \rho_1 P_1 D^{-1}(\beta_2 P_2 X + \beta_1 P_4 Y) \]

with

\[ D = \alpha_1 \alpha_2 \beta_2 \Theta^2 Z^2 + \alpha_2 \beta_2 P_2 P_2 X^2 + \alpha_1 \beta_1 P_3 P_4 Y^2 + \rho_1 \rho_2 (\beta_1 \alpha_2 P_1 P_4 + \alpha_1 \beta_2 P_2 P_3) \]
\[ + q^2 \Theta^2 P_1 P_2 P_3 P_4 \]
\[ q = 2(\rho_2 \beta_2^2 - \rho_1 \beta_1^2), X = \rho_2 - q \Theta^2, Y = \rho_1 + q \Theta^2, Z = \rho_2 - \rho_1 - q \Theta^2, \Theta = \sin \phi / V_i. \]
\[ V_1 = \alpha_1, V_2 = \beta_1, V_3 = \alpha_2, V_4 = \beta_2, P_i = (1 - V_i^2 \Theta^2)^{1/2} \quad \text{for } i = 1, 2, 3, 4. \]

Indices 1 and 2 correspond to \( P \) and \( SV \) in the first medium and indices 3 and 4 correspond
to \( P \) and \( SV \) in the second medium (Fig. 4).

All velocities \( \alpha, \beta \) and densities \( \rho \) are to be taken at the point \( N_i \). It follows that the angles
of emergence can become imaginary and that the resulting displacement amplitudes must be
complex valued.

For
\[ \Theta > 1/V_i, \]
\( P_i \) becomes imaginary:
\[ P_i = -j(V_i^2 \Theta^2 - 1)^{1/2} \]
where \( \phi_i \) in \( \Theta \) is the angle to the normal to the interface and \( j \) is the imaginary unit.

In the case of a liquid–solid interface it is only necessary to insert the correct seismic
parameters (and \( \beta = 0 \)).

\[ \text{Figure 4. Explanation of indices used in equations (2) and (3) for the amplitude calculation. (a) Incident} \]
\[ P \text{-wave and its reflected P-wave component } (V_1 = \alpha_1). \text{ (b) Incident} P \text{-wave and its transmitted P-wave com-} \]
\[ \text{ponent } (V_3 = \alpha_2). \text{ (c) Head wave propagating along a boundary with the } P \text{-velocity of the lower layer} \]
\[ (V_3 = \alpha_2) \text{ and transmitting energy into the upper layer.} \]
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With the first medium a vacuum, the free surface reflection coefficient for a P-wave is
\[ C_{FS} = \left[-1 + 2\beta_2^2\Theta^2 + 4\Theta^2P_3P_4\gamma_2\beta_2^2\right]/D, \gamma_i = \beta_i/\alpha_i \]
and
\[ D = (1 - 2\beta_2^2\Theta^2)^2 + 4P_3P_4\Theta^2\gamma_2\beta_2^2. \]

Other symbols are as in (2) and \( D \) is the Rayleigh function.

If the receiver does not lie inside the medium, but is located on the Earth’s surface, the surface conversion coefficients \( q_x \) and \( q_z \) must be evaluated:
\[ q_x = 4P_3P_4\Theta\beta_2/D, \]
\[ q_z = 2P_3(1 - 2\beta_2^2\Theta^2)/D. \]

In this way not only the incident wave is considered, but also the wave reflected from the free surface, which interferes with the incident wave.

The set of formulae given in (2) can be extended to include coefficients for head waves, which exist in higher-order terms of the ray series. For practical purposes, only head waves at solid-solid interfaces with the incident wave on the side of the lower \( P \)-velocity are considered.

According to Červený & Ravindra (1971) this head wave coefficient is
\[ \Gamma_{131} = C_{13}^*M_{31} \]
where
\[ M_{31} = \frac{1}{2}(C_{31}/P_3)\Theta = 1/\nu, \]
and \( D \) and \( q \), present in \( C^* \), are given in (2). Indices are explained in Fig. 4. \( C^* \) is the \( C \)-coefficient at the critical point on the interface and must be multiplied by \( l_h^{3/2} \) where \( l_h \) is the distance from the critical point to where the ray is emitted from the interface. From this point onwards the remaining portion of the ray to receiver can be calculated in the normal way. Since the ray method is only approximate, it does not give exact results near the critical point although the accuracy increases for higher frequencies (Červený & Zahradník 1972; Červený 1979).

As mentioned earlier, special care must be taken near singular regions, such as the critical region and the neighbourhood of caustics. Near caustics, or focusing points, the ray tube cross-section vanishes and the large amplitudes associated with this phenomenon must be corrected artificially. This is done by testing amplitude ratios at successive box boundaries and assigning finite displacement values to those rays which show a large and discontinuous amplitude increase.

Synthetic seismograms

Once a method with which to calculate ray paths and displacements in complex media has been developed, one can approach the problem of constructing synthetic seismograms. Initially these consist of a set of time-dependent displacement impulses at individual receiver locations. The selection of the set of elementary rays connecting source and receiver depends on the time window under consideration and the ray types believed to be significant. The search for these rays is done by a straightforward trial and error method.

Individual ray types with incrementing initial angles must be calculated until a receiver point at the surface is surrounded. A new initial angle can then be determined by linear interpolation using \[ A = \Delta_R - \Delta_1, B = \Delta_2 - \Delta_1 \text{ and } \phi_{\text{new}} = \phi_1 + (\phi_2 - \phi_1)A/B \] (Fig. 5).
Figure 5. Search method for rays to given receiver locations. Rays are generated from the source S under take-off angles $\phi_1$ and $\phi_2$ until the receiver R is straddled. Then the distances A and B and angles $\phi_1$ and $\phi_2$ are used for a linear interpolation to determine a new take-off angle. This process is repeated until a ray emerges within a given distance of R.

Should the resulting ray not arrive within a given precision of distance (e.g. 0.2 km) the step will be repeated. This method has been described in Cassell (1978) where a linear interpolation is shown to converge more rapidly than a simple bisection of angles. The search procedure for an array of receiver points is facilitated by storing initial angle values for previous receivers and using them as starting angles for subsequent receivers. In the examples shown in Figs 6–8, the average number of steps for individual arrivals has been between 3 and 9. A similar technique applies to rays associated with head waves. The basic difference is that once the critical angle has been established, rather than modifying initial angles, one must modify positions where rays leave interfaces.

The sets of complex valued displacements or impulse responses at a given receiver location can now be superimposed to form an impulse seismogram. The amplitudes of the impulses are given in terms of the desired component (vertical or radial) and the surface conversion coefficient can be included. As the wave progresses along the ray, the form of the source time function $f(t)$ remains the same only if the phase $\psi$ does not change. Phase distortion occurs due to complex transmission or reflection coefficients and is frequency-independent.

For sufficiently high frequencies, the phase in each frequency component can be shifted by $n\pi/2$ where $n$ is the number of internal caustics touched by the ray (Choy & Richards 1975). In order to synthesize these effects, the signal can be expressed by a linear combination of a unit impulse with its Hilbert transform (Červený & Ravindra 1971; Červený & Zahradník 1975),

$$F(t, \psi) = f(t) \cos \psi + g(t) \sin \psi$$

where $g(t)$ is the Hilbert transform of $f(t)$. Convolution of $F(t, \psi)$ with an apparent source function supplies the final seismogram at the receiver input which must be subjected to the response of the receiving equipment. The choice of a realistic source waveform is a basic problem in explosion seismology. It cannot be obtained from observed records since these contain the combined effects of the Earth, receiver and surface reflections. In the case of explosions at sea the sea-surface reflected waveform must be added to the source function for an underwater explosion (Kennett 1977; Fowler 1976a, b). The prediction of realistic waveforms has been attempted by O’Brien (1960) and analytical expressions are given in Müller (1970) and Červený et al. (1977). An important factor which may influence the form of the seismic wave as it travels through the Earth is absorption, especially where wave propagation through sedimentary layers is to be synthesized. A description of the application to a reflection modelling system is given in Smith (1977).
Comparison with the reflectivity method

The accuracy of the synthetic seismograms obtained by the box method is verified by a comparison with synthetics from the reflectivity method for a laterally homogeneous medium. Since the reflectivity method produces the full wave response from a layered model, apart from testing the consistency of the overall amplitude distribution, it provides a good way of checking whether all relevant ray types have been included in the ray method.

For an initial comparison a five-layered model was chosen with boundaries represented by first-order discontinuities (Fig. 6). The net response for the ray method contains primary reflections from each of the four interfaces, multiples generated within the low-velocity layer by both downwards and upwards travelling rays, and head waves off all interfaces with positive velocity gradients. Both computations preclude surface reflections. The travel times for reflections off the top of the low-velocity layer and head waves from the shallowest interface coincide within the distance range of 60–100 km (t-t branch 2-2, Fig. 6). The resulting interference effect causes the amplitudes of the first arrivals to decay rapidly with distance. Close scrutiny of waveforms and amplitude distribution in Fig. 6 reveals good agreement between the two methods. Considering that the box method required 1/5 of the CPU time necessary for the reflectivity method on an IBM 370 it promises to be more than adequate as a tool for fast structural modelling. Increasing the number of layers or introducing velocity gradients and laterally varying interfaces does not increase the computing time significantly. This is demonstrated in the calculation for Fig. 7 where the HILDEERS model (Fuchs & Landisman 1966) is used as a basis for comparison. The model, which has been discussed by Červený et al. (1977), McMechan & Mooney (1980) and Fuchs & Müller (1971) has six layers, the fourth of which has a strong velocity gradient of 6.15–6.93 km s\(^{-1}\) over 16 km. In the reflectivity method such a layer must be represented by a stack of thin segments with increasing velocities (here 10 such segments). Since the CPU time is proportional to the number of boundaries, processing time for the reflectivity method exceeded that of the box method by a factor of eight. The HILDEERS model, which contains similar ray types to those of the previous example, illustrates the interference of the phase-shifted, super-critically reflected rays off the fourth interface with the diving rays within layer 4 (Fig. 7). These prominent arrivals cease in the distance range around 160 km where the velocity gradient, combined with the depth of the fourth interface, produces a cusp beyond which no rays reach the surface. The two last rays at this point are the first reflection off this interface and the last diving ray in the fourth layer. For convenience the lowest interface has been taken to be a first-order discontinuity, although the true HILDEERS model has a small gradient zone at this depth. Arrivals from this interface are very weak in both methods. Multiples were generated within the low-velocity zone as in the previous model. Fig. 7 shows the box method to produce slightly stronger amplitudes with distance for reflections off the top of the low-velocity layer. These differences, however, are consistent with the comparison of reflectivity and ray theory in Červený et al. (1977) and are to be expected when comparing a frequency-dependent with a frequency-independent calculation, especially where thin layers are concerned.

Using the Christie & Sclater (1980) North Sea model a synthetic record section was calculated using a marine pressure response as a source signal (Fig. 8). The response for an underwater explosion with two bubble pulses was calculated as described in Fowler (1976a, b) and convolved with the model's response. Although it was primarily intended to use this realistic model to test the program a comparison with the observations is of interest. Details of the model and its construction are discussed in Christie & Sclater (1980). Basically it consists of six layers with P-velocities increasing with depth. The bottom layer has a strong velocity gradient of 0.05 s\(^{-1}\). The ray types included in the calculation are primary reflections...
Figure 6. Comparison of synthetic seismograms calculated by the Reflectivity Method (Fuchs & Müller 1971) and the box method. The original pulse for both calculations has two extrema and a dominant frequency of 4.3 Hz. The arrivals of primary reflections are numbered for layers from top to bottom and low-velocity layer multiples and head waves are included. The arrows indicate critical distances.
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Figure 7. Comparison of synthetic seismograms by the reflectivity and box methods for the HILDERS model. The most prominent arrivals are from the reflections off the fourth interface and the diving rays within layer 4 which terminate around 160 km where the strong velocity gradient produces a cusp (t-t branch 3–4). Arrows indicate the critical distances and the star denotes the outer cusp. The reduction velocity is 8 km s$^{-1}$. 
off the bottom four interfaces, head waves from the two lowest ones and diving rays in the bottom layer. Also included are reflections off the bottom discontinuity (Moho) with a surface multiple in the top layer. These arrivals appear within the first 100 km between 8 and 9 s and interfere with the primary reflection from the second interface. By calculating both head waves and diving rays from the Moho it is possible to demonstrate their difference in amplitudes. This is apparent in the last two seismograms where there are no diving ray arrivals due to the limited depth of the model. The validity of calculating head waves across interface corners could, of course, be argued. But as long as successive interface segments turn downwards these head waves approximate waves which propagate directly beneath interfaces.

The overall response does not take ringing into account which is caused by multiple reflections in a shallow water layer such as the North Sea, although such an effect can be included. The lack of arrivals around 110 km is caused by a shadow zone produced by interface corners. Layer velocities are as indicated on the right side of Fig. 8.

Figure 8. Synthetic section for the Christie & Sclater (1980) North Sea model. The source signal has a dominant frequency of 3.2 Hz and represents the pressure response and bubble pulse interference for an underwater explosion. The shadow zone at 110 km is caused by interface corners. The velocities are as indicated on the right. Arrivals from the diving rays cease after 180 km due to the limited depth of the model. Included are reflections from the bottom interface with surface multiples in the top layer. The reduction velocity is 8 km s$^{-1}$. 
Unfortunately the observed data are of poor quality and it is very difficult, if not impossible, to identify later arrivals. It should also be pointed out that the model has primarily been constructed by means of a time-term analysis, hence the structural detail is based on 'circumstantial evidence' and should only be taken as a starting model. Nevertheless, with a strong velocity gradient beneath the Moho to produce strong first arrivals at large distances the overall amplitude distribution is not inconsistent with the data.

Conclusions

The box method presented here proves to be sufficiently accurate in comparison with the reflectivity method and is consistent with results expected using zero order asymptotic ray theory. The problem of specifying the types of elementary rays to be computed is an artifact of any ray method and can be overcome with increasing experience of the user.

The main advantage, however, is the speed with which synthetic seismograms can be produced, which in turn allows a large turnover of trial models at low cost. This provides a means of generating models which fit the observations reasonably well before resorting to more expensive methods, should such be available. In order to keep the modelling process as straightforward as possible, the propagation of $S$-waves and therefore of $P$-$S$ converted ray paths has not been programmed and interfaces are purposely limited to being represented by linked linear segments. The algorithm could, however, be modified to suit specific requirements. Additionally, shallow structures can be treated effectively by reducing the side lengths of boxes. If necessary, the increase in disk storage associated with the creation of more boxes could be offset by decreasing the dimensions of the model. By moving the source along the surface, the method can be used for near-vertical reflection modelling in laterally varying media. Although it would require extensive programming, the box structure could be extended to three dimensions. It is planned to use the method presented here to investigate models of some deep-water marine refraction experiments in which the possibility of lateral variations has so far, perforce, been neglected.

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References


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