Dynamic modelling of in-hole mounts for seismic detectors

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Summary. A Dynamic Finite Element Model (DFEM) of the interaction of a stress wave with a cylindrical hole is shown to yield good agreement with analytical solutions based on both Integral Transform and Fourier-Bessel techniques. Experimental results were obtained for the interaction of an explosion-generated stress wave with accelerometers mounted in a cylindrical hole drilled into massive rock. Both theory and experiment indicate a similar distortion in the rise time of the onset of the acceleration pulse travelling past the cylindrical hole. The dynamic model is also used to obtain amplitude transfer functions for a range of inclusion-type mounts that may be used to locate a seismic detector within a cylindrical hole. The modelling results indicate that if only partial contact is made between an in-hole mount and the walls of the cylindrical hole, then significant distortions arise in the amplitude response of the mounted detector to an incident stress wave. For wavelengths greater than 10 times the cylindrical hole diameter the effect of any in-hole mount is shown to be negligible, thus the results have little relevance for the long wavelengths commonly encountered in earthquake seismology. The results are, however, relevant to the study of stress wave propagation at ultrasonic frequencies in rock masses.

Introduction

A sensitive measure of rock conditions is obtained from a knowledge of the attenuation of seismic waves within the volume of rock (Johnston & Toksöz 1980). It is for this reason that ultrasonic frequency acoustic techniques are increasingly being employed to classify the quality of a rock mass. For example, Paulsson & King (1980) performed a seismic experiment at ultrasonic frequencies (up to 60 kHz) in order to assess the attenuation in a volume of rock between a source mounted in a cylindrical hole and a detector mounted in a neighbouring cylindrical hole. In underground mining situations this cross-hole, ultrasonic frequency technique can also be applied to monitor any changes that may occur in the rock mass conditions due to the mining process. In all such cases, the response of the seismic detector mounted in a cylindrical hole is used to infer particle motions in the surrounding material and so an assessment must be made of any perturbations caused by the local environment of the detector.
White (1965) and Pao & Mow (1973) give a comprehensive review of the analytical theory describing the interaction of a stress wave with both empty and fluid-filled cylindrical boreholes in an infinite medium. The interaction of a stress wave with an elastic cylindrical inclusion embedded in an infinite medium is also given by Pao & Mow (1973). However, it is not possible to obtain an analytical solution to stress wave interaction with a seismic detector mount of irregular shape within a cylindrical hole. In this case numerical dynamic methods (such as Finite Difference or Finite Element schemes) must be employed and approximate solutions sought. The applicability of the Finite Element method, in particular, has been demonstrated by Shipley, Leistner & Jones (1967), who made a comparison between a Dynamic Finite Element method (DFEM) and exact solutions for selected cases of simple wave propagation problems. There have been many recent applications of DFEM to seismic wave interaction with given structures (Lysmer & Drake 1972; Smith 1975). Quite often such applications involve the solution of wave propagation in an infinite medium. In order to simulate this realistic situation using a finite model it is necessary to absorb the stress wave energy arriving at the boundaries of the model. The absorption of wave energy at the boundaries of the finite model may be achieved by use of either superposition boundaries (Smith 1974; Cundall et al. 1978), or viscous damping (Lysmer & Kuhlemeyer 1969; White, Vallappan & Lee 1977). An extensive review, together with a further development of such energy absorbing boundaries, is given by Kunar & Rodriguez-Ovejero (1980).

Shipley et al. (1967) have observed that all discrete, dynamic models behave like low-pass filters in so far as such models exhibit dispersion and have cut-off frequencies dependent upon the wave type and element size in the finite element mesh. A digital filter approach to the low-pass filtering action of a finite element mesh is given by Holmes & Belytschko (1976). Furthermore, the non-uniformity of the element size within a given model introduces spurious effects not present in the continuum from which the discrete model is derived. An estimation of the magnitude of these spurious effects in DFEM as well as mesh design criteria for minimizing such effects is given by Bazant (1978).

This study involves the use of a DFEM to analyse the interaction of a stress wave with a seismic detector mounted in a cylindrical hole. In the modelling process emphasis is placed on the problems of both energy absorbing boundaries and spurious effects due to the discretization of the model. Various types of inclusion mounts are modelled in order to assess the perturbation, due to the mount, of an incident P-wave. Two-dimensional plane strain models only are considered for the interaction of a plane compressional wave with an infinitely long cylindrical discontinuity in an infinite medium.

The general model

The interaction of a plane compressional wave with a cylindrical hole may be modelled using the half-structure shown in Fig. 1, for which symmetry considerations require that the vertical displacements along AD be held fixed at zero. In order to generate a plane compressional (P) wave travelling horizontally as indicated in Fig. 1, a uniform pressure load was applied over the face DE. Since there can be no particle displacement normal to the direction of P-wave propagation, the vertical displacements along the boundary EF were held fixed at zero. The boundary conditions discussed for the faces AD and EF of the model were imposed in all subsequent analyses; various constraints on the remaining boundaries are discussed in the relevant sections. The relative dimensions of the model were chosen such that all reflections off the model boundaries reached the central hole at approximately the same time, hence DE = EF/2. Furthermore the length, AD, of the model was fixed at 10 times the diameter, BC, of the central cylindrical hole. The loading of the model as well as
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Figure 1. Half-structure (shown to scale) used to model the interaction of a plane compressional wave with a cylindrical cavity of diameter BC. Displacements along the lines AD and EF are fixed at zero in the direction normal to the front.

the specified boundary conditions and relative dimensions permit an accurate determination of the particle motions induced on both the wave-incident side (C) and the wave-shadow side (B) of a cylindrical hole under the action of the incident P-wave. Numerical solutions to the displacements induced throughout the model were obtained using the Finite Element program ADINA (Bathe 1978).

The dynamic model consisted of 656 elements of the 4-node quadrilateral type. The maximum linear dimension of the largest element was set at \( \lambda/4 \) for the smallest wavelength, \( \lambda \), propagated through the model. According to the results of White, Valliappan & Lee (1979) such discretization should yield a mean error of about 7 per cent in the solution; the error is due to the low-pass filtering action of the finite element mesh. In order to obtain the fine discretization required for the modelling of any general inclusion mount within the cylindrical hole, the half-cavity of Fig. 1 was constructed of 96 elements.

The element material properties for the host region were chosen to match the rock properties of present interest for which Young’s modulus, \( E \), was \( 1.16 \times 10^{11} \, \text{N m}^{-2} \), Poisson’s ratio, \( \nu \), 0.25 and the density \( 3.3 \times 10^3 \, \text{kg m}^{-3} \). These elastic properties yield a P-wave velocity of 6.50 km s\(^{-1}\).

Model comparisons with some analytical solutions

Boundary displacements due to an applied pressure load

If \( \sigma(t) \) is the time-dependent pressure (stress) in an elastic continuum then the corresponding particle displacement, \( u(t) \), is given by

\[
\frac{\partial u(t)}{\partial t} = -\frac{\sigma(t)}{\rho v_p}
\]

where \( \rho \) is the density and \( v_p \) the compressional wave velocity of the elastic material. In the case of a sinusoidal pressure load given by \( \sigma(t) = \sigma_0 \sin 2\pi ft \) the amplitude, \( u_0 \), of the displacement is given by:

\[
u_0 = \sigma_0/(2\pi f \rho v_p)
\]

where \( \sigma_0 \) and \( f \) are the amplitude and frequency, respectively, of the applied load. If \( \mu \) is the shear modulus of the elastic medium and Poisson’s ratio, \( \nu \), is taken as 0.25, then \( \mu = \rho v_p^2/3 \). Furthermore, if \( a \) is the radius of the cylindrical cavity and \( \lambda \) the wavelength of the compressional wave then from equation (2) a dimensionless displacement may be defined as

\[
\frac{\mu u_0}{a \sigma_0} = \frac{1}{3} \left( \frac{2\pi a}{\lambda} \right)^{-1}
\]

where \( 2\pi a/\lambda \) is a dimensionless wavenumber.
Equation (3) provides a simple yet valuable check on the amplitude, $u_0$, of the boundary displacements induced in the model due to an applied sinusoidal pressure load. Fig. 2 shows a comparison of the analytical expression with the DFEM results, the latter being obtained with both vertical and horizontal displacements fixed at zero on the far boundary, AF, of Fig. 1. With this particular model, displacements at frequencies, $2\pi a/\lambda$, less than 0.37 could not be studied since reflections from both the cylindrical hole and the fixed boundaries of the model arrived at the free boundary DE before steady state conditions could be achieved. The far boundary, AF, introduced large amplitude reflections, the cylindrical hole and the boundary EF introduced only small amplitude reflections, as expected from the geometry of the model. However, for dimensionless frequencies greater than 0.37 it was noted that steady state conditions were achieved on the boundary DE after the first few cycles of sinusoidal motion. Naturally, the use of an energy absorbing boundary along the face AF could significantly reduce reflections within the model. However, such boundaries were not implemented at this stage of the modelling since it is obvious from Fig. 2 that the boundary displacements induced in the model are in good agreement with the analytical solution for the frequency range considered. These results indicate that the loading boundary is sufficiently remote from the cylindrical hole since, at the time of loading, this boundary appears as part of an infinite continuum.

The upper frequency limit of the model is dependent upon the maximum linear dimension of the largest element. This dimension was chosen equal to the hole radius $a$. Thus the minimum wavelength, $\lambda_{\text{min}}$, propagated in the model is given by the criteria of White et al. (1979) as $\lambda_{\text{min}} = 4a$. This corresponds to a maximum dimensionless wavenumber given by:

$$2\pi a/\lambda_{\text{min}} = \pi/2.$$  \hspace{1cm} (4)

Alternatively, the maximum frequency, $f_c$, in Hz is given by

$$f_c = \nu_p/4a.$$ \hspace{1cm} (5)

**CAVITY WALL DISPLACEMENTS DUE TO AN INCIDENT 'SHOCK' WAVE**

Integral Transform techniques have been used by Baron & Parnes (1962) to obtain the early time displacements, $u(t)$, of the walls of a cylindrical cavity in an infinite medium subjected
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Figure 3. Displacement response of the cavity walls due to a unit step input pressure wave.

to an incident 'shock' wave. The 'shock' wave was defined as a unit step function in the pressure applied in regions remote from the cavity. Their results are reproduced in Fig. 3 which shows a plot of the dimensionless displacement, $\mu u(t)/a_0$, as a function of the time in units of the cavity transit time, $2a/u_p$. The relevant locations studied within the cavity are described by the angle $\theta$ as shown in the inset of Fig. 3.

The finite element model of Fig. 1 was loaded with an approximation to a unit step function in pressure, and cavity wall displacements for $\theta = 0^\circ$, $90^\circ$ and $180^\circ$ plotted at each time step of $1\mu s$. Fig. 3 shows a comparison of the analytical and DFEM results. In this example, reflections off the model boundaries do not influence the early time response of the cavity walls, since such reflections reach the cavity at times $u_p/2a > 10$, approximately. The small discrepancy between the two methods is partly due to the finite time step of $1\mu s$ employed for the DFEM pressure load in modelling a unit step function. The results of Fig. 3 indicate that any filtering effects due to the size of the finite elements do not significantly distort the early time displacement response of the cavity to an applied unit step load at the boundary.

Seismic response of cylindrical cavity

The previous section dealt with the steady state response of the model boundary to an applied pressure load in the frequency range given by $0.37 < 2\pi a/\lambda < 1.57$, as well as the early time response of the cavity to an incident unit step wave. The present section deals with the steady state response of the cavity walls in the range $0 < 2\pi a/\lambda < 1.57$, the upper frequency limit being given by equation (4) or equation (5).

The displacement transfer function, which gives the steady state displacement response, is defined as the ratio of the spectrum of the displacement measured in a given direction on the cavity wall to the spectrum of the displacement measured in the same direction in the absence of a cavity. Naturally this definition can also be applied to the measurement of either velocity or acceleration; furthermore, the transfer functions for displacement, velocity and acceleration are identical.

Energy absorbing boundaries

In order to simulate an infinite region using a finite dynamic model the unified boundary of the standard viscous type described by White et al. (1977) was used to absorb energy on the boundaries AF and DE of Fig. 1.
For the boundary DE it is then necessary to both generate the P-wave and absorb wave reflections off the central cavity. It is not possible to prescribe displacements for the input on the boundary with viscous dashpots connected; this renders the dashpots ineffective as they are no longer free to respond to the reflected waves. For this reason the input is given as a pressure load applied to the dashpots along the boundary DE. A similar method of generating and absorbing on the one boundary has been recently reported by Kunar & Rodriguez-Ovejero (1980).

A sinusoidal pressure input was then applied to the damped boundary DE, a plot of the displacement, \(\mu u_0/a_0\), versus the wavenumber \(2\pi a/\lambda\), was identical to that shown in Fig. 2 with the exception that all the displacements were reduced by 48.8 per cent. This constant reduction for all displacements illustrates both the frequency-independent nature of the dashpots used in the unified boundary formulation of White et al. (1977) and the fact that the viscous boundary is not a perfect absorber of wave energy, since a 50 per cent reduction in displacements is required for a perfect absorber.

A detailed study of the effects of all the boundaries of the present model showed that most of the input energy was absorbed on the far boundary AF and the majority of the remaining energy was reflected off the hole and absorbed at the input boundary DE. Only a small portion of the energy was ultimately reflected off the non-absorbing boundary EF. These results are expected from geometrical considerations.

A significant reduction in unwanted reflections was thus achieved by absorbing on the boundaries AF and DE. However, an attempt to use a steady state DFEM was found to give inaccurate solutions for the present model since small-scale reflections off both the boundary, EF, and the damped boundaries (which are not perfect absorbers) were found to contaminate the response function before steady state conditions were achieved. Moreover, such steady state modelling is costly, requiring a separate calculation for each frequency of interest. An accurate and efficient method of obtaining a solution to the cavity problem is to determine the transient response of the model to a pressure pulse applied to the boundary DE. Spectral ratio techniques can then be used to obtain the displacement transfer function (hence steady state response) of the cavity.

**INPUT PRESSURE FUNCTION**

There are two restrictions to be placed on the characteristics of the displacement pulse produced by the applied pressure pulse. Firstly, the pulse should include significant amplitudes for all frequencies in the range from zero to the maximum frequency given by equations (4) and (5), and have negligible amplitude contribution at higher frequencies. Secondly, the time duration of the pulse should be short enough to avoid the hole response being significantly contaminated by boundary reflections. The considerable reduction of boundary reflections in this model has enabled the use of a pulse width large enough to avoid the introduction of large amplitudes at frequencies above the design maximum.

A simple pulse shape for the displacements is given by

\[ u(t) = \exp \left[-\pi b^2 t^2 \right] \]  

where \( t \) is the time in units of the cavity transit time \( 2a/v_p \), and \( 2b^{-1} \) the pulse width illustrated in Fig. 4(a). The pressure pulse required to produce this displacement pulse is given from equations (1) and (6)

\[ p(t) = -2\pi p_v b^2 t \exp \left[-\pi b^2 t^2 \right]. \]  

The pressure pulse is illustrated in Fig. 4(b).
Figure 4. Boundary displacement pulse induced by the indicated pressure pulse. The pulse width employed is given as $2b^{-1}$.

The Gaussian pulse shape has been chosen for the displacement since its Fourier Transform is also a Gaussian. Thus it is a simple matter to ensure that there is insignificant energy input to the model above the mesh cut-off frequency $f_c$. The Fourier Transform, $U(f)$, of the displacement pulse is given by

$$U(f) = b^{-2} \exp\left[-\pi f^2/b^2\right].$$

(8)

The width of this spectral shape is $2b$ and so the mesh cut-off frequency is given by $f_c = b$. The width, $T$, of the time pulse is then $2f_c^{-1}$ and substituting into equation (5) yields

$$T = \frac{8a}{v_p}$$

= 4 units of the cavity transit time.

(9)

The relative size of the model (Fig. 1) was also chosen such that reflections off the boundaries contaminated the response function of the cavity only after a time of approximately $2.5T$ had elapsed since the initial response of the cavity. The use of a Gaussian displacement pulse, $u(t)$, of width $T$, thus fulfills the two requirements mentioned earlier.

TRANSFER FUNCTION OF CYLINDRICAL HOLE

A pressure pulse given by equation (7) was applied to the loading boundary, DE, and the resulting displacement pulses were recorded at various locations within the model. Fast Fourier Transform (FFT) techniques were used to obtain the spectral ratio and hence transfer function for both the wave incident side ($\theta = 180^\circ$) and the wave shadow side ($\theta = 0^\circ$) of the cylindrical cavity surface. These results (uncorrected) are shown in Fig. 5 as a plot of the displacement ratio $A(f)$ versus the wavenumber $2\pi a/\lambda$. 
The Fourier-Bessel theory outlined in Pao & Mow (1973) was used by the author to obtain the steady state solution for the interaction of a single frequency plane P-wave with a cylindrical cavity in an infinite medium. This analytical solution was re-evaluated for each frequency in the range of interest and the resulting amplitude spectrum of the displacement ratio is also plotted in Fig. 5. These particular Fourier-Bessel evaluations do not appear to have been reported in the literature. This is possibly due to the fact that this theory has been primarily used to investigate the effect of cylindrical discontinuities on dynamic stress concentrations rather than on particle motions.

At very low frequencies the cavity should have no influence on the particle displacements. At very high frequencies the wave incident side of the cavity approximates to a free, plane surface to the incoming waves and so displacements on this surface should be twice as large as those in the incident wave. The analytical solution fulfils these limiting conditions. For wavenumbers \(2\pi a/\lambda\), up to a limit of 0.6 or so the DFEM results are in good agreement with the analytical solution. This limit corresponds to wavelengths, \(\lambda\), being greater than about 10 times the greatest dimension of the largest element. This result agrees with the criterion of Bazant (1978), who reasons that the presence of wavelengths smaller than the above limit will overshadow the correct dynamic response. The criterion of White et al. (1979) was used as a basis for the present model in which the smallest wavelength propagated was 4 times the greatest dimension of the largest element. For this criterion they obtained a mean error of 7 per cent for the response function obtained using a uniform, rectangular finite element mesh having all elements of equal size. However, the present model has a large contrast in element sizes and so is also subject to the effects of spurious reflections (Bazant 1978) which increases the error above the lower bound estimate of White et al.

In order to improve the model predictions at high frequencies, a study was made of the response of two related models to the pressure input, \(p(t)\), given by equation (7). The first
model consisted of the finite element mesh with the cavity introduced while the second model consisted of the same mesh in which the elements within the cavity region were assigned elastic properties identical to those of the host rock. For the ideal situation of a cavity-free infinite continuum the amplitude response is unity for all frequencies, and hence the second model provides the spurious effects of both the finite elements and the boundaries on the pulse propagation in the absence of a cavity. Fig. 5 shows a corrected amplitude response of the cavity walls obtained by forming the ratio of the amplitude response of the first model to the amplitude response of the second model. This method of correction is only approximate since different wave paths, and thus different elements, are involved for the pulse propagation in each model. For example, the first arrival at the cavity wall in the wave shadow location ($\theta = 0^\circ$) is a diffracted wave that travels through elements different to those involved in the first arrival path at this point in the absence of a hole. It is for this reason that the corrected DFEM results for the wave shadow location are still in error by about 13 per cent. On the other hand, the first arrival wave path elements on the wave incident side ($\theta = 180^\circ$) are not significantly different for both models and thus the correction process should be more successful. This is in agreement with the corrected results of Fig. 5. Both analytical and corrected DFEM results for the wave incident location clearly show the small amplitude peaks which are due to the resonances of a cavity in an infinite medium.

The results of Fig. 5 enable some simple guidelines to be given for the installation of seismic detectors in cylindrical holes. If the wavelengths of interest are greater than 10 times the hole circumference ($2\pi a/\lambda < 0.1$) then the hole has negligible influence on the particle motions recorded by a seismic detector irrespective of the detector location within the hole. If the wavelengths of interest are greater than 2.5 times the circumference of the hole ($2\pi a/h < 0.4$) then the detector should be placed on the wave shadow side of the hole if the hole is to have negligible influence.

**EXPERIMENTAL RESULTS**

Experimental results were obtained for the interaction of a seismic pulse with a cylindrical hole drilled into massive rock in the Urquhart shale on 11 Level South at Mount Isa Mines, Queensland, Australia. The seismic source consisted of a N8 electric detonator fired in the centre of the cross-section of a water-filled hole of 0.1 m diameter. The seismic pulse was detected using Bruel and Kjaer type 8309 accelerometers. An epoxy resin was used to bond the accelerometers directly on to the rock 0.5 m down-hole from the collar of a 0.1 m diameter detector hole. Previous tests verified that the hardened resin did not attenuate the high-frequency accelerations expected from a blast source. The detector hole accommodated two accelerometers bonded on the wave incident side ($\theta = 180^\circ$) and two accelerometers bonded on the wave shadow side ($\theta = 0^\circ$). A source-detector separation of 5 m was chosen in order to ensure that the source wavefront was approximately planar when considering its interaction with a detector hole of 0.1 m diameter.

An explosive source generally produces both P- and S-waves and the transmission of these waves through non-uniform material, such as rock, will generate further waves. Consequently many waves travelling various paths are usually recorded by the detector. It is for this reason that Fourier analysis of the complete waveform recorded by the in-hole detectors cannot be compared with the results of a model that involves a P-wave only, travelling a single path from the source to a particular detector. It is also very difficult to completely isolate the initial P-wave since the onset of other waves generally occurs before the end of the initial pulse. However, it is reasonable to assume that the rise time of the seismic pulse is that due solely to a single P-wave travelling a single path. The rise time is defined as the ratio of the maximum ordinate to the maximum slope of the pulse onset (Gladwin & Stacey 1974).
Figure 6. Seismic pulse rise times (\( \tau \)) determined for repeated firings of electric detonators placed in a water filled hole. The lower values were recorded by an accelerometer located on the wave-incident side (\( \theta = 180^\circ \)) of the detector hole.

Thus meaningful comparisons may be made between the pulse rise time determined experimentally and that determined from the modelling procedure.

The blast waveforms recorded by the detectors were amplified, tape recorded and subsequently analysed using a HP21MX minicomputer. The amplitude-frequency response of the complete measurement system (comprising accelerometer, preamplifier, amplifier and tape recorder) was determined to be uniform within 0.5 dB for the frequency range 0.1 - 60 kHz. The 3 dB roll-off point was located at 80 kHz.

Fig. 6 shows the seismic pulse rise times detected for repeated detonator firings. The lower values were recorded by detector 1 on the wave incident side of the hole (\( \theta = 180^\circ \)) and the upper values were recorded by detector 3 on the wave shadow side (\( \theta = 0^\circ \)). These results show that the rise time on the wave shadow side of the hole is significantly larger than the rise time on the wave incident side. The average of the rise times recorded by detector 1 was 25.1 \( \mu \)s and the average for detector 3 was 35.5 \( \mu \)s. In units of the cavity transit time (\( 2a/v_p \)) these rise times become 1.63 and 2.31 respectively. Furthermore, the rise time average for detector 2 (for which \( \theta = 180^\circ \)) was found to be 1.80 and the average for detector 4 (for which \( \theta = 0^\circ \)) found to be 2.00. All averages were taken for 20 firings. If all the results are split into two groups, \( \theta = 180^\circ \) and \( \theta = 0^\circ \), then the average rise times of the groups are 1.72 and 2.16 respectively. The pooled standard deviation of either mean is also found to be 0.11. Using the basic significance test (Student's \( t \)), the 80 per cent confidence limits for the mean are 1.50 and 1.93 for \( \theta = 180^\circ \) and 1.94 and 2.37 for \( \theta = 0^\circ \). The 90 per cent confidence limits for the mean are 1.38 and 2.03 for \( \theta = 180^\circ \) and 1.82 and 2.49 for \( \theta = 0^\circ \).

COMPARISON OF THEORY AND EXPERIMENT

The theory of Kjartansson (1979) gives the spectrum, \( B(f) \), for an attenuated seismic pulse in an infinite, uniform, isotropic material. The DFEM results give the transfer function \( H(f) \), for a cylindrical hole in such a material. Both \( B(f) \) and \( H(f) \) are complex and the inverse transform of the complex multiplication \( H(f)B(f) \) gives the theoretical seismic wavelet, \( w(t) \), resulting from the interaction of an attenuated pulse with a cylindrical hole. In this study Fast Fourier Transform techniques were used to obtain \( w(t) \).

Fig. 7 shows the rise times of the wavelet \( w(t) \) as a function of the rise time of the attenuated pulse in regions free of the hole influence. The plots are shown for pulses on the wave incident side (\( \theta = 180^\circ \)) and the wave shadow side (\( \theta = 0^\circ \)) of the hole. For the range of rise times of interest (between 1.63 and 2.31) the theoretical results show that the
average of the wave incident pulse rise time and the wave shadow pulse rise time is within 3 per cent of the value of the rise time of the pulse in regions free of the hole influence. The average of the experimental rise times for all four detectors is 1.94 and so is considered to be the best estimate of the average rise time of the pulse in the absence of the detector hole. Assuming this best estimate, Fig. 7 shows the group average of the experimental rise times for $\theta = 180^\circ$ and $0^\circ$. The 80 per cent confidence limits for the mean in each case are also indicated.

Inclusion mounts for seismic detectors

The theory and experiment of the previous section dealt with seismic detectors located at a point on the wall of a cylindrical hole. In general such a situation can only be experimentally achieved for detectors located in-hole reasonably close to the hole collar where it is possible to bond the detector directly to the wall of the borehole. For detector locations at greater and variable depths in-hole an inclusion-type device is generally required, using a remotely controlled mechanism, either hydraulic or mechanical to load the detector against the borehole wall (see, for example, McDonal et al. 1958).

For such in-hole clamping techniques it is necessary to determine the response of the detector mount to an incident stress wave. To a first approximation, the detector and its clamp may be modelled as a cylindrical inclusion embedded in an infinite, elastic host rock.

DFEM solutions were obtained for various types of commonly used inclusion materials; the elastic properties assumed for these materials are given in Table 1, in which $\rho$ is the density, $E$ the dynamic Young's modulus and $\nu$ Poisson's ratio.

<table>
<thead>
<tr>
<th>Material</th>
<th>$\rho$ (kg m$^{-3}$)</th>
<th>$E$ (N m$^{-2}$)</th>
<th>$\nu$</th>
<th>Acoustic impedance (kg m$^{-2}$s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Host rock</td>
<td>3.3 ($\times 10^3$)</td>
<td>1.16 ($\times 10^{11}$)</td>
<td>0.25</td>
<td>2.14</td>
</tr>
<tr>
<td>Brass</td>
<td>8.5</td>
<td>1.05 ($\times 10^{11}$)</td>
<td>0.33</td>
<td>3.64</td>
</tr>
<tr>
<td>Stainless steel</td>
<td>7.8</td>
<td>1.90 ($\times 10^{11}$)</td>
<td>0.31</td>
<td>4.53</td>
</tr>
<tr>
<td>Aluminium</td>
<td>2.7</td>
<td>0.69 ($\times 10^{11}$)</td>
<td>0.33</td>
<td>1.66</td>
</tr>
</tbody>
</table>
The acoustic impedance is defined as the product, $\rho v_p$, where $v_p$ is the velocity of the P-wave. From standard elasticity theory, the acoustic impedance is given by

$$\rho v_p = \left( \frac{\rho E(1-\nu)}{(1+\nu)(1-2\nu)} \right)^{1/2}.$$  

(10)

The amplitude transfer function, $A(f)$, for wavenumbers $2\pi a/\lambda$, up to 1.0 for a brass cylindrical inclusion embedded in the host rock, is shown in Fig. 8. Figs 9 and 10 show the results for stainless steel and aluminium inclusions, respectively. These amplitude transfer functions were obtained by applying the correction (described earlier) to minimize the spurious effects of the finite element mesh. The results show that, in all cases, the inclusion produces a smaller perturbation to amplitudes than does the cylindrical hole. Furthermore, the perturbation due to detectors mounted within the inclusion is a lot less for location on

\[ \text{Figure 8. Amplitude transfer function for a brass cylindrical inclusion embedded in the host rock.} \]

\[ \text{Figure 9. Amplitude transfer function for a stainless steel cylindrical inclusion embedded in the host rock.} \]

\[ \text{Figure 10. Amplitude transfer function for an aluminium cylindrical inclusion embedded in the host rock.} \]
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The wave shadow side than it is for location on the wave incident side. A comparison of Table 1 with the results of Figs 8, 9 and 10 clearly shows that the smaller the mismatch in acoustic impedance between inclusion and host rock, the smaller the perturbation due to the inclusion mount.

It is not possible to ensure that any cylindrical inclusion has perfect contact with the host material at all points on the inclusion circumference. In fact, mechanical or hydraulic in-hole expanding devices generally provide contact with the host material over only a portion of the hole circumference. Thus a non-cylindrical inclusion having partial contact is expected to be a more realistic model of an expandable in-hole seismic mount than that based on a perfect cylindrical inclusion. The non-cylindrical inclusion modelled in this study is shown to scale in Fig. 11.

Fig. 12 shows the corrected amplitude transfer functions for non-cylindrical inclusions constructed of either aluminium or a material having elastic properties matched to those of the host rock. These results show that the amplitude effect of only partial contact of the mount with the cavity walls in quite significant even if care is taken to match perfectly the elastic properties of both the inclusion and host material. The amplitude distortion is a lot less for detectors mounted in the non-cylindrical inclusion on the wave shadow side than it is for detectors mounted on the wave incident side. This result was also noted for the cylindrical inclusion.

Conclusions

The agreement between the Dynamic Finite Element Model (DFEM) and the analytical solutions as shown in Figs 2, 3 and 5 indicate that the discrete model is a good representation of the continuum model from which it was derived.

The transfer function for a cylindrical hole, as shown in Fig. 5, shows that the wave incident side of the hole has an amplitude increase similar to a high-pass filter whilst the
wave shadow side acts as a low-pass filter. This filtering process has the effect of decreasing pulse rise times on the wave incident side whilst increasing pulse rise times on the wave shadow side. Experimental results obtained using an explosive source in massive rock yielded overall average pulse rise times of 1.72 for the wave incident side of the hole and 2.16 for the wave shadow side. The corresponding theoretical results obtained were 1.67 and 2.20 respectively. The best estimate for the rise time of the seismic pulse in the absence of the hole is 1.94 and so illustrates the significant effect of detector mount location on measured rise times. All rise times are given in units of the cavity transit time.

It has been shown that for wavelengths larger than 10 times the hole diameter, the effect of any in-hole mount can be neglected. Thus the present work is not relevant to the study of wavelengths commonly encountered in seismology; such wavelengths are much larger than the diameter of the detector hole. However, for the smaller wavelengths relevant to ultrasonic seismic experiments, the detector mount location has a significant influence on the amplitude transfer function as shown by Figs 5, 8, 9, 10 and 12. For both cylindrical and non-cylindrical inclusion mounts the perturbation due to the in-hole mount is reduced if the detector is placed on the wave shadow side of the hole rather than on the wave incident side. In this study the experimental results were compared with the predictions of a dynamic model. However, it should be appreciated that the amplitude transfer function obtained by modelling a given detector mount may be used to correct the experimental results obtained for the detection of high-frequency seismic waves made with the particular mount.

In general, only partial contact is made between an in-hole seismic mount and the walls of the borehole. This lack of complete contact has been shown to introduce significant distortions in the amplitude transfer function of the mounted seismic detector. It is for this reason that research has been aimed at analysing the interaction of a seismic wave with a hydrophone located in the centre of the cross section of a fluid-filled hole. Both experimental and theoretical work of this nature is nearing completion and will be reported shortly.

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