Spin Structure in the bcc Spin-Lattice with Four-Spin Cyclic Exchange Interactions

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Possible stable spin configurations are investigated for the bcc spin lattice on which spins are coupled with two-spin and four-spin exchange interactions.

§ 1. Introduction

Since it has been suggested by McMahan and Wilkins and Hetherington and Willard that in solid $^3$He four-spin exchange interaction plays an important role in its spin ordering, many investigations have been made on possible spin structures on the basis of the spin Hamiltonian consisting of the usual two-spin exchange interaction and four-spin cyclic exchange interaction.

Recently, Osheroff, Cross and Fisher (OCF) have concluded on the basis of the analyses of the results obtained by NMR experiments that the low temperature spin state of solid $^3$He is the state in which ferromagnetic planes arrange in up-up-down-down sequences along one of the cubic axes, and Roger, Delrieu and Hetherington have shown that this new type of spin structure is realized when the planar four-spin interaction is larger in its magnitude. Up to the present, possible spin structures have been searched for by investigating the modification of spin structures expected for large two-spin exchange interactions by four-spin exchange interactions.

Here in this paper, possible spin structures are investigated from the opposite side by starting with the assumption that four-spin interaction is larger than two-spin interactions.

§ 2. The energy level of the four spins coupled with four-spin cyclic exchange interaction —quantum treatment—

First of all, we consider the energy level of the four spins coupled with the four-spin cyclic exchange interaction

$$-\frac{K}{4} \left[ (\sigma_1 \cdot \sigma_2) (\sigma_3 \cdot \sigma_4) + (\sigma_1 \cdot \sigma_3) (\sigma_2 \cdot \sigma_4) - (\sigma_1 \cdot \sigma_4) (\sigma_2 \cdot \sigma_3) \right],$$

where $\sigma_i$ is the Pauli matrix of spin $i$ and $K$ is the coupling constant which
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is assumed to be negative in this paper. If one pays attention to the z-component, namely, the diagonal part of Eq. (1), the four spin states are classified by the z-component of the total spin, \( M_z \), which can take an integer from 2 to -2. There is one state with \( M_z = 2 \), and there are four independent states with \( M_z = 1 \) and six states with \( M_z = 0 \). The diagonal energies of the states with \( M_z = 2, 1, 0 \), are given, respectively, by \(+1, -1, +1\) aside from the factor \(-K/4\). Among six independent states with \( M_z = 0 \) one comes from quintet state, three come from triplet states and two from singlet states.

Solving the eigenvalue equation of Hamiltonian (1) for these six \( M_z = 0 \) states we obtain the following energy eigenvalues:

\[
\begin{align*}
E_{s1} &= 13 \quad \text{singlet 1}, \\
E_{t1} &= 1 \quad \text{triplet 1}, \\
E_{t2} &= 1 \quad \text{triplet 2}, \\
E_q &= 1 \quad \text{quintet,} \\
E_{s2} &= -3 \quad \text{singlet 2}, \\
E_{t3} &= -7 \quad \text{triplet 3}.
\end{align*}
\]

Although diagonal energy takes a value of 1 or -1 in the same way as two-spin exchange interaction, the energy eigenvalues for four spins are spread from 13 to -7. For a positive value of \((-K/4)\), the lowest state is triplet and the highest is singlet.

§ 3. The stable orientation of the four spins—classical treatment—

In this section we treat four spins as classical vectors.

Hamiltonian (1) can be rewritten as

\[
(\sigma_1 \cdot \sigma_2) (\sigma_3 \cdot \sigma_4) + (\sigma_1 \cdot \sigma_3) (\sigma_2 \cdot \sigma_4) - (\sigma_1 \cdot \sigma_4) (\sigma_2 \cdot \sigma_3)
\]

\[
= (\sigma_1 \cdot \sigma_2) (\sigma_3 \cdot \sigma_4) + (\sigma_1 \cdot \sigma_3) (\sigma_2 \cdot \sigma_4) - (\sigma_1 \cdot \sigma_4) (\sigma_2 \cdot \sigma_3)
\]

\[
= \cos \Omega_{12} \cos \Omega_{43} + \sin \Omega_{12} \sin \Omega_{43} \cos \phi,
\]

where \( \Omega_{12} \) and \( \Omega_{43} \) are, respectively, the angle between \( \sigma_1 \) and \( \sigma_2 \) and that between \( \sigma_4 \) and \( \sigma_3 \), and \( \phi \) represents the angle made by two planes, one including \( \sigma_1 \) and \( \sigma_2 \) and the other including \( \sigma_4 \) and \( \sigma_3 \) as shown in Fig. 1. The extremum of Eq. (2) with respect to \( \phi \) occurs at \( \cos \phi = 1 \) or \(-1\), namely \( \phi = 0 \) or \( \phi = \pi \). Therefore, we can see that the most stable orientation of the four spins occurs when they are in planes parallel to a certain plane. In such a plane, the angle of each spin measured from an arbitrarily chosen axis is denoted by \( \phi_i \). Then, we have \( \Omega_{12} = \phi_2 - \phi_1 \), \( \Omega_{43} = \phi_3 - \phi_4 \) for \( \phi = 0 \) and \( \Omega_{43} = \phi_4 - \phi_3 \) for \( \phi = \pi \). Equation (2) is equal to
Thus, we can conclude that the most stable spin orientation of the four spins coupled with the four-spin exchange interaction is such that four spins are all within planes parallel to a certain plane and their angles $\phi_i$ satisfy the relation

$$\phi_1 - \phi_2 + \phi_3 - \phi_4 = \pi .$$  \hspace{1cm} (4)

It is seen from this relation that if we fixed $\phi_1$, two angles, for example, $\phi_1 - \phi_2$ and $\phi_3$ can still be taken arbitrarily.

§ 4. The stable spin configuration for bcc lattice when $|K_F|$ is large and $K_F=0$

In this section we consider the stable spin configuration for bcc spin lattice where spins are coupled with four-spin cyclic folded exchange interaction. Here, we first consider the case in which the planar interaction is absent. For such a case, if we take one folded four-spin ring and specify each angle of these four spins by $\phi_1$, $\phi_2$, $\phi_3$, and $\phi_4$, the angles of all the spins in other lattice points are determined uniquely by the relation (4). This is shown in Fig. 2. Thus, the most stable state is not uniquely determined by the folded four-spin exchange only, because we have still arbitrary angles $\phi_1 - \phi_2$, and $\phi_3$.

Now we here introduce two spin exchange interactions between first nearest neighbor spins, 2nd nearest neighbor spins and 3rd nearest neighbor spins as follows:

$$H_z = -\frac{J_1}{2} \sum_{i<n} (\sigma_i \cdot \sigma_j) - \frac{J_2}{2} \sum_{i<n} (\sigma_i \cdot \sigma_j) - \frac{J_3}{2} \sum_{i<n} (\sigma_i \cdot \sigma_j) .$$ \hspace{1cm} (5)
Fig. 2. The stable spin orientation of bcc lattice whose spins are coupled by the folded four-spin interaction.

These two-spin exchange interactions will remove the degeneracy due to arbitrary angles $\phi_1 - \phi_2$ and $\phi_3 - \phi_4$. For the spin structure shown in Fig. 2, the energy of Eq. (5) can easily be calculated as

$$E_s = -\frac{N}{2} \left[ J_1 \{ \cos(\phi_2 - \phi_3) + \cos(\phi_4 - \phi_3) + \cos(\phi_2 - \phi_4) + \cos(\phi_4 - \phi_3) \} \right. \begin{array}{c} \scriptstyle+ \frac{3}{2} J_s \{ \cos(\phi_1 - \phi_3) + \cos(\phi_1 - \phi_2) \} + \frac{1}{2} 12 J_z \end{array} \right].$$ \hspace{1cm} (6)

Introducing the relation (4) into Eq. (6), we have

$$\frac{E_s}{N} = -\frac{3}{4} J_s \left[ \cos(\phi_1 - \phi_3) - \cos(\phi_1 - \phi_3 + 2\phi) \right] - 3 J_z,$$ \hspace{1cm} (7)

where $\phi$ is defined as

$$\phi = \phi_2.$$

(8)

For this case, the sum of nearest neighbor interactions vanishes identically.

The lowest energy with respect to $\phi_1 - \phi_3$, and $\phi$ is obtained by

$$\cos(\phi_1 - \phi_3) = 1, \quad \cos(\phi_1 - \phi_3 + 2\phi) = -1, \quad J_z > 0,$$ \hspace{1cm} (9)

$$\frac{E_s}{N} = -\frac{3}{2} J_z - 3 J_z,$$ \hspace{1cm} (10)

and

$$\cos(\phi_1 - \phi_3) = -1, \quad \cos(\phi_1 - \phi_3 + 2\phi) = 1, \quad J_z < 0,$$ \hspace{1cm} (11)

$$\frac{E_s}{N} = +\frac{3}{2} J_z - 3 J_z, \quad J_z < 0.$$ \hspace{1cm} (12)

For $J_z > 0$, we obtain $\phi = \phi_3, \phi = \pi/2$.

The spin structure consists of two sublattices, body center and corner,
which have parallel spins perpendicular to each other. Tentatively we call this state scf\_⊥.

For \( J_1 < 0 \), we have

\[
\phi_1 - \phi_3 = \pi, \quad \phi_1 - \phi_3 + 2\phi = 0, \\
\phi = \frac{\pi}{2}.
\]

This structure is usually called as simple cubic antiferro perpendicular, scf\_⊥. These two structures are both most favourable to \( K_F \) and so \( K_F \) energy is given by \((N/2) \cdot (6/2) \times 4 \times (\frac{1}{4}) \cdot K_F = N(\frac{3}{2} K_F)\).

As for \( K_p \)-part, scf\_⊥ is also favourable but scf\_⊥ is not favourable to \( K_p \). The \( K_p \) energy of scf\_⊥ is calculated as \(-N\frac{3}{2} K_p\). Thus, we obtain the total energy as

\[
\frac{E}{N} = -\frac{3}{2} J_1 - 3J_4 + \frac{3}{2} (K_F + K_P), \quad \text{scf\_⊥}, \quad (13)
\]

\[
\frac{E}{N} = \frac{3}{2} J_1 - 3J_4 + \frac{3}{2} (K_F - K_P), \quad \text{scf\_⊥}. \quad (14)
\]

**§ 5. The stable configuration for bcc lattice when \( |K_F| \) is large and \( K_F = 0 \)**

When four-spin interaction \( K_F \) is large and \( K_F \) is negligible, the situation becomes somewhat complicated compared with the opposite case because if one takes one planar four-spin ring and specifies each angle of these four spins by \( \phi_1, \phi_2, \phi_3 \) and \( \phi_4 \) as before, the angles of other spins are not determined uniquely. In order to determine angles of other spins uniquely, we must specify one more angle, for example, \( \phi_5 \) as shown in Fig. 3. Thus, if we fixes angles \( \phi_1, \phi = \phi_2 - \phi_1, \phi_3 \) and \( \phi_5 \), then all the angles of spins on bcc lattice are determined in terms of these four fixed angles. Results are shown in Fig. 3.

For the spin structure shown in Fig. 3, the sum of the first neighbor interactions is given by

\[
-\frac{J_1}{2} \left[ \cos (\phi_1 + \phi - \phi_3) + \cos (\phi_4 - \phi_3) + \cos (\pi + \phi) + \cos (\phi_1 - \phi_3 - \pi) \\
+ \cos (\phi_4 - \phi_3) + \cos \phi + \cos (\phi_3 - \phi_3 - \pi) + \cos (\phi_4 - \phi_3 - \phi - \pi) \right].
\]

This identically vanishes also. \( E_2 \) is calculated as

\[
\frac{E_2}{N} = -\frac{J_1}{2} \{ \cos (\phi_1 - \phi_2) + \cos (\phi_1 - \phi_3 + \phi) - \cos (\phi_3 - \phi_3 - \phi) \}
\]

\[
-\frac{J_1}{2} \{ \cos (\phi_2 - \phi_3 + \phi) + \cos (2\phi_1 - \phi_2 - \phi_3 - \phi) - \cos (\phi_1 - \phi_3 + 2\phi) \}.
\]
Fig. 3. Spin arrangement of bcc lattice by \(K_p\) four-spin interaction when \(\phi_1, \phi_2=\phi_1+\phi, \phi_3\) and \(\phi_4\) are fixed. The angle of the spin on each lattice site is listed as follows:

1. \(\phi_1\)
2. \(\phi_1+\phi\)
3. \(\phi_2\)
4. \(\phi_3-\phi-\pi\)
5. \(\phi_4\)
6. \(\phi_4+\phi_3-\phi_1-\pi\)
7. \(-\phi_1+\phi_3-\phi_1-\phi\)
8. \(\phi_5-\phi\)
9. \(\phi_6+\phi-\pi\)
10. \(\phi_1-\phi_3+\phi+\phi-\pi\)
11. \(\phi_1+\phi_3-\phi_1+\phi\)
12. \(2\phi_1-\phi_3+\phi\)
13. \(2\phi_3-\phi_3+\phi+\phi-\pi\)
14. \(\phi_1+\phi_3-\phi_1+\phi-\pi\)
15. \(-\phi_1+\phi_3+\phi\)
16. \(-\phi_1+2\phi_3-\phi_1-\phi+\phi-\pi\)
17. \(-\phi_1+2\phi_3-\phi_1-\phi\)
18. \(\phi_3+\phi\)
19. \(-\phi_1+2\phi_3+\phi_3-2\phi_3-2\phi-\pi\)
20. \(-\phi_1+2\phi_3+\phi\)
21. \(-\phi_1+2\phi_3+\phi-\phi+\phi_3-\phi_3-\phi\)
22. \(-\phi_1+3\phi_3-\phi_3-\phi\)
23. \(-\phi_1+3\phi_3-\phi_3+\phi\)
24. \(-2\phi_1+2\phi_3+\phi_3-\phi\)
25. \(-\phi_1+3\phi_3-\phi_3-\phi\)
26. \(-2\phi_1+2\phi_3+\phi_3-\phi\)
27. \(-\phi_1+3\phi_3-\phi_3+\phi\)
28. \(-\phi_1+3\phi_3-\phi_3-\phi\)
29. \(-2\phi_1+2\phi_3+\phi_3-\phi\)
30. \(-\phi_1+3\phi_3-\phi_3+\phi\)
31. \(-2\phi_1+2\phi_3+\phi_3-\phi\)
32. \(-\phi_1+3\phi_3-\phi_3-\phi\)
33. \(-2\phi_1+2\phi_3+\phi_3-\phi\)
34. \(-\phi_1+3\phi_3-\phi_3+\phi\)
35. \(-2\phi_1+2\phi_3+\phi_3-\phi\)
36. \(-\phi_1+3\phi_3-\phi_3-\phi\)
37. \(-2\phi_1+2\phi_3+\phi_3-\phi\)
38. \(-\phi_1+3\phi_3-\phi_3+\phi\)
39. \(-2\phi_1+2\phi_3+\phi_3-\phi\)
40. \(-\phi_1+3\phi_3-\phi_3-\phi\)

These angles can be described by a single formula:

\[\phi_{nz, nv, n, i} = r/J + nzr/Jz + \frac{nنز}{2} + \frac{(nز)}{2} + \frac{( nz + nز + nز)}{2} + \frac{( nz + nز + nز + nز)}{2},\]

where each lattice site is denoted by a set of integers \(n_z, n_v, \) and \(n_i;\) corner sites by even \(n\) and body-center sites by odd \(n.\)

\[\cos (\phi_1 + \phi_3 - 2\phi_3) - \cos (\phi_1 - 2\phi_3 + \phi_3 + \phi) - \cos (\phi_1 - \phi_3 - \phi).\] (15)

This can be rewritten as

\[E_2 = -\frac{1}{2} \left[ -J_2 \cos (\phi_3 - \phi_3 - \phi) + (J_2 - 2J_2 \cos (\phi_3 - \phi_3 - \phi)) \right] \{\cos (\phi_1 - \phi_3) + \cos (\phi_1 - \phi_3 + \phi)\} + \cos (\phi_1 - 2\phi_3 + 2\phi) + 2J_2 \cos (\phi_3 - \phi_3) \cos ((\phi_1 - \phi_3 + 2\phi) + (\phi_3 - \phi_3 - \phi)).\] (16)

Equation (16) includes three functions of cosine. The extremum value of the energy can be obtained by putting cosine as +1 or -1.

Case I. First we put \(\cos (\phi_3 - \phi_3 - \phi) = 1,\) namely, \(\phi_3 = \phi_3 - \phi.\) Then, Eq. (16) becomes

\[E_2 = -\frac{1}{2} \left[ -J_2 + (J_2 - 2J_2) \{\cos (\phi_1 - \phi_3) + \cos (\phi_1 - \phi_3 + 2\phi)\}\right].\]
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\[ +2J_3 \cos(\phi_1 - \phi_2) \cos(\phi_1 - \phi_3 + 2\phi) \].

There are three extrema; the first one is given by

(Ia) \[ \cos(\phi_1 - \phi_2) = \cos(\phi_1 - \phi_3 + 2\phi) = 1, \]

namely, \( \phi_1 = \phi_2, \phi = 0, \phi_3 = \phi_1, \) or \( \phi = \pi, \phi_2 = \phi_1 - \pi. \)

The energy is given by

\[ \frac{E_1}{N} = -\frac{1}{2} [-J_2 + 2(J_4 - 2J_5) + 2J_3] = -\frac{1}{2} (J_2 - 2J_3). \] (19)

This structure is up-up-down-down structure which was found by Osheroff, Cross and Fisher.\(^7\)

The second extremum is given by

(Ib) \[ \cos(\phi_1 - \phi_2) = \cos(\phi_1 - \phi_3 + 2\phi) = -1, \]

namely, \( \phi_3 = \phi_1 - \pi, \phi = 0, \phi_2 = \phi_1 - \pi \) or \( \phi = \pi, \phi_2 = \phi_1. \)

The energy of this case is given by

\[ \frac{E_2}{N} = -\frac{1}{2} [-J_2 - 2(J_4 - 2J_5) + 2J_3] = +\frac{3}{2} (J_2 - 2J_3). \] (21)

The spin structure consists of two interpenetrating simple cubic antiferromagnetic sublattices whose sublattice moments are parallel to each other and is usually called simple cubic antiferro parallel, scaf \( //. \)

The third one is given by

(Ic) \[ \cos(\phi_1 - \phi_2) = 1, \cos(\phi_1 - \phi_3 + 2\phi) = -1 \]

or \[ \cos(\phi_1 - \phi_2) = -1, \cos(\phi_1 - \phi_3 + 2\phi) = 1, \]

namely, \( \phi_1 = \phi_2, \phi = \pi/2, \phi_3 = \phi_1 - \pi/2 \)

or \( \phi_3 = \phi_1 - \pi, \phi = \pi/2, \phi_2 = \phi_1 + \pi/2. \)

The energy of this state is given by

\[ \frac{E_3}{N} = \frac{1}{2} (J_4 + 2J_5). \] (23)

This structure was found and called as simple square antiferromagnet ssqaf by Roger, Delrieu and Hetherington.\(^6\) In this structure, all the spins are parallel along one of the cubic axes and on the perpendicular plane to this axis it is constructed from two penetrating antiferromagnetic square sublattices whose sublattice moments are perpendicular.

Case II. Next, we consider the case in which \( \cos(\phi_3 - \phi_5 - \phi) = -1, \) namely,
\( \phi_5 = \phi_3 - \phi - \pi \). Equation (16) becomes

\[
\frac{E_2}{N} = -\frac{1}{2} [J_1 + (J_2 + 2J_3) \{\cos(\phi_1 - \phi_3) - \cos(\phi_1 - \phi_3 + 2\phi)\} \\
- 2J_3 \cos(\phi_1 - \phi_3) \cos(\phi_1 - \phi_3 + 2\phi)].
\]  

(24)

In this case also, there are three extreme cases. The first one is given by

\[ \text{(Iia)} \quad \cos(\phi_1 - \phi_3) = 1, \quad \cos(\phi_1 - \phi_3 + 2\phi) = -1, \]

namely, \( \phi_1 = \phi_3, \quad \phi = \pi/2, \quad \phi_5 = \phi_1 + \pi/2. \)

The energy of the state is calculated as

\[
\frac{E_2}{N} = -\frac{1}{2} [J_1 + 2(J_2 + 2J_3) + 2J_3] = -\frac{3}{2} (J_1 + 2J_3).
\]  

(26)

This structure consists of two ferromagnetic sublattices whose sublattice moments are perpendicular to each other. This has appeared in the case that \( |K_\parallel| \) is large and was called simple cubic ferromagnet perpendicular, scf \( \perp. \)

The second one is given by

\[ \text{(Iib)} \quad \cos(\phi_1 - \phi_3) = -1, \quad \cos(\phi_1 - \phi_3 + 2\phi) = 1, \]

\( \phi_1 = \phi_3 - \pi, \quad \phi = \pi/2, \quad \phi_5 = \phi_3 - \pi/2. \)

The energy is given by

\[
\frac{E_2}{N} = -\frac{1}{2} [J_1 - 2(J_2 + 2J_3) + 2J_3] = +\frac{1}{2} (J_1 + 2J_3).
\]  

(28)

This structure is equivalent to structure Ic, namely, sqaf.

The last one is given by

\[ \text{(Iic)} \quad \cos(\phi_1 - \phi_3) = 1, \quad \cos(\phi_1 - \phi_3 + 2\phi) = 1, \]

\( \phi_1 = \phi_3, \quad \phi = 0, \quad \phi_5 = \phi_1 - \pi \text{ or } \phi = \pi, \quad \phi_5 = \phi_1 \)

or

\[ \cos(\phi_1 - \phi_3) = -1, \quad \cos(\phi_1 - \phi_3 + 2\phi) = -1, \]

\( \phi_1 = \phi_3 - \pi, \quad \phi = 0, \quad \phi_5 = \phi_1 \text{ or } \phi = \pi, \quad \phi_5 = \phi_1 - \pi. \)

The energy of the state is given by

\[
\frac{E_2}{N} = -\frac{1}{2} (J_1 - 2J_3).
\]  

(30)

The structure is equivalent to Ia, namely, up-up-down-down state.

By the calculations made so far, it has been shown that four stable spin structures exist when the planar four-spin interaction is assumed to be
large, up-up-down-down state, ssqaf, scaf// and scf⊥. The energies of these four states are

1. \[ \frac{E_1}{N} = \frac{1}{2} (J_2 - 2J_3), \quad \text{up-up-down-down,} \]
2. \[ \frac{E_2}{N} = \frac{1}{2} (J_2 + 2J_3), \quad \text{ssqaf,} \]
3. \[ \frac{E_3}{N} = \frac{3}{2} (J_2 - 2J_3), \quad \text{scaf//,} \]
4. \[ \frac{E_4}{N} = \frac{3}{2} (J_2 + 2J_3), \quad \text{scf⊥.} \]

The stable regions in \( J_2-J_3 \) parameter plane for these four states are shown in Fig. 4.

![Fig. 4. Stable regions of the four states, up-up-down-down, ssqaf, scaf//, scf⊥.](https://academic.oup.com/ptps/article-abstract/doi/10.1143/PTP.69.475/1895922)

These four states are all favourable to \( K_P \) and they have \( \frac{3}{2}K_F \) as the planar four-spin energy. As mentioned before, scf⊥ state is also favourable to \( K_F \) and therefore this state has \( \frac{3}{2}K_F \). However, other three states are not favourable to \( K_F \). Up-up-down-down state has \( \frac{1}{2}K_F \), ssqaf, \(-\frac{1}{2}K_P\) and scaf// \(-\frac{3}{2}K_F \). Thus, the total energies of the above four states are given by

1. \[ \frac{E}{N} = \frac{1}{2} (J_2 - 2J_3) + \frac{3}{2} K_P + \frac{1}{2} K_F, \quad \text{uudd,} \quad (31) \]
2. \[ \frac{E}{N} = \frac{1}{2} (J_2 + 2J_3) + \frac{3}{2} K_P - \frac{1}{2} K_F, \quad \text{ssqaf,} \quad (32) \]
3. \[ \frac{E}{N} = \frac{3}{2} (J_z - 2J_s) + \frac{3}{2} K_p - \frac{3}{2} K_F, \quad \text{scf/}, \] (33)

4. \[ \frac{E}{N} = -\frac{3}{2} (J_z + 2J_s) + \frac{3}{2} K_p + \frac{3}{2} K_F, \quad \text{scf/}. \] (34)

Scf\(\perp\) state is favourable both to \(K_p\) and \(K_F\). This state has a strong ferromagnetic moment of \(1/\sqrt{2}\) times as large as complete ferromagnetic value. This is not a new one but linked to what is called as weak ferro by Okada and Ishikawa\(^9\) when \(|K_p + K_F|\) decreases.

§ 6. Conclusions

We have investigated possible stable spin structures of the bcc spin lattice in which spins are coupled each other with usual two-spin exchange interactions and the four-spin cyclic exchange interactions. Our approach is as follows. We first assume that the four-spin interaction planar or folded is most important and search for the most stable configurations under the condition that only the planar or folded four-spin interaction exists. The most stable configuration is not uniquely determined only by the four-spin interaction and degeneracy remains. Then, we introduce the two-spin interactions to remove this arbitrariness and derive the most stable structure with respect to both four-spin and two-spin interactions.

Stable spin structures we have obtained are found to be already proposed ones. Iwahashi and Masuda\(^9\) presented a table of the energies for several spin structures. AF (110) state in this table did not appear in our case. The reason for this is as follows. The four spin energy of this state is shown to be equal to \(\frac{1}{2} (K_F - 3K_P)\) and this value is neither lowest with respect to \(K_F\) nor to \(K_P\). Therefore, this state is unstable with respect to both \(K_F\) and \(K_P\).

Our approach is based on the standpoint that the four-spin cyclic exchange interaction is primarily important. However, as far as this condition is satisfied, our results are definite.

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