Explaining differential sources of zoonotic pathogens in intensively-farmed catchments using kinematic waves

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ABSTRACT

Surveys in streams draining intensively farmed catchments can, during flood events, indicate differential time-concentration patterns between a bacterial health-risk indicator (E. coli) and a major zoonotic pathogen that it seeks to indicate—Campylobacter. The indicator’s peak concentration at a monitoring station can arrive ahead of the flood peak (the pollutograph leads the hydrograph), whereas the peak pathogen concentration arrives with the flood peak (the pollutograph and hydrograph peaks coincide). In other cases the E. coli pollutograph can lag the hydrograph. These observations have generated the hypothesis that such behaviour reflects three different possible (predominant) sources of pathogens in the floodwater: (i) by sediment entrainment, (ii) via local land runoff, or (iii) from upstream releases (e.g., from dams, inflows, or upstream floods). A general theory for contaminants in idealized stream floods has been developed, considering all three sources, based on kinematic wave theory. It can explain the observed differential time-concentration patterns. The calculation procedures and associated results are intended to inform public policy, by identifying predominant pathogen sources and therefore helping to focus attention on the important delivery mechanisms. This will better inform quantitative health risk assessments for downstream water users (recreational uses, water supplies, food production and processing industries).

Key words | campylobacter, E. coli, kinematic waves, models, runoff, sediments, streams

INTRODUCTION

Campylobacteriosis has been reported at up to 400 cases per 100,000 persons per annum in New Zealand (Till & McBride 2004) and is the subject of a major modelling research effort (Lake et al. 2007). Animals are the major “reservoir” of Campylobacter, and so identifying effective intervention strategies to minimize their transmission to humans includes consideration of its transport characteristics over land and in freshwater streams. Herein we consider how the transport of this pathogen and its indicator (E. coli) respond to changing hydrological conditions.

Consider two local mechanisms of microbial (bacterial) transport through stream systems during rainfall. Firstly, rainwater flowing overland toward stream banks can entrain bacteria from fecal material previously deposited on adjacent land (Nagels et al. 2002; Muirhead et al. 2004). Secondly, enhanced stream velocity during floods drives entrainment of any faecal material in the stream sediments and on its banks. While sediments and stream banks are a substantial source of E. coli, they are much less so for the Campylobacter bacterium. This is because the sediments may be depauperate in the pathogen yet rich in E. coli, as has been found in sediment surveys in a Waikato stream draining intensive pastoral agriculture (Snowsill 2007)—a result predicted by Donnison et al. (2006). These observations, coupled with considerations of non-local (upstream) mechanisms, provide the motivation for studying the differential timing of peaks of Campylobacter versus E. coli during storm flood events, as shown in Figure 1, using kinematic waves.

The appropriateness of kinematic wave theory, at least for smaller steeper streams, has been endorsed by Li et al. (1975) and by Krein & De Sutter (2001) who observed that “there is a lag in the arrival of the flood water behind the rise in stage”.

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Its huge advantage, as we shall see, is to replace a complex momentum balance with a simple power law. More generally, other observations (pers. comm., Dr R. Davies-Colley, NIWA, Hamilton) have shown that a microbial stream “pollutograph” can lag, lead or be in harmony with the stream’s hydrograph.

CONCEPTUAL MODEL

Following a speculation made by Wilkinson et al. (2006), we seek to mimic differential peak timing behaviour using a mathematical model based on kinematic wave theory, described in some detail by McBride & Mittinity (2007). The development was assisted by a clear exposition of the mathematical formulation of the problem and the use of mathematical wave theory—its application to surface water contamination is rare (Singh 2002). It therefore seems prudent to examine such a simple case, to help identify any problems in mathematical formulation of the problem and the use of

Assumptions about lateral inflow

The first runoff flush can be expected to result from higher bacteria entrainment rates than later in the storm, as stores become depleted. However, the lateral land distance over which the former entrainment has occurred will be smaller than that occurring later in the storm when the entrainment rate at a point is lower, but the distance over which entrainment occurs is longer—compensating for the higher point-wise entrainment rate. Accordingly, during the constant overland flow period the delivery of bacteria to the stream could possibly be taken as zero-order (constant) during the runoff period; that is,

\[ M_t = rC_t \]  

where \( M_t [\# T^{-1} L^{-1}] \) is the land-derived bacteria delivery rate per unit channel length (“#” denotes numbers of bacteria), \( r [L^2 T^{-3}] \) (a constant) is the overland flow rate per unit channel length, and \( C_t [\# L^{-3}] \) is the concentration of bacteria in the lateral inflow.

Assumptions about sediment entrainment

We assume that the finiteness of the sediment microbial store demands that the bacterial entrainment rate is first-order and proportional to the remaining store after entrainment onset. Also, following Valentine & Wood (1979) (see also Rutherford 1994), that rate is taken to be proportional to the flood-flow velocity excess, that is,

\[ M_s = e_s \left( \frac{U - U_b}{U_b} \right) s \]  

where \( M_s [\# T^{-1} L^{-1}] \) is the delivery rate of entrained bacteria per unit channel length, \( e_s [T^{-1}] \) is the stream entrainment coefficient, \( U [LT^{-1}] \) is the cross-section average flow velocity (with value \( U_b \) at baseflow, from the dam outlet), and \( s [\# L^{-1}] \) is the store of bacteria in the stream sediments and banks per unit channel length. Entrainment occurs when \( U > U_b \) but there is deposition when \( U < U_b \).

Why analyse such a simple case?

Firstly, this approach involves a novel application of kinematic wave theory—its application to surface water contamination is rare (Singh 2002). It therefore seems prudent to examine such a simple case, to help identify any problems in mathematical formulation of the problem and the use of...
associated equation solution techniques. Secondly, the equation solution procedures generally require the use of approximate numerical methods, because analytical solutions have not been found. This makes “benchmarking” of the solution procedure difficult. However, we have obtained an analytical solution for one simple case (lateral inflow to a wide channel with positive baseflow, as summarised in the Appendix 1), which provides for some concrete benchmarking. Finally, simple cases such as proposed can actually provide more general insight into contaminant hydrology than is the case for much more elaborate models, which tend to focus on particular cases.

Solving the equations

McBride & Mittinity (2007), following Chapra (1997), used an explicit numerical scheme for wide channels (width >> depth). This has the advantage of relative simplicity (compared with an implicit scheme), but also has a drawback in that it isn’t fully mass-conserving, and introduces some “numerical dispersion” in which pollutograph waves tend to be artificially smeared out. Implicit numerical schemes have been developed, based on the “four-point Preissman” scheme (Liggett & Cunge 1975; Abbott & Basco 1989; Martin & McCutcheon 1999) using a time weight \( \omega \) (0 ≤ \( \omega \) ≤ 1). These also permit calculations for the narrow channel case, in which the channel side slope (\( \theta_c \)) must also be specified. An attraction of the implicit approach is that its computational molecule doesn’t involve any nodes outside the local grid cell which, on physical grounds, are only needed in the presence of second-order diffusion terms. Space precludes presenting further details here. The algorithms have been developed in Microsoft Visual Basic.

TRANSPORT EQUATIONS

Here we extend the model to include bacterial inactivation.

Continuity (water balance) equation

Application of mass conservation principles leads to

\[ \frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = r \]  \hspace{1cm} (3)

where \( A \) [L²] is the stream cross-section area, \( Q \) [L³ T⁻¹] is the stream discharge (with baseflow value \( Q_b \) at the dam), \( x \) [L] is distance along the channel, and \( t \) [T] is time since flood commencement. Each term in Equation (3) has units \([L^2 T^{-1}]\). Following Chapra (1997), we adopt the key assumption of kinematic wave theory—that the discharge is a function of depth alone—and use Manning’s equation

\[ Q = \frac{\rho A^{5/3}}{n P^{2/3} \sqrt{b}} \]  \hspace{1cm} (4)

where \( \theta_b \) [dimensionless] is the channel bed slope, \( P \) [L] is the wetted perimeter, \( n \) [dimensionless] is the Manning roughness coefficient and \( \rho \) is the units adjustment factor (see Appendix 2). This equation can be solved for the fundamental kinematic wave equation (which obviates the need to develop a much more complex momentum balance equation)

\[ A = x Q^\beta \]  \hspace{1cm} where \( \beta = \frac{3}{5} \) and \( x = \left( \frac{n P^{2/5}}{\sqrt{b}} \right)^{3/5} \]  \hspace{1cm} (5)

demonstrating that \( \alpha \) is variable, not constant (because the wetted perimeter changes with discharge). However, the value of \( \alpha \) becomes nearly constant for a rectangular channel much wider than it is deep—because we then have \( P \approx B_0 \) where \( B_0 \) [L] is the channel width. The constant value of \( \alpha \) is then

\[ x \approx \left( \frac{n B_0^{2/3}}{\sqrt{\theta_b}} \right)^{3/5} \]  \hspace{1cm} (6)

Differentiating Equation (5) with respect to time and substituting the result into Equation (3) we have the kinematic wave equation (Chapra 1997) augmented by lateral inflow

\[ \frac{\partial Q}{\partial x} + \alpha \beta Q^{\beta-1} \frac{\partial Q}{\partial t} = r \]  \hspace{1cm} (7)

By substituting explicit forward-time/backward-space differences, Equation (7) can be represented by the approximate difference equation

\[ Q_i^n - Q_i^{n-1} + \alpha \beta (Q_i^{n})^{\beta-1} \left( \frac{Q_i^{n+1} - Q_i^n}{\Delta t} \right) \approx r \]  \hspace{1cm} (8)

where, for example, \( Q_i^n = Q(i \Delta x, i \Delta t) \), such that \( i = 1, 2, \ldots i_{\text{max}} \) and \( n = 0, 1, 2, \ldots n_{\text{max}} \) (\( i \) counts downstream distance steps;
\( n \) counts time steps. This can be simply solved for the unknown stream discharge

\[
Q_i^{n+1} \approx Q_i^n + (U_c)^n \left( \frac{Q_i^n - Q_i^{n-1}}{\Delta x} + r \right) \Delta t.
\]

where \( U_c \approx \frac{Q_1 - Q}{x \beta} \) (9)

and \( U_c \) is the celerity of the kinematic wave \([L \cdot T^{-1}]\). This is always greater than the cross-section average velocity of the stream water (Chapra 1997), defined by \( U = Q/A \).

### Sediment bacteria equation

We assume that the bacteria (and sediment) in the stream bed do not move downstream whilst in the bed, but can be entrained into the water column and so be convected downstream in the water (so there is no \( \partial / \partial x \) term). In that case we simply have

\[
\frac{\partial S}{\partial t} = -e_s \mu S: \mu = \frac{U - U_b}{U_b} \quad (10)
\]

where all variables are as defined earlier. Each term in Equation (10) has units \([\# \cdot L^{-1} \cdot T^{-1}]\). Using backward-time differencing for the temporal derivative, the approximating difference equation is

\[
S_{i+1}^{n+1} \approx (1 - e_s \mu \Delta t) S_i^n \quad (11)
\]

The initial density of bacteria in the bed, \( S_0 \ [# \cdot L^{-1}] \), is taken to be constant down the channel. For input to the model we typically have data for the number of bacteria per unit area of stream bed \((s)\). This is simply related to the \( S \) variable by \( S = sB_0 \).

### Aquatic bacteria equation

From mass conservation principles we obtain

\[
\frac{\partial AC}{\partial t} + \frac{\partial QC}{\partial x} = rC_i + e_s \mu S - kAC \quad (12)
\]

where \( C(x, t) [# \cdot L^{-3}] \) is the concentration of bacteria in the stream water, \( k \) is the bacteria inactivation coefficient; all other variables have been defined previously. Each term in Equation (12) has units \([\# \cdot L^{-1} \cdot T^{-1}]\). Using backward-time differencing for the temporal derivative and defining a segment volume as \( V = A \Delta x \), the approximating explicit difference equation is

\[
\frac{(VC)_{i+1} - (VC)_i}{\Delta t} \approx \frac{(QC)_{i+1} - (QC)_i}{\Delta t} + \frac{rC_i + e_s \mu S_i}{A} \Delta x \quad (13)
\]

Now, noting that the lateral inflow adds a volume of \( r \Delta x \Delta t \) to each segment during each time step, we can write the term \((VC)_{i+1}^{n+1}\) as

\[
(VC)_{i+1}^{n+1} = (VC)_{i+1}^n + (Q_i^n - Q_i^{n-1}) \Delta t [C_i^{n+1}] \quad (14)
\]

Substitution this result into Equation (13) gives

\[
C_i^{n+1} \approx \frac{V_i^n C_i^n + (Q_i^n - Q_i^{n-1}) - (Q_i^n + e_s \mu S_i^n) \Delta x \Delta t}{V_i^n + (Q_i^n - Q_i^{n-1}) + r \Delta x \Delta t} \quad (15)
\]

The numerator represents the mass in the segment \( i \) at the previous time step, while the denominator represents its volume.

### Indicative results

In Figure 2 we show predictions of the downstream hydrograph for conditions similar to those pertaining to the Tenepi Stream, for both "Cases I and II" (as defined in Appendix 1). The former occurs when the inflow duration \( t_0 \) exceeds the catchment’s "time of concentration" \( t_c \), defined as the instant at which the stream discharge at the monitoring site first reaches its maximum possible value. Case II arises when \( t_0 < t_c \).

In Figure 3 we show predictions of the downstream hydrograph Campylobacter versus E. coli for the same stream conditions studied in Figure 2a. In Figure 3a there is no sediment store \((s_0)\) is the initial number of bacteria per unit area of sediment\), but a constant concentration of Campylobacter in the lateral inflow \((500 \text{ per } 100 \text{ mL})\). In Figure 3a there is both a lateral inflow concentration of E. coli and it also has a substantial sediment store \((s_0 = 10^7 \# \cdot m^{-2})\).

### Discussion and conclusion

The results shown in Figure 2 show that for the wide channel, the explicit and implicit numerical schemes agree well with the analytical solution. Both exhibit some "clipping" at the
time of first reaching the peak (Case I) or at the time of first descent from the peak (Case II). The results further demonstrate that for Case I (time of concentration exceeds the inflow duration, i.e., $t_0 > t_d$) the discharge remains at its maximum value until the inflow ceases (at $t_0 = 15$ h). This is seen for both wide and narrow channels, as expected. In Case II ($t_0 < t_d$) the discharge remains at the value it attained at time $t_0$ (always less than the maximum attained for Case I) for a subsequent time which we call the “recession overhang”. In this case the flood wave for the narrow channel is higher and of shorter duration than for the wide channel, as may be expected.

Figure 3 demonstrates that the differential timing of peaks of Campylobacter and E. coli are able to be predicted by the model, merely by switching a sediment store off (for Campylobacter) and on (for E. coli), via the prescription of appropriate values for $s_0$ (the initial sediment areal storage). The Figure indicates that this differential timing pattern can occur even in the presence of some inactivation—field and laboratory studies show that the inactivation of Campylobacter under solar radiation is somewhat faster than the equivalent rate for E. coli (Obiri-Danso et al. 2001; Sinton et al. 2007a,b). The E. coli pollutograph’s narrowness reflects depletion of the sediment stores of that bacterium. Using an implicit scheme for bacteria concentrations should make these pollutants thinner.

This analysis indicates that Campylobacter may enter streams through storm runoff, rather than via sediment entrainment, and this could have important practical implications for the choice of effective Best Management Practices (BMPs) on farms, a topic of increasing interest in New Zealand (Collins et al. 2007), Lake et al. (2007), McBride & Chapra (in preparation). These results demonstrate the potential for simultaneous monitoring of indicators and pathogens, and associated kinematic wave modeling, to provide more informed understanding of the sources of the microbes that we are most interested in, yet seldom measured—the pathogens. Furthermore, the rapidly evolving field of microbial source tracking should provide even more discrimination on the differential infectivity pathogens according to their source. For example, Campylobacter from wild birds may be much less infective to humans than those originating from poultry, ruminant or human sources (French 2008).
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REFERENCES

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### NOMENCLATURE

<table>
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<tr>
<th>Term</th>
<th>Meaning</th>
<th>Units (^\d)</th>
</tr>
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<tr>
<td>(\alpha)</td>
<td>Proportionality factor in the kinematic wave equation, (A = \alpha Q^\beta)</td>
<td>(L^2 \cdot T^\beta)</td>
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<tr>
<td>(\beta)</td>
<td>Kinematic wave equation exponent ((\beta = 3/5) for Manning's equation)</td>
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</tr>
<tr>
<td>(\theta_b)</td>
<td>Channel bed slope</td>
<td>–</td>
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<tr>
<td>(\theta_s)</td>
<td>Channel side slope</td>
<td>–</td>
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<td>(\mu)</td>
<td>Relative stream water velocity excess, (\mu = (U - U_b)/U_b)</td>
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<td>(\rho)</td>
<td>Units conversion factor in Manning's equation, (Q = \rho [A^{5/3} q^{1/2}/(nP^{2/3})])</td>
<td>(L^{2-1/\beta} \cdot T^{-1})</td>
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<tr>
<td>(A)</td>
<td>Stream cross-section area</td>
<td>(L^2)</td>
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<tr>
<td>(B_0)</td>
<td>Stream bed width</td>
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<td>(C)</td>
<td>Bacteria concentration in the stream water</td>
<td># (L^{-3})</td>
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<tr>
<td>(C_b)</td>
<td>Bacteria concentration at baseflow</td>
<td># (L^{-3})</td>
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<tr>
<td>(C_l)</td>
<td>Bacteria concentration in the lateral inflow</td>
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<td>Stream sediment entrainment coefficient</td>
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<td>Distance counter for numerical scheme: (x = i\Delta x)</td>
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<tr>
<td>(k)</td>
<td>Bacteria inactivation coefficient</td>
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<td>(L)</td>
<td>Stream length from dam to downstream boundary</td>
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<tr>
<td>(M_l)</td>
<td>Delivery rate of land-derived bacteria per unit channel length</td>
<td># (T^{-1} \cdot L^{-1})</td>
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<td>(M_s)</td>
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<td>Stream channel wetted perimeter</td>
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<tr>
<td>(Q_b)</td>
<td>Stream discharge at baseflow</td>
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<tr>
<td>(Q_{\text{max},I})</td>
<td>Maximum discharge at downstream boundary for Case I ((\text{see Appendix 1}))</td>
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<td>Store of bacteria in stream sediments per unit channel length</td>
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<td>Steam segment volume, (V = A\Delta x)</td>
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<td>(x)</td>
<td>Distance along the channel from the dam</td>
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\(^1\) The symbol "\#" denotes numbers of bacteria; "L" denotes spatial dimension; "T" denotes time.
APPENDIX 1: Analytical flow solutions for a wide channel with positive baseflow

Henderson & Wooding (1964) developed an elegant analytical solution for an overland flow kinematic wave arising from steady rainfall of finite duration, given zero initial depth (see also Streeter 1966). This solution can be extended to the case of a lateral inflow of finite duration to a wide channel with positive baseflow and constant upstream boundary flow (such as may be obtained at a dam outlet). This provides a "benchmark" for associated numerical solutions—at least for the flow component in a wide channel. The key equation is Equation (5), i.e., \( A = \alpha Q^b \).

First we must first define two key time periods: the duration of lateral inflow \( t_0 \), and the time \( t_s \) at which a steady-state maximum flow is reached at the downstream boundary (at \( x = L \)). The latter time is also commonly called the catchment "time of concentration" (e.g., Eagleson 1970). It may be calculated as:

\[
t_s = \frac{2}{\alpha} \left[ (\alpha L + Q_b)^b - Q_b^b \right] \tag{A.1}
\]

where \( L \) is the length of the stream segment (from the dam to the downstream boundary), and all other terms are as defined previously (and are also listed in the immediately preceding Nomenclature section).

Following Henderson & Wooding (1964) we then define two cases—"Case I" and "Case II"—according to whether or not the lateral inflow duration exceeds the time of concentration (i.e., \( t_0 > t_s \) or \( t_0 < t_s \)). Their associated water surface profiles are depicted on Figure A.1. In Case I, the profile \( A'DC \) will be attained after a time \( t_s \). This will be maintained, with constant depth \( BC \) at the point \( B \), until lateral inflow ceases (at the later time \( t_0 \)). In Case II a water surface profile such \( A'DE \) will apply at time \( t_0 \), in which a plateau extends some way upstream of \( B \) with constant depth \( BE \) (less than \( BC \)). This depth will be maintained at \( B \) for some time after lateral inflow ceases, until a time \( t_{II} \), which we can calculate. This time increment \( t_p = t_{II} - t_0 \) we call the "recession overhang".

The analytical solutions for stream discharge are as follows:

Case I \( (t_0 > t_s) \):

\[
Q = \begin{cases} \left( \frac{rt}{\alpha} + Q_b \right)^{1/b} & : 0 < t \leq t_s \\ Lr + Q_b (= Q_{\text{max},I}) & : t_s < t \leq t_0 \\ Q_{\text{rec},I} & : t_0 < t \leq t_{II} \\ Q_b & : t \geq t_0 + L/U_{c,b} \end{cases} \tag{A.2}
\]

where \( U_{c,b} = Q_b^{1/b}/(\alpha \beta) \) is the celerity of the kinematic wave at baseflow (a known quantity), and

Case II \( (t_0 \leq t_s) \):

\[
Q = \begin{cases} \left( \frac{rt}{\alpha} + Q_b \right)^{1/b} & : 0 < t \leq t_0 \\ Lr + Q_b (= Q_{\text{max},II}) & : t_0 < t \leq t_{II} \\ Q_{\text{rec},II} & : t_{II} < t \leq t_{II} \\ Q_b & : t \geq t_0 + L/U_{c,b} \end{cases} \tag{A.3}
\]

The recession discharges \( Q_{\text{rec},I} \) & \( Q_{\text{rec},II} \) are solutions of \( f(Q) = L - [(Q - Q_b)/r + U_c(t - t_0)] = 0 \), which is implicit in \( Q \) (because \( U_c \propto Q^{1-b} \)). This equation can be readily solved, to high accuracy, using the standard Newton-Raphson iterative method.

The time variable \( t_{II} \) in Equation (A.3) is defined (for Case II only) as the last time for which the flow at \( B \) remains constant \( (= Q_{\text{max},II}) \). That is, \( t_{II} = t_0 + t_p \), where \( t_p = \beta(t_0 - t_0) \) is the afore-mentioned recession overhang, and \( t_{II} = \alpha(Q_{\text{max},II} - Q_b^{b-1})/r \) is the time to the (instantaneous) peak discharge that occurs when the lateral inflow duration equals the time of concentration (i.e., when \( t_0 = t_s \), occurring when Cases I and II coincide.

APPENDIX 2: Manning’s equation and its units

According to Henderson (1966), Flamant (in 1891) wrongly attributed this equation to Manning. He noted that it was derived by Gaucker in 1868 and Hagen in 1881.
In Europe the equation is often known as the Gauckler-Strickler equation, or just Strickler equation, although Strickler's contribution was made in the 1920s. The value of $n$ is calibrated such that the numerical value of $r$ is 1 in metres-second units (i.e., $r=1 \text{ m}^{1/3} \text{ s}^{-1}$). In contrast, were we to use the similarly popular Chézy resistance equation (in which case $\beta = 2/3$, whereas Manning's equation has $\beta = 3/5$), the value of $r$ is 1 in feet-second units (i.e., $r=1 \text{ ft}^{1/2} \text{ s}^{-1}$). Chow (1959, sec. 5–4) notes that even though a French engineer (Chézy) developed this equation (in 1769) it has become customary to report it in Imperial units.