

## ANFIS-based approach to scour depth prediction at abutments in armored beds

Mohammad Muzzammil and Javed Alam

### ABSTRACT

An accurate estimation of the maximum possible scour depth at bridge abutments is of paramount importance in decision-making for the safe abutment foundation depth and also for the degree of scour countermeasures to be implemented against excessive scouring. Most of the scour depth prediction formulae available in the literature have been developed based on the analysis of laboratory and field data using statistical methods such as the regression method (RM). The alternative approaches, such as artificial neural network (ANN) and adaptive neuro-fuzzy inference system (ANFIS), are generally preferred to provide better solutions in cases where the available data is incomplete or ambiguous in nature. In the present study, an attempt has, therefore, been made to develop the ANFIS model for the prediction of scour depth at the bridge abutments embedded in an armored bed and make the comparative study for the performance of ANFIS over RM and ANN in modeling the scour depth. It has been found that the ANFIS model performed best amongst all of these methods. The causative variables in raw form result in a more accurate prediction of the scour depth than that of their grouped form.

**Key words** | ANFIS, artificial neural network, bridge abutments, local scour, regression analysis

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### NOTATION

$d$	median size of the sediment	$g$	acceleration due to gravity
$d_a$	median diameter of armored-layer particles	$h$	depth of the approaching flow
$d_{ar} = d_a/d$	relative size of armored-layer particles	$h_r$	relative water depth = $h/L$
$d_{sa}$	scour depth in armored layer	$K$	coefficient of regression equation
$d_{sar}$	relative scour depth = $d_{sa}/L$	$n_1, n_2, n_3, n_4$	exponents of regression equation
$F$	Froude number	$r$	correlation coefficient;
$F_e$	$U_e/(\Delta gL)^{0.5}$	$\beta$	mean absolute percentage error
$L$	width of abutment normal to the approaching flow	$\gamma$	root mean square error
$T_r$	relative armored-layer thickness	$x, y$	inputs to the ANFIS
$U$	mean velocity of approaching flow	$z$	output of the ANFIS
$U_c$	mean velocity of approaching flow at the incipient motion of sediment	$A_i^k$	fuzzy set for input variable $x$
$U_{ca}$	critical velocity for armored-layer particles	$B_j^k$	fuzzy set for input variable $y$
$U_e = U - 0.5U_c$	the excess approaching flow velocity	$p_k, q_k, r_k$	parameters of consequent part
		$s$	relative density of sediment particles
		$f_k$	consequent function

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$a_i, c_i$	parameters of premise
$\mu_{A_i}(x)$	membership value of $x$ in $A_i$
$w_k$	weight of $k$ th inference rule
$\bar{w}_k$	firing strength of $k$ th inference rule
$k$	rule number
$K_s$	abutment shape factor
$R_k$	$k$ th rule
$z_k$	output of $k$ th inference rule
$\Delta$	$s-1$

## INTRODUCTION

Failure of bridges due to scour at their foundations consisting of abutments and piers is a common occurrence. Local scour at foundations has long been of concern for engineers (Cardoso & Bettes 1999). In the safety evaluation of bridges, local scouring of bridge foundation material near piers/abutments is, therefore, an important issue (Huber 1991; Dey & Barbhuya 2004a).

Riverbeds are commonly composed of a mixture of different sizes of sands and gravels in the upper reaches. A process of armoring on the riverbeds commences under the varied stream flow velocities, resulting in an exposure of coarser particles due to washing out of the finer fraction. Melville (1975) is probably the first who recognized the scouring potential for the armoring on the riverbeds. Ettema (1980) studied the scour at circular piers in armored beds in the context of the collapse of the Bulls Bridge over the Rangitikei River in New Zealand. They studied scour at piers in thin armored layers and stratified beds. The thickness of the stratified bed is greater than that of the natural armor layer thickness. Froehlich (1995) reported the natural armor-layer thickness as being one to three times the armoring particle sizes. Dey & Barbhuya (2004a, b) studied the scour at abutments embedded in an armored bed. They found that a larger scour depth develops at an abutment embedded in an armored bed (unless a secondary armored layer developed within the scour hole) than if the abutments were embedded in a bed of uniform sediments. Dey & Raiker (2007) investigated clear-water scour at circular and square piers, experimentally embedded in a sand bed overlain by a thin armored layer of gravels. They showed that the scour depth at a pier

with an armored layer under the limiting stability of the surface particles is greater than that without an armored layer for the same bed sediments, if the secondary armoring formed within the scour hole is scattered. On the other hand, the scour depth with an armored layer is less than that without an armored layer for the same bed sediments, when the compact secondary armor layer shields the scour hole.

Most of the available scour depth prediction equations for bridge piers and abutments are based on the regression analysis of laboratory data. It has been found that the laboratory data-based regression equations do not accurately predict the prototype conditions and hence they mostly give conservative results and overestimate the scour depth. This is attributed to the fact that the conventional analysis of data cannot include the correct influence of the set of influential parameters on scour depth. There is a lack of reliable formulae for prediction of the scour depth to cover different ranges. The results from the existing methods greatly differ from each other, thus resulting in a major controversy in the design and cost of the protection methods. A great deal of research effort has, therefore, been devoted to exploring and refining the methods for improving traditional physically based analysis in such situations. Recently, artificial neural networks (ANN) and adaptive neuro-fuzzy inference systems (ANFIS) are commonly used as alternative approaches to the traditional regression analysis. The fuzzy inference system (FIS) has been employed in the prediction of uncertain systems because its application does not require knowledge of the underlying physical process as a precondition (Bateni & Jeng 2007; Bateni *et al.* 2007).

Fuzzy logic has been widely used in rainfall-runoff modeling (Şen & Altunkaynak 2004), modeling uncertainty in the prediction of bridge pier scour (Johnson & Ayyub 1996), reservoir operation control (Shrestha *et al.* 1996), optimal water allocation (Kindler 1992), modeling the infiltration and water movement in the unsaturated zone (Bardossy & Disse 1993), regional drought analysis (Pongracz *et al.* 1999), modeling of time series (Altunkaynak *et al.* 2004a, b), fuzzy logic model for equilibrium scour downstream of a dam's vertical gate (Uyumaz *et al.* 2006), ANFIS-based approach for the prediction of pile group scour (Bateni & Jeng 2007), prediction of wave parameters (Ozger & Sen 2007), stream flow prediction (Ozger 2009) and an ANFIS-based approach to predict the scour location of spillway (Azmathullah *et al.* 2009).

Almost all of these studies indicate that the ANFIS results are a more accurate prediction compared with the nonlinear regression approach.

The available literature on the abutment scour revealed that the exact scour mechanism and the effects of different parameters on scour depth are yet to be fully understood or explored (Barbhuya & Dey 2004). The literature on scour at abutments in armored beds is very scanty, though the problem of scour at abutments in uniform and non-uniform sediments has been well explored by various investigators (Dey & Barbhuya 2004a, b). Further there is no single analytically derived equation which is valid for a wide range of flow conditions, bed material properties and abutment shape configurations, because of the difficulties in precisely modeling the phenomenon in a laboratory medium. Lack of understanding of complex flow conditions and simplified modeling of the phenomenon would lead to pronounced modeling uncertainty. On the other hand, reliable field data are scarce, leading to calibration problems. Engineering solutions concerning estimation of the safe foundation depth and selection of scour protection measures would be subject to uncertainty (Yanmaz & Kose 2007). The use of fuzzy set theory allows the user to include unavoidable imprecision in the data. Fuzzy inference is the actual process of mapping from a given set of input variables to an output based on a set of fuzzy rules. A fuzzy inference system based on fuzzy IF-THEN rules has the ability to deal with ill-defined and uncertain systems. The fuzzy modeling approach is a weighted average of several linear models, which are introduced by the rules.

The available literature on the application of ANN and ANFIS to scour depth prediction at abutments in uniform sand beds in general and abutments in armored beds in particular is limited. Further, it has also been reported that the ANNs are associated with difficulties such as success in a given problem and an unpredictable level of accuracy that could be achieved. The usefulness of ANNs and ANFIS compared to the traditional methods must therefore be checked for every application and their performance should also be ascertained by trying out different combinations of network architectures and learning schemes (Azmathulla et al. 2008). As far as the problem of scour at abutments in an armored bed is concerned, it involves various types of uncertainties resulting from flow, geometry, sediments and statistically based prediction models. The ANFIS-based

approach for scour depth prediction in such situation appears to be more appropriate.

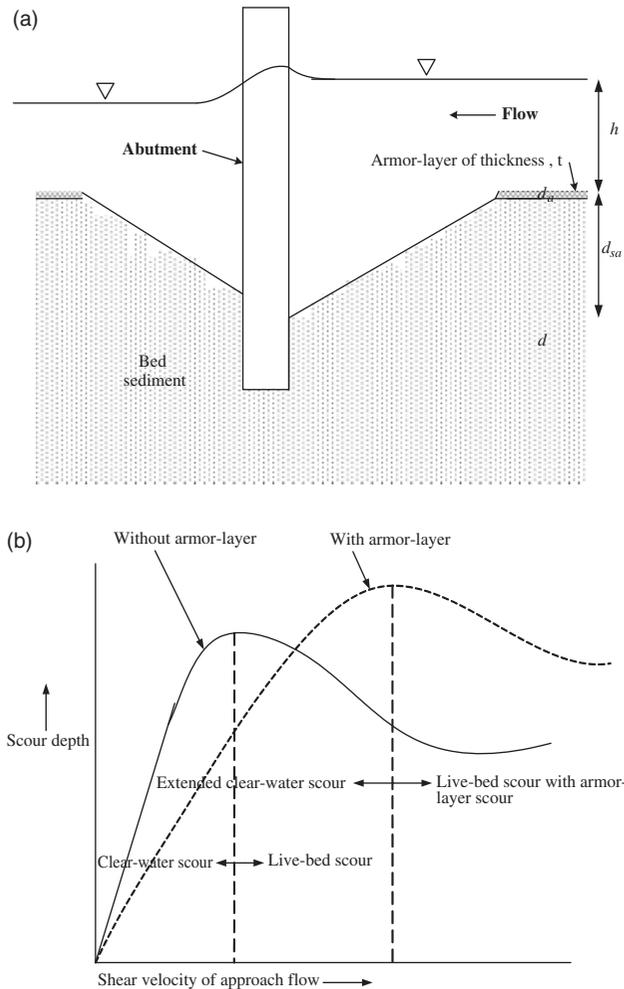
The main objective of the present study is, therefore, to develop an ANFIS model for scour depth prediction at abutments in armored beds. The performance assessment of the ANFIS model is then compared with that of the regression model and the ANN model using laboratory data from the literature.

## SCOUR IN ARMORED BED

When the abutment is founded in a bed of relatively fine sediment overlain by a layer of coarse sediment, the armored layer is formed due to the sorting of non-uniform sediment (Figure 1(a)). The armored layer extends the magnitude of critical shear velocity for the threshold motion of bed particles, maintaining an extended clearwater scour condition up to the limiting stability of surface particles. A relatively greater magnitude of scour depth develops in the vicinity of an abutment placed in an armored bed than if the abutment were embedded in a bed without an armored layer, due to the surface particles at the threshold condition (Figure 1(b)).

Ettema (1980), Raudkivi & Ettema (1985) and Kothiyari (1989) investigated the effect of stratification of the bed material on scour depth at cylindrical piers in the case of clear-water scour. It has been reported that the stratification, in which a relatively thin coarse top layer covers a thick fine bottom layer, is the critical condition. Once the top coarse layer is scoured away, scour depth will increase rapidly.

Dey & Barbhuya (2004b) carried out a detailed investigation on clear-water scour at abutments in thinly armored beds. The experimental data in the clear-water scour condition under the limiting stability of armored-layer particles was used to develop the equation of maximum equilibrium scour depth through regression analysis as provided in Equation (1), where  $d_{sar}$  = relative scour depth =  $d_{sa}/L$ ;  $d_{sa}$  = scour depth in armored bed;  $L$  = length of abutment perpendicular to the flow direction;  $K_s$  = abutment shape factor;  $F_e = U_e/(\Delta gL)^{0.5}$ ;  $U_e = U - 0.5U_c$  = the excess approaching flow velocity;  $U$  = mean flow velocity;  $U_c$  = critical velocity of bed sediment;  $\Delta = s - 1$ ;  $s$  = relative density of sediment particles;  $h_r$  = relative water depth =  $h/L$ ;  $h$  = depth of approach flow;  $t_r$  = relative armored-layer thickness =  $t/L$ ;  $t$  = armor-layer thickness;



**Figure 1** | (a) Definition sketch of scour at an abutment in an armored layer and (b) variation of scour depth with the approaching-flow shear velocity with and without an armored layer.

$d_{ar} = d_a/d =$  relative size of armored-layer particles;  $d_a =$  median diameter of armored-layer particles;  $d =$  median diameter of bed sediments:

$$d_{sar} = 6.18K_s F_e^{0.26} (h_r)^{0.18} (t_r)^{-0.19} (d_{ar})^{-0.15} \quad (1)$$

The non-dimensional parametric representation has been interpreted as:

- The term  $F_e$  is a measure of the ratio of excess approaching flow velocity  $U_e$  to  $(\Delta gL)^{0.5}$ . It represents the mobility of the submerged sediment particles in the vicinity of the abutment during scouring.  $U_e$  is less than or equal to zero when there is no scour.

- The term  $h/L$  refers to the effect of approaching flow depth  $h$  on the scour depth  $d_{sa}$ .
- The term  $d/L$  indicates the role of particle sizes of bed sediment on scour depth  $d_{sa}$ .

## RESULTS AND DISCUSSION

### Dataset of scour parameters

Laboratory data on the scour parameters relating to equilibrium scour depth around a vertical-wall, 45° wing-wall and semicircular abutments for the case of the clear-water condition in uniform sediments were obtained from the literature (Dey & Barbhuya 2004b). These data have been used herein for the development of the various models for scour depth prediction at bridge abutments. The 33 data values are available for each type of abutment with different approaching depths and sediment sizes, maintaining the conditions at approximately  $U/U_c = 0.95$ . The overall sample size of the data is 99. Table 1 shows the range of parameters for these datasets.

### Regression method for the scour depth prediction using non-dimensional dataset

Dey & Barbhuya (2004b) used all of the dataset to obtain Equation (1) using regression analysis. In the present study, a nonlinear regression method was used to get the regression parameters of the scour prediction model using 80% (79) of

**Table 1** | Data range of scour parameters

Parameters	Range
Abutment length, $L$ , (m)	0.06–0.100
Flow depth, $y$ , (m)	0.099–0.156
Mean velocity, $U$ , (m/s)	0.377–0.836
Sediment size, $d$ , (mm)	0.26–0.91
Scour depth, $d_{sa}$ (m)	0.102–0.29
Armor-layer thickness, $t$ (mm)	4.0–15.0
Size of armor-layer, $d_a$ (mm)	1.15–5.45
Critical velocity ratio, $U/U_{ca}$	0.90–0.965
Shape factor, $K_s$	0.75–1.00

the available entire data selected randomly after removing the outlier. It leads to Equation (2) for the estimation of scour depth at the bridge abutment. Validation of this equation was made with the help of the remaining unseen 20% (20) of the data, which were not involved in their derivation.

$$d_{sar} = 6.18K_s F_e^{0.26} (h_r)^{0.21} (t_r)^{-0.22} (d_{ar})^{-0.15}. \quad (2)$$

A comparison between observed and predicted values of scour was made in terms of three performance indices, namely (i) correlation coefficient ( $R$ ), (ii) mean absolute percentage error ( $mape$ ) and (iii) root mean square error ( $rmse$ ). The performance indices of the regression model during calibration and validation dataset are provided in Table 2. It may be observed that the performance of the regression model is satisfactory. It may be noted that the value of  $R$  for Equation (1) as proposed by Dey & Barbhuja (2004b) is 0.861, which is almost the same as in the present study.

The scour depth prediction Equation (2) has been obtained using the regression method (RM). The following drawbacks in any RM application have been pointed (Sun & Cheng 2005; Uyumaz *et al.* 2006).

- (i) The deviations of scatter points from the fitted curve have zero value with assumed constant variance. However, in an actual scatter diagram most often the variance is not constant but changes, depending on the independent variable value.
- (ii) The regression curve may pass close to a certain percentage of points in the scatter diagram, but this cannot account for the validity of the method.
- (iii) The prediction errors are expected to obey a Gaussian distribution function, which is not the case in many practical studies.
- (iv) The prediction errors are also expected to be independent from each other, i.e. completely random (noise).

**Table 2** | Performance of regression method

Stages	$R$	$mape$	$rmse$
Calibration	0.86	7.09	0.092
Validation	0.87	7.40	0.094

$R$  = correlation coefficient;  $mape$  = mean absolute percentage error;  $rmse$  = root mean square error

In order to avoid such problems in the application of the regression method, an alternative approach is generally advocated (Johnson & Ayyub 1996; Uyumaz *et al.* 2006). Recently ANN and ANFIS are commonly employed as alternative approaches to the traditional regression analysis (ASCE Task Committee 2000b; Azmathullah *et al.* 2005, 2006; Bateni & Jeng 2007).

### ANN approach for scour depth prediction using grouped dataset

Artificial Neural Networks (ANN) are considered to be a flexible modeling tool capable of learning the mathematical mapping between input and output variables of nonlinear systems (Bateni & Jeng 2007). The concepts involved in ANN along with their applications in water resources engineering are well described in ASCE Task Committee (2000a, b). Various applications of ANN in the field of hydraulic engineering have been presented by Azmathullah *et al.* (2005, 2006), Bateni *et al.* (2007) and Muzzammil & Siddiqui (2003).

The majority of ANN applications in hydraulic and water resources engineering involve the employment of the conventional feed-forward type (FF) of architecture, where there are no backward connections, trained using the error back propagation scheme. The network architectures of the regular feed-forward type are commonly trained using the standard back-propagation (FFBP) and cascade correlation (FFCC) training schemes to ensure proper network training. The radial basis function (RBF) network is also similar to the architecture of FFBP but it uses only one hidden layer, in which each neuron operates as per the Gaussian transfer function, as opposed to the sigmoid function of the common FFBP.

The ANN with training algorithms such as the Feed-Forward Back-Propagation (FFBP), Feed-Forward Cascade Correlation (FFCC) and Radial Basis Function (RBF) have been developed in the Matlab environment for scour depth prediction modeling in the present study. The Levenberg-Marquardt algorithm was used for faster training. This method involves a significant training parameter,  $\mu$ . When  $\mu$  is zero, the method corresponds to just Newton's method. When  $\mu$  is large, this becomes a gradient descent with a small step size. Newton's method is faster and more accurate near an error minimum, so the aim is to shift towards Newton's

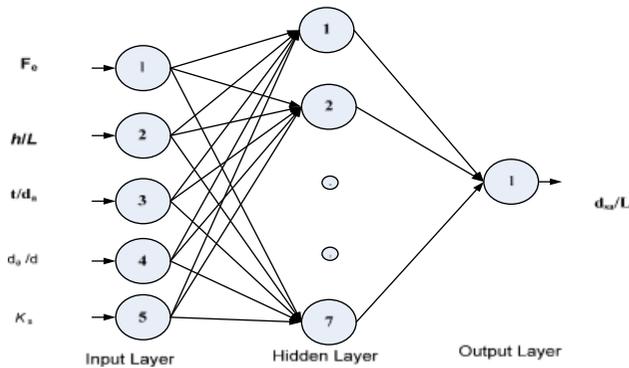


Figure 2 | A typical architecture for artificial neural networks.

method as quickly as possible. Thus,  $\mu$  is reduced after each successful step (reduction in performance function) and is increased only when a tentative step would increase the performance function. In this way, the performance function will always be reduced at each iteration of the algorithm. The optimum value of  $\mu$  in the present case has been found to be 0.001. The method involves the training of ANN with excess Froude number ( $F_e$ ), relative flow depth ( $h_r$ ), relative armored-layer thickness ( $t_r$ ), relative particle size of the armored layer ( $d_{ar}$ ) and shape factor ( $K_s$ ) as inputs and the relative scour depth ( $d_{sa}/L$ ) as output (Figure 2). The training data was the same randomly selected 80% of the entire available data for the network that was used in the regression analysis. The remaining 20% of data was used for validation. Table 3 shows the details of the network architecture for various training algorithms. The performance of various ANN models against the regression models was assessed quantitatively in terms of performance indices, as shown in Table 4.

Table 4 indicates that the performance of the FFBP training algorithm is the best among the other training algorithms of ANN. It may further be observed that all the ANN

Table 3 | Details of the ANN architecture

ANN models	Optimum number of neurons		
	Input layer	Hidden layer	Output layer
FFBP	5	6	1
FFCC	5	6	1
RBF	5	7	1

Table 4 | Performance of ANN models

Models	Training			Validation		
	<i>R</i>	<i>mape</i>	<i>rmse</i>	<i>R</i>	<i>mape</i>	<i>rmse</i>
FFBP	0.90	5.55	0.074	0.88	5.75	0.074
FFCC	0.92	4.34	0.063	0.85	6.93	0.090
RBF	0.88	6.21	0.082	0.85	7.49	0.092

*R* = correlation coefficient; *mape* = mean absolute percentage error; *rmse* = root mean square error

models with the all-training algorithms are superior to the regression model (Table 2).

### ANFIS-based models for scour depth prediction using grouped dataset

The fuzzy logic system has been widely applied to modeling, control, identification, prediction etc. (Sun & Cheng 2005; Ahmad & Ayyub 2006; Tiwari & Ayyub 2006). But the fuzzy model lacks in self-learning and adaptive ability. The neural network has been shown to possess both learning and adaptive ability to input-output data. It is proved to have a good approximation capability for a wide range of nonlinear functions and has been modeled for nonlinear dynamic systems. But in system modeling, network training results in a black-box representation. The model developed is difficult to interpret in human language (Sun & Cheng 2005).

The adaptive neuro-fuzzy inference system (ANFIS) is basically an integration of the techniques of fuzzy systems and artificial neural networks (ANN) (Ayyub 2006). The ANN provide connectionist structures and learning abilities to the fuzzy systems whereas the fuzzy systems offer ANN a structured framework with high level IF-THEN rule thinking and reasoning.

There are two types of fuzzy inference systems: (i) Mamdani type and (ii) Takagi-Sugeno (TS) type. Mamdani's fuzzy inference method is the most commonly seen fuzzy methodology and was among the first control systems built using fuzzy set theory. The Mamdani approach provides the outcome of the fuzzy rule as a fuzzy set for the output variable and hence the step of defuzzification is essential to get a crisp value of the output variable whereas the TS approach does not require a classical defuzzification procedure and the outcome of the fuzzy rule is a scalar

rather than a fuzzy set for the output variable. The main problem associated with TS fuzzy logic modeling is related to the selection of the parameters. An effective method is, therefore, required to tune the membership functions so as to minimize the error measures. Jang (1993) proposed the ANFIS approach to optimize the parameters of the membership functions and the consequent part by using a hybrid-learning algorithm. The fuzzy model parameters may be estimated by the various approaches such as clustering techniques, genetic algorithms, gradient decent algorithms and numerical analysis. The neural network back-propagation learning algorithm and the least-squares method are, however, simple and efficient methods and are generally employed to estimate the membership function parameters and the consequent part of the parameters, respectively (Uyumaz *et al.* 2006). Details of the ANFIS architecture have been provided in Appendix 1.

In the present study, ANFIS is used to get the fuzzy parameters for the prediction of scour depth at the bridge abutments in a armored bed of sediment. As in the previous case, here also only 80% of the available data was used for model prediction and remaining unseen 20% of data was used for testing of the model. This was done in the Matlab environment. ANFIS along with a subtractive clustering method was used for the scour depth prediction with excess Froude number ( $F_e$ ), relative flow depth ( $h_r$ ), relative armored-layer thickness ( $t_r$ ), relative particle size of the armored layer ( $d_{ar}$ ) and shape factor ( $K_s$ ) as inputs and the relative scour depth ( $d_{sa}/L$ ) as output.

The optimum value of cluster radius was determined by trial and error based on the criterion of maximum correlation coefficient and minimum root mean square error. The optimal radius of cluster has been found to be 1.0.

Figures 3 and 4 depict the details of the membership functions (MFs). The initial and final MFs of each of the input parameters ( $F_e$ ,  $h_r$ ,  $t_r$ ,  $d_{ar}$ ,  $K_s$ ) may be compared with each other. The change in the shapes of the MFs for an input after training reflects its influence on the output.

The summary of the results of the ANFIS model has been given in Table 5. It may be observed that the number of fuzzy rules corresponding to the cluster radius of 1.0 is four.

The performance of the ANFIS model was assessed and performance indices are given in Table 6 (last row) for training as well as validation processes. An overall

assessment of the various scour depth prediction models has also been made in this table. It may be observed that the ANFIS model shows the best performance among the other prediction methods under consideration. A qualitative comparison among the various prediction models may also be observed in Figure 5. It also shows a similar conclusion. The treatment of nonlinearity in the scour data based on the

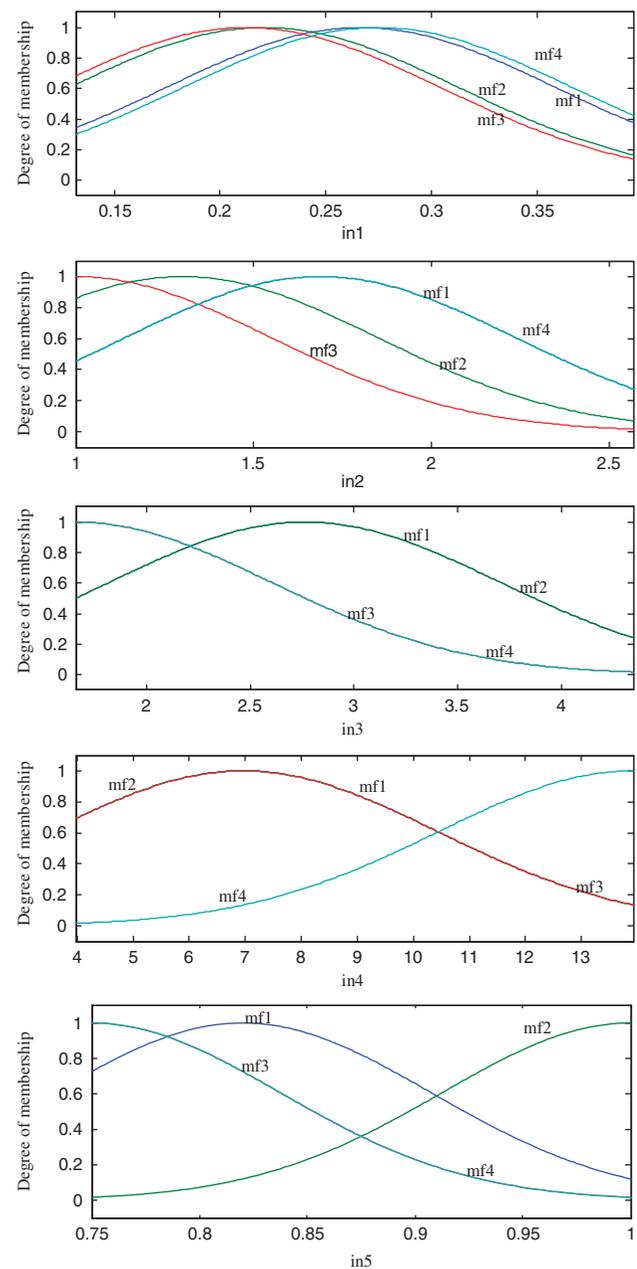


Figure 3 | Initial membership function for inputs (in1 =  $F_e$ , in2 =  $h_r$ , in3 =  $t_r$ , in4 =  $d_{ar}$ , in5 =  $K_s$ ).

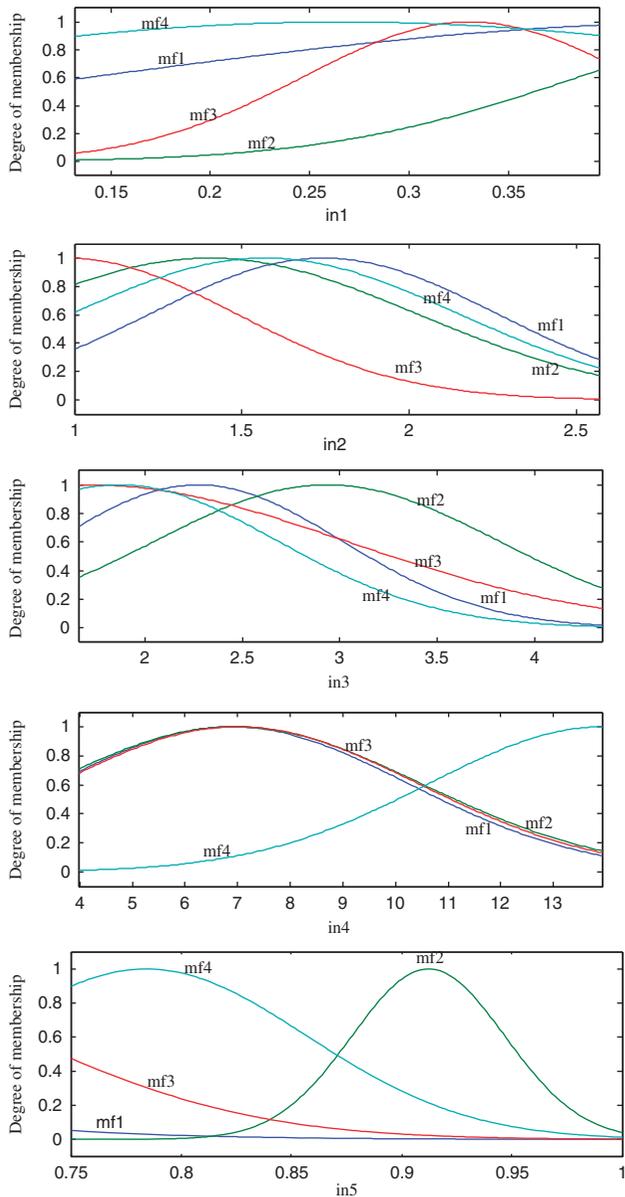


Figure 4 | Final membership function for inputs (in1 =  $F_e$ , in2 =  $h_r$ , in3 =  $t_r$ , in4 =  $d_{gr}$ , in5 =  $K_s$ ).

ANFIS approach worked much better than in the other schemes. It means that the scour data are more amenable to fuzzy if-then rules rather than crisp value processing.

**ANFIS models for scour depth prediction using original dataset**

The pattern of the data presented for training is considered to be one of the important aspects of the ANFIS network

Table 5 | ANFIS model parameters for grouped data

Parameters	Number
Nodes	56
Linear parameters	24
Nonlinear parameters	40
Total parameters	64
Training data pairs	79
Checking data pairs	20
Fuzzy rules	4
Cluster radius	1.0

approach. In this case, the ANFIS model, along with a subtractive clustering method, was used for the scour depth prediction with the original variables such as the excess approach velocity ( $U_e$ ), the abutment length ( $L$ ), flow depth ( $h$ ), thickness of armored layer ( $t$ ), particle size of armored layer ( $d_a$ ), sediment size ( $d$ ) and shape factor ( $K_s$ ) as inputs and the scour depth ( $d_{sa}$ ) as output.

The details of the parameters for the ANFIS model for raw data are provided in Table 7. A comparison of the performance of the models based on grouped data and raw data has been made in Table 8. A close observation of this table indicates that the raw data provide better performance than that of the grouped dataset. These results are in line with those of Bateni *et al.* (2007).

**A comparison of the proposed scour prediction models with the existing equilibrium scour depth prediction equations**

Most of the previous empirical equations have been developed for the case of the maximum equilibrium scour depth in

Table 6 | A comparative assessment of various models

Models	Training			Validation		
	R	mape	rmse	R	mape	rmse
RM	0.86	7.09	0.092	0.87	7.40	0.094
FFBP	0.90	5.55	0.074	0.88	5.75	0.074
ANFIS	0.95	4.16	0.05	0.91	5.48	0.075

R = correlation coefficient; mape = mean absolute percentage error; rmse = root mean square error

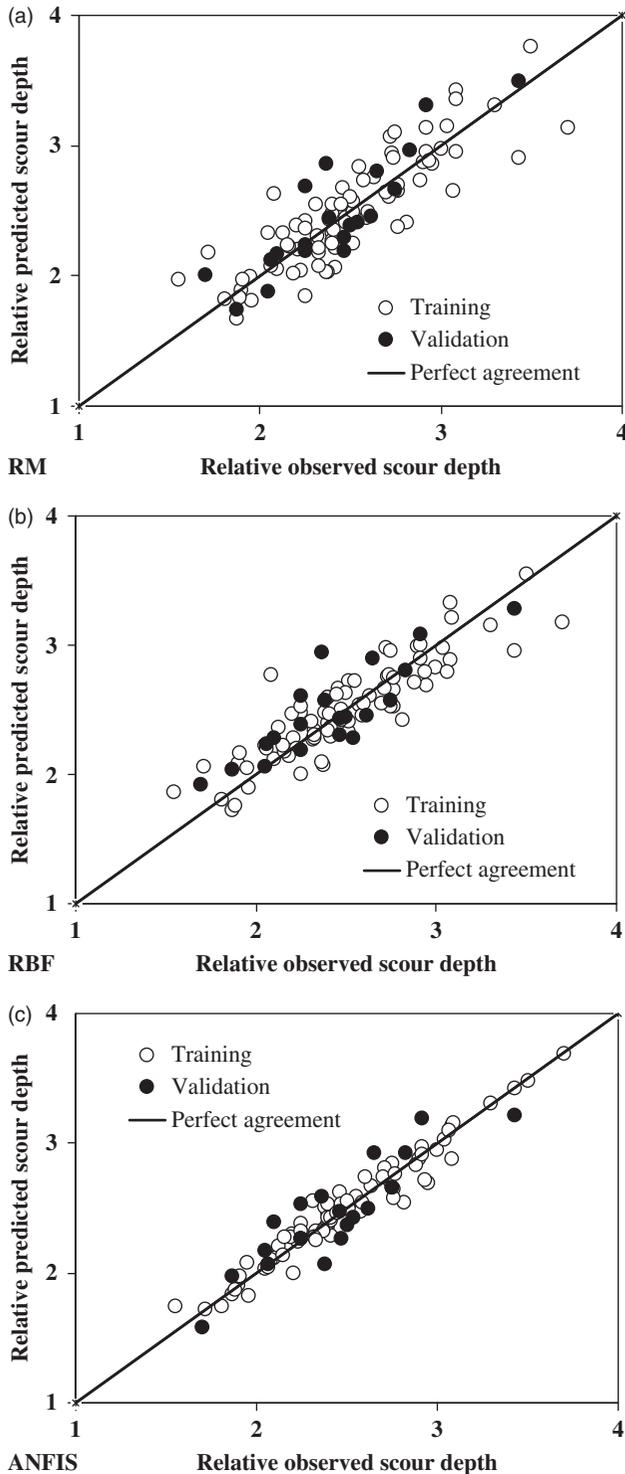


Figure 5 | Scatter diagrams of relative observed and predicted scour depths for (a) RM, (b) ANN (RBF) and (c) ANFIS.

Table 7 | ANFIS model parameters for raw data

Parameters	Numbers
Nodes	37
Linear parameters	14
Nonlinear parameters	24
Total parameters	38
Training data pairs	79
Checking data pairs	20
Fuzzy rules	2
Cluster radius	1.5

uniform sediments, which corresponds to the condition of the initiation of sediment motion, i.e.  $U = U_c$ . In order to compare the present models for the scour depth predictions with the scour predictions of previous empirical formulae, the proposed regression equation may, therefore, be reduced for the expression of the maximum scour depth at  $U = U_{ca}$  as given below. Equation (2) may be written as

$$d_{sam} = 5.16K_s F_{ca}^{0.26} (h_r)^{0.21} (t_r)^{-0.22} (d_{ar})^{-0.15} \quad (3)$$

where  $d_{sam} = d_{sam}/L$  and  $F_{ca} = U_{ca}/(\Delta gL)^{0.5}$ ; that is the critical Froude number. For maximum scour depth  $d_{sm}$  at the abutment with uniform sediments ( $d_{ar} = 1$  and  $t_r = 1$ ; for no armored layer), Equation (3) is reduced to

$$d_{smr} = 5.16K_s F_c^{0.26} (h_r)^{0.21} \quad (4)$$

where  $d_{smr} = d_{sm}/L$  and  $F_c = U_c/(\Delta gL)^{0.5}$ . A comparison of Equation (4) can now be made with corresponding empirical equations available in the literature (Table 9). In order to compare the estimated values of scour depth at a short vertical-wall abutment aligned with the approaching flow in a rectangular channel with a uniform sediment bed using the

Table 8 | Performance of ANFIS for data presentation

Data	Training			Validation		
	R	mape	rmse	R	mape	rmse
Raw	0.97	3.81	0.048	0.96	5.59	0.071
Group data	0.95	4.16	0.054	0.91	5.48	0.075

R = correlation coefficient; mape = mean absolute percentage error; rmse = root mean square error

**Table 9** | Scour depth prediction models for various investigators

Investigators	Scour prediction models	$d_{sm}$ (m)
Melville (1992)	$d_{se} = K_s K_d$	4.00
Lim (1997)	$d_{smr} = 1.80 K_s (h_r)^{0.5}$	4.03
HEC-18 (Froehlich 1989)	$d_{smr} = 2.27 K_s \Delta^{0.305} (h_r)^{0.265} F_c^{0.61} + h_r$	3.88
Richardson & Davis (2001)	$d_{smr} = 7.273 K_s \Delta^{0.165} (h_r)^{0.835} F_c^{0.53}$	8.90
Oliveto & Hager (2002)	$\frac{d_{st}}{L} = 0.085 \sigma_g^{-0.5} \left(\frac{h}{L}\right)^{0.33} F_c^{1.5} \left(\frac{L}{d_{50}}\right)^{0.75} \log T$	9.90
Dey & Barbhuya (2004a, b)	$d_{smr} = 5.16 K_s F_c^{0.26} (h_r)^{0.18}$	5.90
Regression model (present study)	$d_{smr} = 5.16 K_s F_c^{0.26} (h_r)^{0.21}$	5.95
ANFIS model (present study)		5.28

equations of different investigators, the following example is considered herein:

- abutment length,  $L = 2$  m;
- approaching flow depth,  $h = 2.5$  m;
- uniform sediment size,  $d_{50} = 1$  mm;
- standard deviation of particle size,  $\sigma_z = 1.10$ ;
- relative density of sediment particles,  $s = 2.65$ ;
- mean approaching flow velocity during flood peak,  $U = 0.5$  m/s; and
- critical flow velocity,  $U_c = 0.57$  m/s.

The estimated scour depth using the given data from the various investigators has been provided in Table 9. This table shows that the results of the present study are comparable with the scour prediction models available in the literature.

Appendix 2 shows (a) the portability of the proposed ANFIS-based scour prediction model and (b) the consistency of the calibrated model based on parametric study.

## CONCLUSIONS

An attempt was made to assess the performance of the various prediction models such as the regression method (RM), artificial neural network (ANN) and adaptive neuro fuzzy inference system (ANFIS) using an adequate size of laboratory data for scour depth at bridge abutments placed in an armored bed.

In the case of ANN models, the Feed-Forward Back-Propagation (FFBP) method was found to be the best among

the other training algorithms of ANN. The performance of ANN with all three training algorithms (FFBP, FFCC and RBF) has been found to be better than that of the regression method.

As far as the performance of the ANFIS model is concerned, it has been found that ANFIS is the best among the regression as well as ANN models. It has further been found that the ANFIS approach predicts scour depth better when it is trained with raw data rather than with grouped data

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## APPENDIX 1

### Architecture of the ANFIS

ANFIS was first put forward by Jang (1993) and is classified into three types according to the types of fuzzy reasoning and fuzzy if-then rule employed. The third of them is Takagi-Sugeno ANFIS. The selection of the fuzzy inference system (FIS) is the major concern in the design of ANFIS. The present study is based on Takagi and Sugeno's fuzzy if-then rule representation, wherein the consequent part of the rule is a linear function of the input variables and the parameters may be estimated by a simple least-squares error method.

Figure A1 depicts a typical architecture of ANFIS for a system with two inputs ( $x$  and  $y$ ) and a single output ( $z$ ). The inference mechanism of ANFIS is mathematically expressed by the set of rules. These rules are generated through the experience of the system operator, design engineer or expert.

The  $k$ th rule is generally expressed in the form (If *premise* THEN *consequence*) and is given by

If ( $x$  is  $A_i^k$  and ( $y$  is  $B_j^k$ ) Then

$$(z \text{ is } f_k = p_k x + q_k y + r_k) \quad (\text{A1})$$

where  $A_i^k$  and  $B_j^k$  are the  $i$ th and  $j$ th fuzzy term sets of representing  $x$  and  $y$ , respectively. The consequent function  $f_k = p_k x + q_k y + r_k$  has parameters  $p_k$ ,  $q_k$  and  $r_k$ , which are adjustable and are tuned in the training phase. A bell-shaped

or Gaussian membership function is commonly considered for each fuzzy subset that has three or two parameters. We have considered a Gaussian membership function in the present study which has only two parameters. Each fuzzy subset (say  $A_i$ ) is defined by a membership function  $\mu_{A_i}(x)$  as in (A2):

$$\mu_{A_i} = \exp\left(-0.5 \left(\frac{x - c_i}{a_i}\right)^2\right) \quad (\text{A2})$$

where  $a_i$  and  $c_i$  are the parameters of antecedent fuzzy set  $A_i$  of the  $i$ th membership function. These parameters control the shape of the Gaussian membership function.

The architecture of the ANFIS in Figure 1 has five layers. The functional details of these layers are:

**Layer 1:** This layer calculates the degree to which the given input  $x$  (or  $y$ ) satisfies the term set  $A_i$  (or  $B_j$  for input  $y$ ). The membership function for each term set  $A_i$  (or  $B_j$ ) may be a generalized bell function or Gaussian function, as described above. The parameters ( $a_i$ ,  $c_i$ ) for the Gaussian function are termed as premise parameters.

**Layer 2:** The nodes in this layer are fixed nodes labelled  $\rho$ . The output of each node is the product of the incoming signals. Thus the output of the  $k$ th node of the layer is given by

$$w_k = \mu_{A_i}(x) * \mu_{B_j}(y) \quad (\text{A3})$$

The output of each node represents the strength of the corresponding rule.

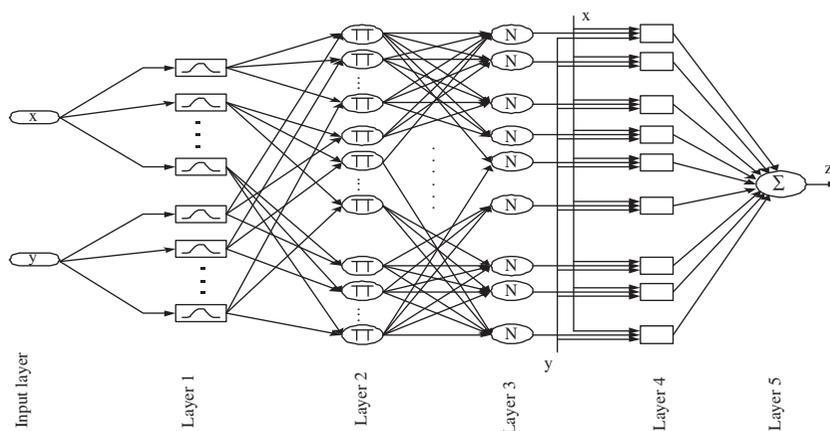


Figure A1 | Structure of a two input and single output ANFIS.

**Layer 3:** The nodes of this layer are fixed nodes labelled  $N$ . The  $i$ th node of the layer calculates the normalized firing strength of the corresponding rule. The normalized firing strength ( $\bar{w}_k$ ) of a rule ( $k$ th) is the ratio of the strength of that rule ( $w_k$ ) and the sum of the strengths of all rules, i.e.

$$\bar{w}_k = \frac{w_k}{\sum_k w_k} \quad (\text{A4})$$

**Layer 4:** The nodes of this layer are adaptive nodes. The node function of the  $i$ th node of the layer is given by

$$z_k = \bar{w}_k * f_k = \bar{w}_k * (p_k x + q_k y + r_k). \quad (\text{A5})$$

Here  $\bar{w}_k$  is the normalized firing strength of the  $k$ th rule, which is obtained in layer 3, and  $(p_k, q_k, r_k)$  is the set of parameters of this layer. These parameters are referred to as consequent parameters.

**Layer 5:** It has one node for single output, which is a fixed node labelled  $\sum$ . This node calculates the sum of all incoming signals. Thus the overall output ( $z$ ) of the ANFIS is

$$z = \sum_k z_k = \sum_k (\bar{w}_k * f_k) = \frac{\sum_k w_k * f_k}{\sum_k w_k} \quad (\text{A6})$$

The ANFIS modelling involves two major phases: (i) structure identification and (ii) parameter estimation. The structure identification amounts to determining the proper number of rules required, i.e. finding how many rules are necessary and sufficient to properly model the available data and the number of membership functions for input and output variables.

Clustering techniques have been recognized as a powerful tool to extract initial fuzzy rules from given input–output data. The purpose of the clustering is to identify a natural grouping of data from a large dataset to produce a concise representation of the system behaviour. Subtractive clustering is commonly used for the initialisation of the parameters for ANFIS training.

The steepest-descent (or back-propagation) method is applied for identification of the parameters. But this usually takes a long time before it converges. In the present case we find that some of the parameters are linearly related to the output (consequent parameters). Thus these linear parameters can be identified by the least-squares error (LSE)

**Table A1** | Basic steps of ANFIS hybrid learning

	Forward pass	Backward pass
Premise parameters	Fixed	Gradient descent
Consequent parameters	LS estimator	Fixed
Signal	Node output	Error signal

method, for fixed values of nonlinear parameters (premise parameters). After identifying the consequent parameters (which are linear) we can apply the steepest-descent method for identification of the premise parameters (which are nonlinear parameters). This hybrid learning approach, which combines the steepest-descent method and LSE method, gives rapid identification of parameters. The hybrid-learning algorithm has two passes, the forward pass and the backward pass. In the forward pass the node output is calculated until layer 4. Then LSE is applied to identify the consequent parameters. After this the backward pass is done. In this, the error signal is the input and is propagated backward. From the error signal the premise parameters are calculated by applying the steepest-descent method. The procedure of hybrid learning is summarized in Table A1.

## APPENDIX 2

### Application of the present ANFIS-based scour depth prediction model at abutment in armored beds

The application of the present model is very straightforward. The trained networks may be saved as a file. The following function of Matlab would then be used to find the output for the given input:

**Function**  $Y = \text{EVALFIS}(U, \text{FIS});$

This function simulates the Fuzzy Inference System (FIS) for the input data  $U$  and returns the output data  $Y$ . For a system with  $N$  input variables and  $L$  output variables,  $U$  is an  $M \times N$  matrix, each row being a particular input vector.  $Y$  is an  $M \times L$  matrix, each row being a particular output vector.

**Example 1**

- (i) Let the input parameters ( $F_e h_r t_r d_{ar} K_s$ ) be denoted as 'input':

$$\text{input} = \begin{bmatrix} 0.1 & 1.25 & 1 & 1 & 1 \\ 0.2 & 1.25 & 1 & 1 & 1 \\ 0.3 & 1.25 & 1 & 1 & 1 \\ 0.4 & 1.25 & 1 & 1 & 1 \\ 0.5 & 1.25 & 1 & 1 & 1 \end{bmatrix}.$$

In this case  $F_e$  has been varied and other parameters have been considered as constant.

- (ii) The relative scour depth ( $d_{sar}$ ) may be computed using the following function:

$$d_{sar} = \text{evalfis}(\text{input}, \text{fismat2})$$

The result of this function has been obtained as

$$d_{sar} = \begin{bmatrix} 2.6423 \\ 3.2300 \\ 3.3014 \\ 3.3165 \\ 3.3263 \\ 3.3363 \end{bmatrix}.$$

- (iii) The scour depth ( $d_s$ ) may be obtained by multiplying  $d_{sar}$  by  $L$  as

$$d_s = d_{sar} * L.$$

The result of this step has been obtained as

$$d_s = \begin{bmatrix} 5.2847 \\ 5.4904 \\ 5.7749 \\ 5.8387 \\ 5.8727 \\ 5.9080 \end{bmatrix}.$$

As such the scour depth may be assessed using the present developed scour prediction models.

**(b) Demonstration of consistency of the present calibrated model**

The consistency of the present calibrated model may be checked by carrying out the parametric study.

**Table A2** | Influence of Froude number on the scour depth

$F_e$	0.1000	0.2000	0.3000	0.4000	0.5000
$d_s$	5.2847	5.4904	5.7749	5.8387	5.9080

**Effect of Froude number on scour depth**

The details of computation of scour depth for some selected values of Froude number have already been explained in Example 1. The influence of Froude number on the scour depth has been shown in Table A2. It may be observed that the scour depth increases with the increase in  $F_e$ .

**Effect of relative flow depth on scour depth**

The computation of scour depth for the selected values of the flow depth has been explained briefly in Example 2.

**Example 2**

- (i) Let the input parameters ( $F_e h_r t_r d_{ar} K_s$ ) be denoted as 'input':

$$\text{input} = \begin{bmatrix} 0.1 & 1.25 & 1 & 1 & 1 \\ 0.1 & 1.50 & 1 & 1 & 1 \\ 0.1 & 1.75 & 1 & 1 & 1 \\ 0.1 & 2.00 & 1 & 1 & 1 \\ 0.1 & 2.25 & 1 & 1 & 1 \\ 0.1 & 2.50 & 1 & 1 & 1 \end{bmatrix}.$$

In this case  $h_r$  has been varied and other parameters have been considered as constant.

- (ii) The relative scour depth ( $d_{sar}$ ) may be computed using the following function:

$$d_{sar} = \text{evalfis}(\text{input}, \text{fismat2})$$

**Table A3** | Influence of relative flow depth on the scour depth

$h_r$	1.250	1.50	1.750	2.000	25000
$d_s$	5.285	5.49	5.660	5.775	5.873

The result of this function has been obtained as

$$d_{sar} = \begin{bmatrix} 2.6423 \\ 2.7452 \\ 2.8300 \\ 2.8875 \\ 2.9194 \\ 2.9364 \end{bmatrix}.$$

(iii) The scour depth ( $d_s$ ) may be obtained by multiplying  $d_{sar}$  by  $L$  as

$$d_s = d_{sar} * L$$

The result of this step has been obtained as

$$d_s = \begin{bmatrix} 5.2847 \\ 5.4904 \\ 5.6600 \\ 5.7749 \\ 5.8387 \\ 5.8727 \end{bmatrix}.$$

The influence of relative flow depth on the scour depth has been shown in [Table A3](#). It may be observed that the scour depth increases with the increase in  $h_r$ .