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Necessary and Sufficient Existence Criteria of Overconstrained Five-Link Spatial Mechanisms With Helical, Cylinder, Revolute, and Prism Pairs[1]

K. H. Hunt.[2] While appreciating the authors' dedicatory remarks, I am bound to throw doubt on their claims as summarized in Table 1. I make no direct comment on the technique used; this may in itself usefully extend Dimentberg's method, but it does not here reveal any new information about exceptional linkage-mobility.

First of all, those nineteen linkages in Table 1 which have C-pairs (Nos. 2, 4, 6, 8, 9, 10, 11, 13, 14, 15, 17, 18, 20, 22, 24, 27, 30, 32, 35), all have mobility 2, while the remaining sixteen have mobility 1. Freedoms can easily be added to any linkage, and thereby its mobility may be increased. Earlier in the paper the authors do in effect point this out (using the word "triviality") when they discuss planar and spherical solutions to the 5-R linkage. But the results for these nineteen $M = 2$ linkages are equally trivial; and certainly it is not true that ". . . . . the results remain unaffected if one of the helical pairs [of an $M = 1$ linkage] is replaced by a cylinder pair" (as the authors claim). The joints of each of the remaining sixteen $M = 1$ linkages all belong to that special form of fourth-order screw-system (four-system) which comprises screws of infinite pitch along all lines in space together with screws of all pitches along all lines parallel to a given *line* [27].[3]

As explained in [27, 28]—and indeed as is implicit in [21] and [22] (quoted by the authors)—, any five-freedom loop, four of whose screws are defining-screws for this special four-system, is not merely transitorily mobile but *automatically* full-cycle mobile, simply because the property of parallelism always leaves the geometry of the system unchanged even after joints within the loop have moved finitely. No complicated analysis of closure conditions (as pursued by the authors) is necessary to establish this fact; moreover, all of these sixteen remaining linkages are among those that have already been revealed [21, 22]. The authors' replacement of an H-pair by a C-pair is bound to raise the loop-mobility of the other nineteen from 1 to 2 because a C-pair is kinematically equivalent to two coaxial H-pairs of different pitches; the added screw that is required for the H-pair-to-C-pair-conversion belongs to the same four-system that had already determined the mobility.

Further, the property of parallelism of all H-pairs of finite pitch (including zero) is, [21, 22], the *only* criterion for the sixteen $M = 1$ linkages in Table 1; no further equations need be satisfied since the added prismatic pairs have merely to be inserted anywhere to close the loop provided that no two of them are ever parallel to one another and that no three are ever parallel to the same plane. The added "existence criteria" in entries 13 to 35 (Table 1) in the form of complicated equations appear to add restrictive conditions that are entirely unnecessary.

Next, it must be stated that the device the authors use to reduce from 2 to 1 the freedom of a certain C-pair in their "parent mechanisms" is highly restrictive, for the two freedoms in a C-pair are, in general, two coaxial screws of different pitches, not one screw of infinite pitch parallel to one of zero pitch. Those $M = 1$ loops that contain four and five H-pairs, and some of those that contain three, are thus bypassed. Yet all such loops have been revealed earlier [21, 22].

Moreover, in [21] and [22] there are other five-link $M = 1$ loops where full-cycle mobility can be explained solely through their screws belonging to the special five-system which comprises screws of infinite pitch along all lines in space, together with screws of all pitches along all lines parallel to a given *plane* [27]. The key to the linkages' automatically maintaining full-cycle mobility lies in dividing the six screws of a 6-H loop into two groups of parallel screws, *either* 3 + 3 *or* 2 + 4. To reduce the number of links to five, one pair of parallel H-pairs can coalesce coaxially to give a C-pair; and some further substitutions (consistent with screw-system geometry) of some H-pairs by R and P-pairs are valid. Yet even though all of these five-link loops have been described earlier they do not appear to be revealed by the authors' "necessary and sufficient existence criteria," and certainly they are not in Table 1.

The paper therefore succeeds only in partially revealing some of the results given in the references the authors acknowledge; and the process of revelation is extraordinarily laborious. Contrary to the clear meaning of the title (and elsewhere) the criteria are *neither* necessary *nor* sufficient. In the field within which the paper turns out to be limited the power of screw system geometry is amply sustained. The screw system approach shows straight away, incidentally, that parallelism and concurrence of axes must be avoided in any $M = 1$ 5-R linkage (a problem the authors admit they have not elucidated). For this linkage, all the R-axes must always belong to a linear congruence (whose properties are well known) and surely this is the tool that must be used by anyone who may try to outdo Myard [10] and Goldberg [7], or perhaps to prove rigorously that no more 5-R's can exist.

With due deference to the authors' opening sentence, the writer does not agree that this area per se is "one of the most . . . important . . . in the science of mechanisms," though it has its fascinations. Restatements of known exceptional linkage-mobilities (by whatever new methods)can only be important, surely, in the context *either* of realistic applications *or* of shedding light on some related field of endeavour.

Additional References

27 Hunt, K. H., "Screw Systems in Spatial Kinematics," Monash University, Department of Mechanical Engineering Report No. MMERS3, 1970.

28 Waldron, K. J., "The Mobility of Linkages," Doctoral dissertation, Stanford University, 1969.

Authors' Closure

The authors appreciate Professor Hunt's keen interest in the paper. His comments formed the basis for some stimulating discussion at the time of the presentation of the paper at the conference in San Francisco.

Professor Hunt has raised several points in his discussion. The authors intend to deal with them in a systematic manner.

At the very outset, the discusser throws doubt on the authors' "claims" (as he puts it) as summarized in Table 1 on the ground that this table does not reveal any new information about exceptional mobility. This is quite surprising because nowhere in the paper have the authors claimed that the mechanisms listed in Table 1 are entirely new. The purpose of the paper, on the other hand, was to demonstrate the power and usefulness of Diment-

[1] By P. R. Pamidi, A. H. Soni, and R. V. Dukkipati, published in the Aug. 1973 issue of the JOURNAL OF ENGINEERING FOR INDUSTRY, TRANS ASME Series B, Vol. 95, No. 3, pp. 737–744.

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[3] Numbers in brackets designate Additional References at end of discussion.

berg's method in obtaining the existence criteria of the 5R mechanism and certain other overconstrained mechanisms with five links and consisting of two or three prismatic pairs. The authors were perhaps guilty of not making this point very clear in the paper. The authors, however, do wish to emphasize one significant point here. It should be noted that the results in [21] and [22] were obtained by considering the five-link H-H-H-H-H mechanism proposed by Voinea and Atanasiu (see authors' additional reference [28]) which is itself an overconstrained mechanism. The results in the present paper have, on the other hand, been obtained by considering the more general zero family mechanisms. The present results, therefore, go beyond those of [21] and [22] and show that there are no mechanisms with two passive couplings and consisting of two or three prismatic pairs other than those obtained in [21] and [22] and confirmed in this study.

The 19 mechanisms in Table 1 that have C-pairs do indeed have, as Professor Hunt has rightly pointed out, a mobility of two and represent essentially trivial cases. The authors feel that these mechanisms should not have been included in the table as they detract from its value. Further, the statement in the latter part of the paper that the results remain unaffected even if one of the helical pairs is replaced by a cylinder pair is certainly not correct. Clearly, the replacement of a helical pair by a cylinder pair will result in an increase in mobility from one to two.

The discusser states that the property of parallelism of the helical pairs of finite pitch (including zero) is the only criterion for the existence of the 16 mechanisms with mobility one listed in Table 1 and that the equations in the entries 13 to 35 in the table appear to add restrictive conditions that are unnecessary. This is, however, not correct. A closer examination of these equations shows that they are not additional restrictions but are merely loop-closure conditions that must be satisfied by the constant kinematic angles of the mechanisms in order that the parallelism of the helical pairs may be preserved. Thus, for example, take the case of the five-link H-H-P-P-H mechanism shown in Fig. 7. If the three helical pairs in this mechanism are parallel to one another, the three twist angles β, γ, and δ and the two constant displacement angles χ_k and ξ_k at the two prismatic pairs cannot all have arbitrary values. A closed configuration with parallel helical pairs will result only when these five kinetic angles satisfy equation (31). Similar considerations apply to equations (35) through (38), equation (41), equation (43), as well as to the latter portions of equations (25) through (27).

The discusser remarks that the way in which the authors reduce the freedom of a cylinder pair from two to one is restrictive. The authors disagree. Whether one regards a cylinder pair as a combination of two helical pairs of different pitches (as the discusser suggests) or as a combination of a prismatic pair and a helical pair of finite pitch (including zero) as the authors have done is really not important. Both are equally valid concepts. The actual concept employed in any given case is decided entirely by the context of the development. The only important point to note in the present context is that a cylinder pair is a joint with two degrees of freedom in which the rotation is independent of the translation. The pair reduces to a joint with one degree of freedom if either the rotation or the translation is suppressed or if the rotation and the translation are forced to have a constant ratio between them. The cylinder pair reduces in these cases to a prismatic, revolute, or helical pair, respectively.

The discusser's statement that the derived criteria are neither necessary nor sufficient is not correct. In the general context of Dimentberg's method, it should be pointed out that the criteria are always necessary. This follows as a direct consequence of the method since the criteria are derived assuming that a mechanism of the desired type exists. This, however, does not guarantee that the derived criteria will always yield a nontrivial mechanism of the desired type. It should, therefore, be emphasized that the existence criteria obtained by Dimentberg's method are always necessary, but not always sufficient.

The discusser complains that the process of revelation is laborious. This is admittedly a drawback which could be quite significant in certain cases. One should, however, not forget that the Dimentberg approach has other compensatory features. First, as already mentioned, the method is capable of yielding the necessary conditions for existence. These include all possible solutions since any and every mechanism of the type under consideration must satisfy these conditions. Second, and this is more important, there is the assurance of finite mobility. This follows from the nature of the method. Since one starts with a parent mechanism of assured finite mobility, the finite mobility of the derived mechanisms is assured. The same cannot be said for other methods including screw theory. These other methods are always concerned essentially with only transitory or instantaneous mobility. Finite mobility results, as the discusser himself has pointed out, only when it can be shown that instantaneous mobility exists in all positions of the mechanism. Further, as mentioned in the paper, Dimentberg's method is particularly well-suited for obtaining the existence criteria of mechanisms in which there are conditions imposed not only on the twist angles, but also on the link lengths. The 5R mechanism considered in this paper and the R-C-R-C mechanism considered in Appendix D of [20] are examples of such mechanisms. Screw theory is certainly not capable of handling such cases.

The authors are sorry to note that the last paragraph of Professor Hunt's comments does not contribute to the discussion of the subject on hand. The discusser is entitled to his opinion on the mobility of space mechanisms, but the authors do not share his view. The authors know that the mobility of mechanisms has been an important and interesting enough topic to have engaged the attention of several classical kinematicians as well as leading modern investigators like Dimentberg, Moroshkin, Sharikov, Voinea, and Atanasiu, and others. The authors are also aware of Professor Waldron's and the discusser's own interest in this area.

The authors also do not agree with the last sentence of the discusser's comments. Surely the application of new and interesting methods of study to known problems has been attempted before in the investigation of mechanism mobilities. Such studies reveal the versatility and usefulness of the methods employed. The authors like to cite references [21] and [22] themselves as examples in this regard. The authors recognize the contribution made by these papers to mobility study, but they do not think that the mechanisms obtained and discussed in these references are as exceptional and unique as the discusser would have us believe.

Additional Reference

28 Voinea, R. P., and Atanasiu, M. C., "Geometrical Theory of Screws and Some Applications to the Theory of Mechanisms," *Revue de Mécanique Appliquée*, Vol. 7, No. 4, 1962, pp. 845–860.

Analysis and Synthesis of Mechanical Error in Linkages—A Stochastic Approach[1]

C. Amarnath[2] and B. K. Gupta.[3] The authors have presented a simple and elegant tool for the analysis and synthesis of errors in linkages. A few comments on certain assumptions are in order. The authors have assumed that the pin may randomly lie anywhere inside the clearance circle. It has been shown [9] that the pin center has a predictable locus, and is restricted to a very small portion of the clearance circle; in other words the probabili-

[1] By S. G. Dhande and J. Chakraborty, published in the Aug. 1973 issue of the JOURNAL OF ENGINEERING FOR INDUSTRY, TRANS. ASME, Series B, Vol. 95, No. 3, pp. 672–676.

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