Journal of Applied Mechanics Discussion

Axisymmetric Elastic Waves Excited by a Point Source in a Plate[1]

J. R. Hutchinson.[2] Drs. Weaver and Pao are to be congratulated for a very thorough study of the elastic waves emanating from a transverse step load on the surface of an elastic plate. The purpose of this discussion is threefold: first, to make the reader aware of a much more simplistic approach taken by the discusser in a recent paper [1]; second, to compared this simplistic approach with the more precise approach of Weaver and Pao, and to point out both similarities and differences in the solution; and third, to suggest that perhaps consideration of even higher thickness modes might be of interest.

In reference [1] the same problem as treated by Weaver and Pao was considered. The approach used was to expand the solution in terms of the normal modes found by using Mindlin's plate theory. Instead of considering the radius approaching infinity, as Weaver and Pao did, the radius of the plate was chosen as just large enough so that for the points considered and for the time intervals of interest the wave would not have time to be reflected back from the boundary to the point considered. The transverse displacement response was found for both a step load and an impulsive load in dimensionless form. Because of the simplicity of the solution it was easy to solve the problem for the specific dimensions and physical properties used by Weaver and Pao. Results of this solution are shown in Figs. 1 and 2.

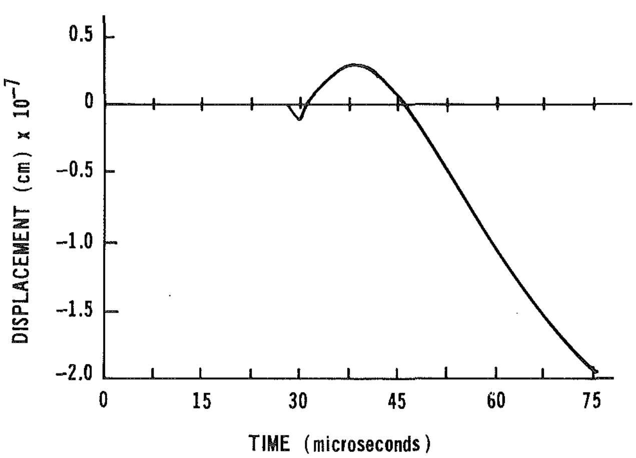

Fig. 1 Transverse displacement at a radius of 10 cm

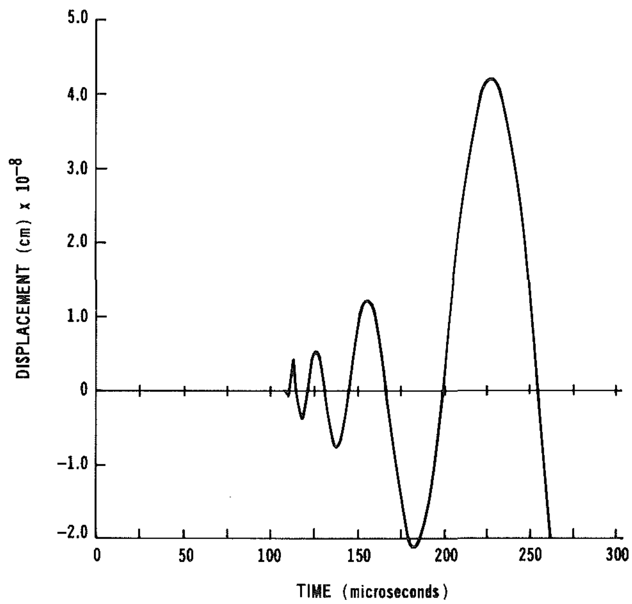

Fig. 2 Transverse displacement at a radius of 40 times the thickness ($r = 80h$)

The first thing to notice is that if Fig. 1 were inverted then both Figs. 1 and 2 would be almost identical in overall shape to Weaver and Pao's Figs. 8(*a*) and 10(*a*), respectively. The reason for the inversion on Fig. 1 is that I took my force as upward (in the same direction as positive transverse displacement) whereas Weaver and Pao took the force downward; however, by the same reasoning, Fig. 2 should also be inverted from Weaver and Pao's, which it is not. The second major discrepancy is in the magnitude of the response. My displacements are about two and half times smaller than those reported by Weaver and Pao.

The difference in sign and amplitude are too large to be ascribed to differences in solution methods, particularly when the solutions match in other important aspects. The simplicity of the method used in reference [1] also allows a very simple check on the accuracy of the modal superposition. The boundary conditions on the (finite) plate in reference [1] were clamped. Use of modal superposition of the particular solution of the modal equations of motion should, therefore, lead to the static solution for a clamped centrally loaded circular plate. This modal superposition check was performed and gave answers that were within three significant figures of the static solution as given on p. 69 of reference [2]. Unless I am misreading their physical and geometric properties I would have to conclude that Weaver and Pao have made some minor numerical error in their analysis which accounts for the sign and amplitude differences.

[1] By R. L. Weaver and Y. H. Pao and published in the December, 1982 issue of the ASME JOURNAL OF APPLIED MECHANICS, Vol. 49, pp. 821–836.

[2] Civil Engineering Department, University of California, Davis, Calif. 95616.

Other differences to note are that the simple solution is incapable of showing the spike caused by the Rayleigh surface wave on the upper surface. The simple solution in fact looks more similar to the response found by Weaver and Pao using only the first antisymmetric branch shown in their Fig. 7(*a*). There is no doubt that their solution contains many refinements that the simple solution cannot possibly show.

In reference [3] axial wave propagation in a circular rod due to laser deposition was studied. It was found that whereas an approach similar to that used in reference [1] yielded results that matched the gross behavior extremely well, certain details were not adequately represented without consideration of the higher thickness modes of the elasticity solution. It was found for instance that modes above the forty-fifth showed group velocities that approached the dilatational velocity and further had a large component of plane-type behavior. Thus these very high thickness modes were able to explain the experimentally observed waves that arrived at the dilatational velocity. Those results would indicate that there might also be something to be learned by investigation of the higher thickness modes in a plate (Weaver and Pao stopped at the tenth thickness mode).

In this brief discussion it has not been my purpose to denigrate the excellent work done by Weaver and Pao. Their investigation goes far beyond the simplistic approach of reference [1] and their response curves show the refinements brought about by the inclusion of many thickness modes. It has only been my purpose to show that the simpler approach is capable of describing the gross response of the plate, and to comment that further study of higher thickness modes might also prove fruitful.

References

1 Hutchinson, J. R., "Wave Propagation Using Mindin Plate Theory," *Proceedings of the Second Cairo University MDP Conference*, Dec. 27-29, 1982, Cairo, Egypt.

2 Timoshenko, S., and Woinowsky-Krieger, S. *Theory of Plates and Shells*, 2nd Ed., McGraw-Hill, New York, 1959.

3 Hutchinson, J. R., and Percival, C. M., "Higher Modes of Longitudinal Wave Propagation in Thin Rods," *Journal of the Acoustical Society of America*, Vol. 44, 1968, pp. 1204-1210.

Authors' Closure

The authors thank Dr. Hutchinson for his comments and for his careful comparison of the plate response determined from Mindlin's theory of a moderately thick plate and that from the Rayleigh-Lamb theory of a plate with arbitrary thickness.

In the preprint version of the authors' paper a factor of π was missing from the denominator of equation (7.1). This error and its ramifications were corrected in the published and reprint versions. The discrepancies in magnitude are explained if Dr. Hutchinson has made his comparisons with the early version. He is undoubtedly correct in pointing out the sign error in Figs. 10. The authors regret the confusion.

Investigation of higher modes is needed if one is interested in the high frequency response of the plate. In that case, and in the near field and at early times, the method of generalized ray is more effective. Based on that method, Ceranoglu and Pao (reference [12] in authors' paper) showed that the earliest arrived signal indeed travels with the speed of dilatational wave. The amplitude of these early arrivals, however, is much less than that of the later signals.

But if the interest is in the far field and with the highest frequency details of the earliest arrivals, a normal mode expansion of far more than 10 branches, as suggested by Dr. Hutchinson, would probably be necessary. One might conjecture however, that there exist asymptotically valid closed expressions for the earliest arrivals in the far field. A normal mode expansion of hundreds of branches would be very time-consuming and possibly prone to round-off errors.