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Longitudinal Grain Sorting by Current in Alluvial Streams

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The longitudinal current sorting of graded sediment is analysed by use of a sediment transport formula developed recently by Engelund and Fredsøe (1976). The analysis yields that for a given longitudinal profile of a river, the longitudinal variation in mean grain diameter and the grain distribution curve is unique determined by the sediment input and water discharge. No requirement concerning an ultimate equilibrium state of the bed elevation is introduced. The analysis yields that the mean grain size will decrease as the slope of the river decreases and further, that originally logarithmic normally distributed sand will tend to be logarithmic hyperbolic distributed after sorting.

Introduction

It is normally observed that the grain sizes in a river decrease in the downstream direction. This can not only be explained by abrasion due to reduction of grain sizes by mechanical processes acting on the grains during their travel through the river, see Rana, Simons and Mahmood (1973). In their paper, Rana et al. alternatively considered the changes of grain sizes due to current sorting: by considering a river where the longitudinal profile is in equilibrium in the sense that the mean level of the bed does not change with time (which involves that the rate of sediment transport is the same in all cross sections in the river) they found, that the mean grain size must decrease as the slope of the river decreases.

However, by applying the continuity equation on each size fraction of sediment, their theory predicted that the median diameter at a given cross section increases with time.

In the present paper the sorting of grain sizes by current is considered from a different point of view. The requirement that the sediment discharge is constant in all cross sections in the river is not maintained, so that the mean level of the bed now is allowed to change with time. However, it will be shown that the time scale for changes in the level of the bed is much larger than the time scale for current sorting of the grains. Hence, the present analysis is very straightforward: by use of a sediment transport formula and a given initial geometry and water discharge of the river, the changes with time of the grain sizes at a certain location is easy to investigate. The present model does not investigate the local sorting (sorting because of presence of dunes and meanders) so the mean grain diameter must in the following be interpreted as a mean value over at least one meanderlength.

Description of the Model

The present model treats the case where the water discharge per unit width is assumed constant along the river. Normally, the water discharge per unit width is slightly increased in the downstream direction, where the river width is increasing, corresponding to the regime formula

$$\frac{Q}{B} \sim B \tag{1}$$

where Q is the total discharge and B the width (see Engelund and Hansen 1972). However, the purpose of the present paper is mainly to investigate the effect of grain sorting by a current, so effects due to changes in river width and discharge are disregarded.

The longitudinal profile of the river bed h is initially described by an exponential function of the form

$$h = h_0 e^{-\alpha x} \tag{2}$$

where x is a coordinate in the flow direction, α is a constant and h_0 the level of the bed at $x=0$.

The boundary conditions of the problem have been chosen as follows:

1) The rate of sediment input at $x=0$ is constant. The bed sediment at $x=0$ is assumed to have a known mean diameter d_{mo}, and the sediment is here logarithmic normally distributed with a given standard deviation.

2) At the end of the river, the level of the water surface is kept constant, which for instance will be the case if the river enters the sea or a large reservoir.

The transportation of sediment is described by use of the sediment transport formula developed by Engelund and Fredsøe (1976). This model, which is based on physical ideas related to those introduced by Bagnold (1954), is able to split the total load up into bed load and suspended load. This is especially important in connection with the present problem, partly because the particle velocity is different if the particles stay in suspension instead of travelling as bed load, partly because the mean size of the sediment travelling in suspension is smaller than the mean size of the bed load sediment. Because of these facts, it is of importance to use an accurate sediment transport model, which takes at least some account of the gradation.

The above mentioned sediment transport model describes the transport in terms of mean quantities, as for instance the dimensionless bed load Φ_b is given as a function of the dimensionless shear stress θ

$$\Phi_b \equiv f(\theta) \equiv 5[1+(\frac{0.267}{\theta-\theta_c})^4]^{-\frac{1}{4}}(\sqrt{\theta}-0.7\sqrt{\theta_c}) \quad (3)$$

where Φ_b and θ is given by

$$\Phi_b \equiv \frac{q_b}{\sqrt{(s-1)g\,d_m^3}} \quad \text{and} \quad \theta = \frac{\tau}{\rho\,g\,(s-1)d_m} \quad (4)$$

q_b is the rate of bed load transportation, ρ the water density, g the acceleration of gravity, τ the shear stress, s the relative density of the sand grains, θ_c the critical Shields' parameter, and d_m the mean grain diameter.

To be able to account for the migration of different sand sizes, the relation (3) is assumed to be valid for each individual fraction of grain sizes, but normalized so that the correct total transport rate Φ_b is obtained. Hence, we may write

$$\Phi_b = \sum_i \Phi_{bi}\,\phi_i \quad ,$$

where Φ_{bi} is the rate of bed load transport for a single fraction and ϕ_i is a factor proportional to the volumetric percentage of that fraction, adjusted so that the total transport is correct. This procedure is close to the one applied by Einstein (1950) and verified by his flume tests with sediment mixtures.

The transport of suspended sediment is calculated in the following way: in the paper by Engelund and Fredsøe, the bed concentration of suspended sediment c_b was found to be a function of θ only. If λ_b is the linear concentration, related to c_b by

$$c_b \equiv \frac{0.65}{(1+1/\lambda_b)^3} \quad (5)$$

it was shown that λ_b is calculated by

$$\theta = \theta_c + 0.267[1+(\frac{0.267}{\theta-\theta_c})^4]^{-\frac{1}{4}} + 0.027s\theta\lambda_b^2 \quad (6)$$

The volumetric bed concentration, c_{bi}, of each fraction is then found as a function of θ_i, where $\theta_i = \theta \cdot d_m/d_i$, d_i is the mean diameter of that fraction.

When the bed concentration is known, the suspended transport Φ_{si} is found from graphs by Einstein (1950) which depends on the flow parameters and the fall velocity w_i of the grains belonging to the individual fractions.

The total suspended load is calculated in a similar manner. The mean bed concentration, c_b is based on θ, and the effective fall velocity is a mean of the fall velocities of the fractions for which the inequality

$$w_i < 0.8\, U_f \tag{7}$$

is fulfilled (Engelund 1973). U_f is the friction velocity, $U_f = \sqrt{\tau/\rho}$

The suspended load, Φ_{si}, of each fraction is then normalized in the same way as the bed load.

Now, the model works as follows: The river is divided into a number of sections of the length ΔL, and the sediment transport into a section from the upstream direction $q_s(x)$ and the sediment transport $q_s\ (x + \Delta L)$ out of the same section is calculated.

This results in changes in the bed elevation, which is calculated by a finite difference method based on the sediment continuity equation

$$\frac{\partial h}{\partial t} \equiv -(1-n)\frac{\partial q_s}{\partial x} \tag{8}$$

where n is the porosity of the bed (Engelund and Hansen 1972).

By use of the sediment continuity equation on each fraction of sediment, the composition of the deposited or eroded sand is calculated as the difference between the incoming and the removed sand. If deposition occurs, the deposited sediment is mixed up with the sand in the bed in case of presence of dunes. This mixing occurs in a layer of thickness equal to the dune height. The variation in the dune height H with different hydraulic conditions is until now not fully understood, but as an approximation, H has been put equal to 15% of the water depth (as done by Rana et al. 1973). As will be shown in the next section, this choice does not affect the variation in the grain sizes along a river, but is only of importance in that sense, that the time, necessary for a river to come into »current sorting equilibrium« from a non-equilibrium situation strongly depends on the dune height.

Changes in Grain Diameter with Time

In Fig. 1, the development in grain sizes, d_m, with time in a specific run is depicted. d_m is defined as the mean grain size in the active layer with the thickness H. The bed is assumed initially to be covered by sand with the same

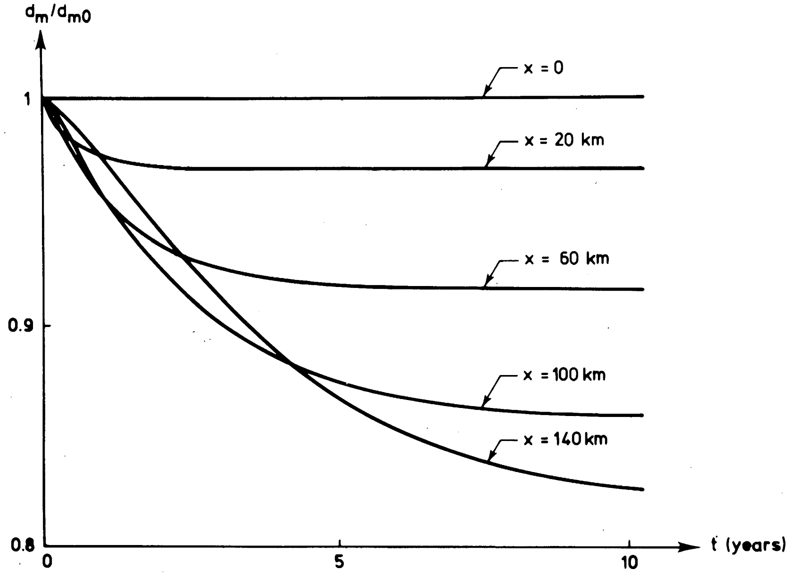

Fig. 1. Variation of the median grain diameter with time and space.
d_{m0} = 0.3 mm, q = 1 m²/s, h_0 = 100 m, α = 5.10^{-6}m^{-1}.

mean diameter and standard deviation on the whole stretch. Further, the bed is assumed to be dune covered. The hydraulic resistance is in this case calculated by use of the method proposed by Engelund (1966):

This involves Einstein's boundary-layer equation (Einstein 1950):

$$\frac{V}{U_f'} = 6 + 2.5 \ln \left(\frac{D'}{k}\right) \qquad (9)$$

V is the mean flow velocity, $U_f' = \sqrt{\tau'/\rho}$, τ' is the shear stress due to skin friction, $D' = D\sqrt{\tau'/\tau}$, where D is the mean flow depth, and k is the equivalent sand roughness defined as 2.5 d_m (Engelund and Hansen 1972).

The relation between the skin friction and the total shear stress is

$$\theta' = 0.06 + 0.4\,\theta^2 \qquad (10)$$

(Engelund 1966), where

$$\theta' = \frac{\tau'}{\tau}\,\theta$$

It is of interest to note from Fig 1, that the grain size at a certain place becomes constant after some time. The more downstream, the section in consideration is located, the more time will it take for the grain size to become constant, due to the fact, that the grain size at a given location is a function of the upstream grain sizes.

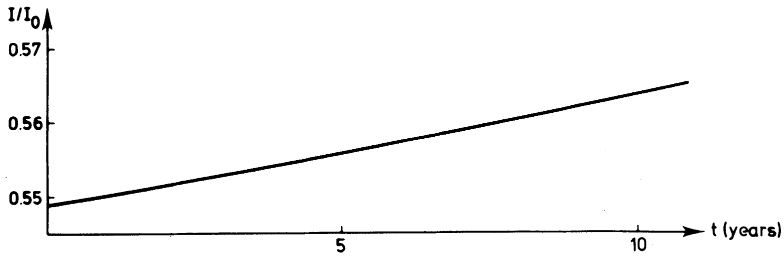

Fig. 2. Variation of the slope with time at x = 120 km.
(same data as in Fig. 1).

Further, it is seen from the figure, that the constant value, which the grain size attains is decreasing in the downstream direction, in which the gradient of the flow also decreases, according to Eq. (2).

In Fig. 2, the simultaneous changes in the gradient of the river at a certain location are shown. Because of the initial exponential form of the longitudinal river slope, the present model always involves that deposition will occur in the bed, because of the fact that the rate of sediment transport at a given water discharge is decreasing with decreasing slope.

At the beginning of the river, the slope I_0 remains constant, which is a consequence of the requirement, that the sediment input to the river is constant.

All other places, the gradient of the river will approach the value I_0 because the equilibrium situation for a river with a constant discharge and constant width is a straight line.

By comparison of Figs. 1 and 2, it is realized that the current sorting of the grain sizes is quasi-stationary: If the original composition of the bed is not in current-sorting equilibrium, this equilibrium will be attained very fast as compared with changes in the bed elevation.

It is seen that the river at $x = 120$ km is nearly in current sorting equilibrium after 10 years, while the slope in the same time only has changed from $0.55 \cdot 10^{-4}$ to $0.565 \cdot 10^{-4}$. The relative difference between the time scales for current sorting and development in slope depends on the length of the river: If we change the initial profile given by Eq. (2) to

$$h \equiv h_1 e^{-\alpha(h_0/h_1)x} \tag{11}$$

the current sorting equilibrium will be obtained even faster, compared with the changes in slope, if $h_1 > h_0$.

Spatial Changes in Grain Diameter

By use of results like those depicted in Figs. 1 and 2 it is easy to obtain the relationship between the local value of the gradient $I = I(x, t)$ and the local value of the mean grain diameter d_m at a certain time. As the changes in the river bed elevation occur very slowly, it is in this way possible to describe the quasi-stationary variation in the mean grain diameter in the longitudinal direction in a river with a known longitudinal profile. The result of such an analysis obtained for the same data as used in Figs. 1 and 2 is depicted in Fig. 3. It is seen that the analysis predicts that the mean grain diameter will decrease in the flow direction. The dotted line indicates the longitudinal distribution of d_m under the assumption, that the sediment transport is the same in all cross-sections in the river.

In order to test the theory against data, it is necessary to have a lot of samples of the sediment placed in the river bed, because the present theory does not describe the local sorting of the sediment, due to the presence of various bed forms. Such data are available in the Nedeco Report on the river Niger (1959), where a part of the lower river Niger is analysed in details. In Fig. 4, the measured and calculated variation in d_m in a about 400 km long stretch has been compared. This stretch has the special advantage, that no tributary is joining the main river, so the discharge is nearly constant through the stretch as is also the width of the river. The discharge has two peaks during the year, and as representative measure of the discharge, the mean value of the largest of these two peaks has been used. It is seen from Fig. 4 that the agreement between theory and observations is fairly good.

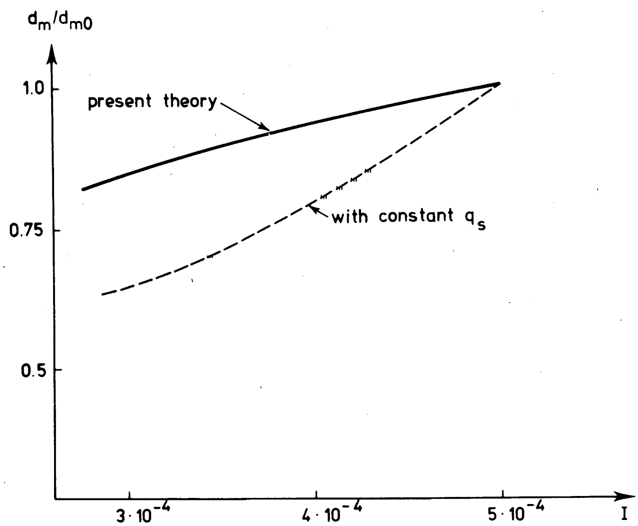

Fig. 3. The median diameter, d_m, as a function of the slope (same data as in Fig. 1). The relation shown by the dotted line is not stationary for a given longitudinal river profile.

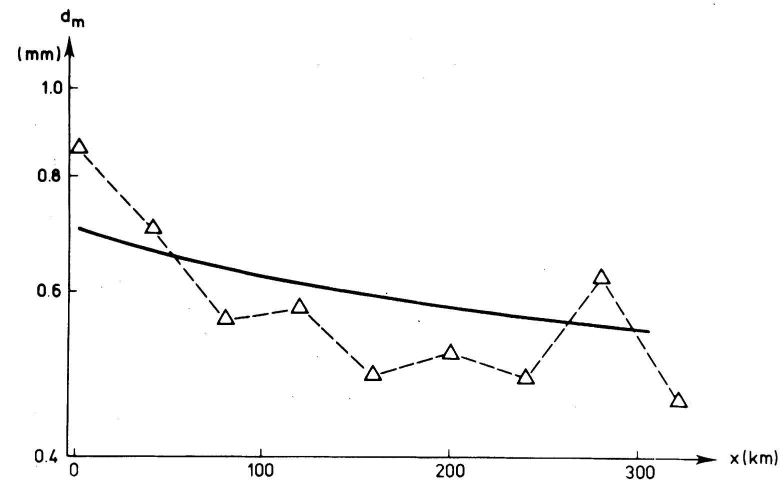

Fig. 4. The distribution of the median diameter along the lower river Niger, theoretical compared with measured. The following data are used: $d_{m\bar{\sigma}}$ = 0.7 mm, q = 10.6 m^2/s, I_0 = $1.07 \cdot 10^{-4}$, α = $1.38 \cdot 10^{-6} m^{-1}$.

Changes in Grain Size Distribution Curve due to Current Sorting

The most widely accepted distribution of sediments is the log-normal distribution. The occurrence of logaritmic normally distributed material can be explained as the outcome of a large block, that has been randomly divided and subdivided.

By analysing natural deposits Bagnold (1971) found that their probality density functions in most cases differ from the log-normal. If the diameter and the density function P is plotted on double logaritmic paper, the logaritmic normal distribution will become a parabola, but the measured distributions of sand will be more like hyperbolas, as pointed out and discussed in detail by Barndorff-Nielsen (1976).

As mentioned previously, the incoming sand at the start of the river is in the present model assumed to be logaritmic normally distributed. Besides the longitudinal changes of mean diameter, sorting in each individual fraction also takes place in the downstream direction, so that increasing deviations from the logaritmic normally distribution will occur. Some of the changes predicted by the present model correspond closely to what Bagnold observed in the laboratory.

In Fig. 5A some measurements obtained by Bagnold (1971) are depicted. These measurements are carried out in a windtunnel, and Bagnold obtained sorting of the sediment by increasing the area of cross section in the flow direction. This results in a decrease in the shear stress in the flow direction as does a decrease in the gradient in a river. The dotted line in Fig. 5a represents the distribution of the

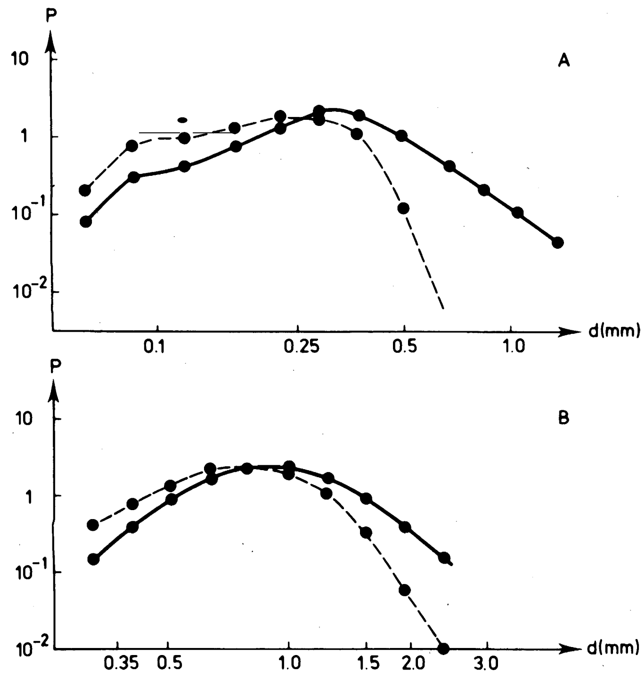

Fig. 5. A: Double-logarithmic plot of sand size distributions from Bagnold's experiments.
B: Double-logarithmic plot of sand size distributions from the present model at $x = 0$ km and $x = 600$ km. $d_{m0} = 1$ mm, $q = 4.7$ m²/s, $I_0 = 6.5 \cdot 10^{-4}$, $\alpha = 1 \cdot 10^{-6}\,\mathrm{m}^{-1}$.

sediment catched in a trap at the end of the windtunnel, while the full line represents the original sediment in the bed.

In Fig. 5B, the development in the grain distribution curve obtained from the present theory is shown. The trend in changes in the grain distribution curve is the same as that in Bagnold's experiments. Comparison with the present analysis is not possible, partly because Bagnold's experiments are made by wind, partly because of the lack of data.

Closing Remarks

In the present analysis, the grain sorting by current is considered under several simplifications of which the following should be emphasized: (i) the river meandering has not been taken into account, (ii) the discharge and width are assumed constant along the river, and (iii) variations in depth and current velocities in the transverse directions are neglected.

Item (i) may be of importance, because the helical motion introduces a transverse grain sorting in a bend, because the suspended load, which consists of finer grain sizes than the bed load, is transported in the inward direction by the helical motion, while the bed load by reasons of continuity must be transported in the outwards direction. Hence, the finer part of the sand occurs at the inner part of a bend. An investigation of this problem is rather complex and has for the present been avoided.

Item (ii) is easy to incorporate in the present analysis, but is of major importance what concerns morphological changes of the river, rather than what concerns the specific problem of current-sorting.

Finally item (ii) may change the presented results slightly: As the current velocity is greater in the middle of the river than close to the banks, the rate of longitudinal grain sorting may vary over the river cross section.

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