Nuclear Polarization of $^{12}\text{B}$ in Muon Capture Reaction

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(Received April 28, 1983)

The average and longitudinal polarizations of the recoil $^{12}\text{B}$ are studied in detail in the reaction, $^{12}\text{C} + \mu^{-} \rightarrow ^{12}\text{B} + \nu_{\mu}$. The contributions from excited states are properly taken into account. They cannot be neglected even if we adopt the low branching ratios for the excited states obtained in the recent experimental data. The effects of core polarization and exchange currents are considerably large. The core polarization enhances the average polarization, while the exchange currents reduce it. Contrary to this, both effects reduce the partial muon capture rate for the ground state of $^{12}\text{B}$. The latest experimental data on the ratio of the average polarization to the longitudinal polarization gives the coupling constant ratio, $g_{e}/g_{s} = 9.1 \pm 1.8$, with the $0\text{p}$-shell wave function of Hauge and Maripuu and the Sussex interaction for the core polarization. Practically the same value of $g_{e}/g_{s}$ is obtained with the Cohen-Kurath type wave function.

§ 1. Introduction

Recent theoretical and experimental investigations on the nuclear beta decays shed much light on the structure of the weak charged currents. However, among the six nucleon form factors involved in these currents, Eqs. (1)~(3) below, the induced pseudoscalar term is difficult to study in nuclear beta decays, since this term is effectively proportional to the ratio of the lepton mass versus the nucleon mass. Many investigations on muon capture reactions have been performed to obtain information on the magnitude of the pseudoscalar form factor. It is generally claimed that the obtained magnitude is consistent with the canonical value, although the accuracy is somewhat limited. In principle, the pseudoscalar form factor can be determined through the muon capture in hydrogen atom without ambiguity due to the nuclear structure, if the other nucleon form factors are well established. The recent experiment on muon capture in liquid hydrogen gives the value

$$g_{e}/g_{s} = 7.1 \pm 1.5,$$

while the previous experimental values scatter in a relatively wide range. This is partly due to the experimental difficulty in measuring the low muon-capture rate and partly due to the ambiguities remaining in molecular physics.

Partial muon capture rates in complex nuclei are sensitive to the nuclear wave functions adopted as well as to the weak nucleon form factors. We cannot derive the pseudoscalar form factor from partial muon capture rates, unless we are confident in the nuclear wave functions. The ambiguity due to nuclear wave functions is greatly reduced if we use relative physical quantities, such as the average or longitudinal polarization of the recoil nuclei in a muon capture reaction. We have studied these polarizations together with the partial muon capture rates in $^{12}\text{C}$. In these investigations we have taken into account the following:

1. The theory of muon capture reactions is formulated as exactly as possible. That is,
we use the relativistic muon wave functions obtained numerically by solving the Dirac equation with the Coulomb potential of the finite size nucleus. The retardation expansion is properly performed so as to follow the allowed and forbidden theory of the muon capture reactions.\textsuperscript{50}

2. Nuclear wave functions of the general $0p$-shell are adopted.

3. Contributions of the excited states to the ground state formation in $^{12}$B are taken into consideration for obtaining polarizations of the recoil nucleus.

The third remark is required since the observed polarizations of $^{12}$B are partly due to the processes in which initially the excited states of $^{12}$B are formed and the subsequent gamma emissions lead to the ground state.

In this paper, we investigate again the muon capture in $^{12}$C, because new experimental data have been published on the ratio of the average polarization to the longitudinal polarization\textsuperscript{61} as well as on the partial muon capture rates.\textsuperscript{7,83} We use the formalism of Refs. 3)\textsuperscript{−}5) to take into account the effects of the core polarization and exchange currents. The importance of the exchange current was pointed out by Kubodera et al.\textsuperscript{9} particularly in the time component of the axial vector current. They estimated that the exchange current enhances the parameter $\gamma$ of the matrix element ratio by about 40\% in beta decays of the $A=12$ system. This contribution added to the naive impulse approximation result leads to disagreement with the experimental data. The discrepancy has been solved by considering the nuclear structure effect on the matrix element of the time component of the axial vector current, that is, the core polarization in the first-order perturbation.\textsuperscript{10} The authors of Ref. 10) showed that both the core polarization and exchange current effects are rather large. However, they cancel each other almost completely, and the resultant value of $\gamma$ is approximately equal to that of the impulse approximation in agreement with the experimental data. In the present work on the muon capture reaction, we shall consider the exchange current effect not only in the time component but also in the space component of the weak vector and axial vector currents. For the core polarization, we include excitations higher than $2\hbar\omega$ so as to obtain a convergent result.

It is worthwhile to make a few comments on the related recent works. Parthasarathy and Sridhar studied the effect of the exchange current to the average polarization in the muon capture of $^{12}$C.\textsuperscript{11} Instead of calculating the exchange currents explicitly, they enhanced the axial-vector coupling constant for the time component by a factor 1.5. As a result they found a smaller value of the polarization. Guichon and Samour have made a study of beta decay, muon capture and gamma decay in the $A=12$ system.\textsuperscript{12} They have also taken into account the effects of the core polarization and exchange currents, and contribution of the excited states in $^{12}$B. Their treatment is, however, different from ours in the following respects. For core polarization, they included excitations only up to $2\hbar\omega$. Their formalism of the muon capture reaction treats muons nonrelativistically so that the nuclear Coulomb corrections are only partially taken into account. Furthermore, the diagrams adopted for the exchange currents are slightly different from ours.

In § 2, we describe the nucleon form factors in the hadron currents and the diagrams for the exchange currents. Using the expressions of the physical quantities given in § 3, and the muon and nuclear wave functions given in § 4, we analyze the experimental data of the average and longitudinal polarizations and the partial muon capture rates in $^{12}$C as functions of the pseudoscalar form factor. The results are discussed in § 5.
current density operators are summarized in the Appendix. We use the same notation as in Refs. 3) and 4) throughout this work.

§ 2. Weak interactions and diagrams for exchange currents

In this section we briefly review the interaction Hamiltonian of the muon capture and the diagrams which we adopt for calculating the effects of exchange currents. The weak interaction is given by the current-current type.

\[ H = -(G/\sqrt{2}) \int L_{\mu}(x)J_{\mu}(x)dx, \]

where \( J_{\mu}(x) \) and \( L_{\mu}(x) \) being the \( V-A \) hadron and left-handed lepton current density operators, respectively. The Fermi coupling constant is taken to be \( 130 \)

\[ G = (1.4122 \pm 0.0043) \times 10^{-4} \text{erg cm}^3. \]

The matrix elements of the hadron currents \( J_{\mu}^V \) and \( J_{\mu}^A \) for the muon capture in a proton are generally expressed under the proper Lorentz transformation as follows:

\[ \langle p_f | J^{V,A} \rangle | p_i \rangle = i\bar{u}(p_f) \left( g_{\gamma} \gamma_\mu + (g_{\mu}/2M) \sigma_{\rho\sigma} q_{\rho} + i(g_{\nu}/m_{\mu}) q_{\nu} \gamma_{\sigma} \right) u(p_i), \]

where

\[ q_{\rho} = (p_f - p_i)_\rho \quad \text{and} \quad \sigma_{\rho\sigma} = (1/2i) [\gamma_\rho, \gamma_\sigma]. \]

The six form factors \( g_\rho \) are functions of the invariant momentum transfer \( q^2 \). They are real if time reversal invariance holds. According to the CVC hypothesis, which we will assume throughout this paper, \( g_{\mu} \) and \( g_{\nu} \) are related to each other, and their \( q^2 \)-dependence is the same as that of the isovector electromagnetic form factors of the nucleons. \( 15 \) Furthermore, we have \( g_\mu = 0 \), by neglecting the nucleon-mass difference.

The \( q^2 \)-dependence of the axial vector form factor \( g_A \) is obtained from the neutrino experiments. \( 16 \) For beta decay, \( q^2 = 0 \), we have

\[ g_A^2 - g_A (q^2 - 0) = 1.254 \pm 0.007. \]

With the PCAC hypothesis, the ratio of \( g_\rho \) and \( g_A \) is given by

\[ g_\rho/g_A = 2Mm_\mu/(m_{\pi^0}^2 + q^2) \approx 7 \]

for \( q^2 \approx m_{\pi^0}^2 \). Finally, the induced tensor form factor \( g_\gamma \) is taken to be vanishingly small. This is consistent with the recent investigation on the beta-ray angular distributions in the \( A=12 \) system. \( 18 \)

We assume the matrix elements (3) for the off-mass-shell nucleons. The nucleon currents in the nucleus are given up to the first order of the nucleon velocity as follows:

\[ \rho^V(x) = \sum_{s=1}^{A} \delta(x-r_s) g_{\nu} \gamma_s \gamma_{i-} \]

\[ \rho^A(x) = i \sum_{s=1}^{A} \delta(x-r_s) [(1/2M) (g_A + g_\gamma) \sigma_s \cdot \vec{r}] \]
\[ J_s^V(x) = -i \sum_{s=1}^{A} \delta(x - r_s) [(i/2M)(g_V + g_A)\sigma_s \cdot \mathcal{P} + (g_V/2M) \mathcal{P} + (g_V/M) \mathcal{P} \mathcal{P}] \tau_s^c, \]
\[ J_s^A(x) = -i \sum_{s=1}^{A} \delta(x - r_s) [(g_A + (W_0/2M)g_V)(1/m_s)(g_V/2M)\sigma_s \cdot \mathcal{P} \mathcal{P}] \tau_s^c. \] (6)

Here the subscript \( I \) refers to the impulse approximation, and the differential operators \( \mathcal{P}_s \) and \( \mathcal{P} \) operate on the \( s \)-th nucleon and lepton wave functions, respectively. The energy \( W_0 \) is the energy difference in the initial and final nuclear states. For evaluating the \( g_V \) terms, which appear in both charge and current densities of the axial vector, we use the Lorentz covariant form in Eq. (3), and transform it into the pseudoscalar form by using the Dirac equations for leptons. Multipole expansion of the one-body operators can be seen in Table 1 of Ref. 3.

Since the nucleus consists of a number of nucleons, the muon capture in the nucleus takes place through not only the one-body currents in Eq. (6), but also the many-body currents. For simplicity, we will consider as many-body currents the one-pion exchange currents in the static approximation. The diagrams adopted are shown in Figs. 1(a) \( \sim (g) \), and the current-density operators for these diagrams are summarized in the Appendix. It should be noticed here that only the negative energy part of the nucleon propagator is included for the nucleon Born diagram (pair current). We use Adler and Dothan’s results\(^{30} \) to calculate the non-Born amplitude for the weak pion-production by the time component of the axial vector current in Fig. 1(e). We assume that only the (3,3) resonance contributes to the non-Born space components of both vector and axial vector currents. The induced pseudoscalar term with \( g_P \) in the one-body currents is represented mainly by the diagram in Fig. 1(h). A correction for this term is given by the two-body current in Fig. 1(g).

In order to derive two-body current densities, we adopt the following \( \pi NN \) and \( \pi N A \) interactions:\(^{31} \)
\[ H_{\pi NN} = ig \int \bar{\phi} \gamma_5 \tau_i \phi \psi d\mathbf{x}, \] (7)
\[ H_{\pi NA} = (f^*/m_\pi) \int \bar{\phi} \tau_i \phi (\partial \phi / \partial x_a) d\mathbf{x} + \text{h.c.} \] (8)

\(^{31} \) Strictly speaking, Eqs. (7) and (8) are the interaction Lagrangians with the reversed signs.
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Here, $\psi$ is the Rarita-Schwinger field of spin 3/2, and $T_i$ is the $i$-th component of the transition isospin operator. Nucleon and pion fields are $\psi$ and $\phi_i$, respectively. For the coupling constants of $\pi N\Delta$, $VN\Delta$ and $AN\Delta$ vertices, we assume the prediction of the quark model. We adopt the following numerical values:

\begin{equation}
(f^*/f)^2 = 72/25,
\end{equation}

where

\begin{equation}
f = m_s g/2M
\end{equation}

and

\begin{equation}
f^2/4\pi = 0.08.
\end{equation}

Exchange-current density operators are expanded into multipoles in terms of the coordinates of the particles 1 and 2. In these density operators, the terms proportional to $\delta(r_1 - r_2)$ are dropped so that the short range correlation is partially taken into account.

§ 3. Formulas for physical quantities

In this section, we simply summarize the formulas for the following three quantities in the muon capture with the transition $0 \rightarrow J$:

a. Partial muon capture rate

\begin{equation}
\Lambda(J) = 2\pi q^2 (dq/dE)(G^2/2)\sum_{x} |\langle J | E_x(x, -1)|0\rangle|^2.
\end{equation}

b. Longitudinal polarization

\begin{equation}
P_L(J) = -\sqrt{J + 1} \langle J | E_x(x_1, -1)|0\rangle + \sqrt{J} \langle J | E_x(x_2, -1)|0\rangle^2
\times [J(2J + 1)\sum_{x} |\langle J | E_x(x, -1)|0\rangle|^2]^{-1}.
\end{equation}

c. Average polarization

\begin{equation}
P_{av}(J) = [(J + 1)|\langle J | E_x(x_1, -1)|0\rangle|^2 - J|\langle J | E_x(x_2, -1)|0\rangle^2]
\times [3J(2J + 1)\sum_{x} |\langle J | E_x(x, -1)|0\rangle|^2]^{-1}.
\end{equation}

In Eq. (11), we have $x_1 = J$ and $x_2 = -J - 1$ with the upper sign, if $\pi_i \cdot \pi_j = (-)^j$, or $x_1 = -J$ and $x_2 = J + 1$ with the lower sign if $\pi_i \cdot \pi_j = (-)^{i+j}$. In Eq. (12), $x_1$ and $x_2$ are the same as those in Eq. (11). The operators $E_x$ of rank $J$ are represented by the sums of operators derived from the impulse (see Table 1 of Ref. 3)) and exchange currents.

The measured value $P^{exp}$ of the longitudinal or average polarization is the weighted sum of the partial muon captures which feed the excited states as well as the ground state of the daughter nucleus. We can derive the polarization $P^{exp}(J)$ of the ground state $J$ from the measured value $P^{exp}$ by subtracting contributions of muon captures leading to the excited states $J_a$.

\begin{equation}
P^{exp}(J) = P^{exp} + \sum_{a} [\Lambda(J_a)/\Lambda(J)] [P^{exp} - A(J_a)P(J_a)],
\end{equation}

with the attenuation factor through gamma decay.
\[ A(J_e) = \frac{J_e(J_e+1)(2J_e+1)}{J(J+1)(2J+1)(2J_e+1)}a(J_e) \]

and

\[ a(J_e) = (2J_e+1)\sum_{L}(-)^{J_e+L+1} W(J_e J_f JJ; 1L)[\gamma(\sigma, L)]^2/\omega(J_e). \]

Here \( \gamma(\sigma, L) \) is the gamma-decay amplitude of the electric or magnetic 2\(^{nd} \) pole, and \( \omega(J_e) \) is the decay rate of the state \( J_e \). In the cascade gamma transitions, the factor \( a(J_e) \) is slightly modified\(^{3,4} \).

In the case of the muon capture in \(^{12}\)C, the daughter nucleus \(^{12}\)B has the level scheme in Fig. 2. We assume that all excited states decay only through dipole gamma emissions. Then, with the measured branching ratios we have \( A(1^+)=0.734 \pm 0.032 \), \( A(2^-)=0.995 \pm 0.005 \) and \( A(2^+)=1 \). The effect of the 0\(^+\) state is neglected.

\section{4. Muon and nuclear wave functions}

The radial wave functions for the bound muon are obtained numerically by solving the Dirac equation with the Coulomb potential of the nuclear charge distribution

\[ \rho_{\text{charge}}(r) = (1/3\pi^{1/2}b^5)[1 + (4/3)(r/b)^5] \exp[-(r/b)^2], \]

with the oscillator parameter \( b = 1.64 \) fm.

For the positive parity states of \(^{12}\)C and \(^{12}\)B, we adopt two types of configurations of the general 0p-shell, one by Hauge and Maripuu\(^{22} \) and the other by Cohen and Kurath.\(^{23} \) Furthermore, for the transition 0\(^+\)\( \to 1^+ \) we take into account the core polarization in the first order perturbation. This effect is expressed by the variation of the nuclear matrix element for the operator \( O \) as

\[ M_{\text{CP}} = \sum_{\pi} \langle f | O | n \rangle \langle n | V | i \rangle + \sum_{\pi} \langle f | V | n \rangle \langle n | O | i \rangle, \]

where \( V \) is the residual interaction, \( | i \rangle \) and \( | f \rangle \) are the initial and final wave functions in the 0p model space, \( | n \rangle \) is the intermediate state orthogonal to the model space, and \( \varepsilon_n \) is the energy denominator. The effect is considerably large for the time component of axial vector. It is also sizable for the space components of axial vector and vector, though it vanishes in the case of the beta decay. The details of the manipulation can be seen in Ref. 10. The residual interaction adopted here is the Sussex interaction\(^{24} \) in the case of the Hauge-Maripuu model, since in this model the effective interaction is derived from the same interaction. In the case of the Cohen-Kurath model, (8-16)POT, the residual interaction adopted here is a phenomenological central force,

\[ V_C(r) = V_0[\alpha_0 + \alpha_\sigma \sigma_0 + \alpha_\tau \tau_0 + \alpha_\sigma_\tau \sigma_\tau + \alpha_\sigma_\tau_0 \sigma_\tau_0]f(r) \]

with \( V_0 = -60 \) MeV, \( \alpha_0 = 0.0875 \), \( \alpha_\sigma = -0.0625 \), \( \alpha_\tau = -0.1625 \), \( \alpha_\sigma_\tau = \alpha_\sigma \), and \( f(r) \) of the Gaussian type with the range of 1.61 fm, plus the tensor part of the Hamada-Johnston
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potential with a radial cutoff at $r_c=0.7$ fm. A similar potential was once adopted by Sagawa et al. in the analysis of the inelastic electron scattering in the $A=12$ system. These two models are referred to as HM and CK, hereafter.

For the $1^-$ and $2^-$ states of $^{12}$B, we assume the configurations given by model I in Ref. 3. These configurations are consistent with the general $1\hbar\omega$ excitations, and the main component is $(2s_{1/2})^2(1p_{3/2})^{-1}$ in both states. The oscillator strength parameter is $b = 1.64$ fm throughout this paper.

§ 5. Results and discussion

In order to study the magnitudes of the induced pseudoscalar form factor, we analyzed experimental data of the average polarization and the ratio of the average polarization to longitudinal polarization, since the polarizations are less nuclear structure dependent than the partial muon capture rate. Furthermore, the ratio, $R = P_A/|r|$, is free from the systematic ambiguities in experiments compared with individual polarizations. In Figs. 3 and 4, we give the theoretical values of $P_A(1^+)$ and $R(1^+) = P_A(1^+)/|r|(1^+)$ as functions of $g_r/g_A$. The experimental values $P_A^{\exp}(1^+)$ and $R^{\exp}(1^+)$ are derived from the experimental data on $P_A^{\exp}$ and $R^{\exp}$:

$$P_A^{\exp}=0.452\pm0.042 \quad \text{Ref. 27}$$

and

$$R^{\exp}=(P_A/|r|)^{\exp}=-0.516\pm0.041 \quad \text{Ref. 6),} \quad (16)$$

Fig. 3. Average polarization $P_A(1^+)$ of $^{12}$B(g.s.) as a function of $g_r/g_A$. The solid line refers to HM. The dotted line referring to CK coincides with the solid line. The shaded region shows the experimental value $P_A^{\exp}(1^+)$ in Eq. (17), based on the experimental data on $P_A^{\exp}$, those on partial muon capture rates and our theoretical values of $P_A(1^+)$, $P_A(2^+)$ and $P_A(2^+)$ in Table II.

Fig. 4. Ratio $R(1^+)$ of $^{12}$B(g.s.) as a function of $g_r/g_A$. The solid (dotted) line refers to HM (CK). The shaded region shows the experimental value $R^{\exp}(1^+)$ in Eq. (17), based on the experimental data on $R^{\exp}$, those on partial muon capture rates and our theoretical values of $P_A(J^\pi)$ and $P_r(J^\pi)$ in Table II.
Table I. Experimental values of partial muon capture rates $\Lambda^{\exp}(J^\pi)$ for four bound states of $^1\text{H}(10^9$ sec$^{-1}$).

<table>
<thead>
<tr>
<th>$J^\pi$</th>
<th>1$^-$</th>
<th>2$^-$</th>
<th>2$^+$</th>
<th>1$^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roesch et al.$^a$</td>
<td>6.28 ±0.29</td>
<td>0.27±0.10</td>
<td>0.12±0.08</td>
<td>0.38 ±0.10</td>
</tr>
<tr>
<td>Giffen et al.$^b$</td>
<td>5.97±0.35</td>
<td>0.06±0.20</td>
<td>1.080±0.125</td>
<td></td>
</tr>
</tbody>
</table>

a) Ref. 7.  b) Ref. 8.  c) Assumed to be zero.

Table II. Average and longitudinal polarization and partial muon capture rates of the excited states.

<table>
<thead>
<tr>
<th>$J^\pi$</th>
<th>2$^-$</th>
<th>2$^+$</th>
<th>1$^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{\mu}(J^\pi)$</td>
<td>0.499 (0.497)</td>
<td>0.477</td>
<td>0.431</td>
</tr>
<tr>
<td>$P_{\lambda}(J^\pi)$</td>
<td>−0.320 (−0.329)</td>
<td>−0.378</td>
<td>−0.988</td>
</tr>
<tr>
<td>$A(J^\pi)(10^9$ sec$^{-1}$)</td>
<td>0.320 (0.292)</td>
<td>0.278</td>
<td>1.41</td>
</tr>
</tbody>
</table>

Nuclear models adopted are HM (or CK) for the 2$^-$ state and Model I in Ref 3) for the 2$^+$ and 1$^+$ states.

by subtracting contributions from the excited states as in Eq. (13), with the experimental data on the partial muon capture rates $\Lambda^{\exp}(J^\pi)$ in Table I and the theoretical values of $P_{\mu}(J^\pi)$ and $P_{\lambda}(J^\pi)$ in Table II. To calculate these, we adopt HM or CK (8-16)POT for the 2$^-$ state and Model I in Ref 3) for the 1$^-$ and 2$^+$ states. We also assume $g_\mu/g_A=7$. Because of the smallness of the branching ratios in muon captures to the excited states as shown in Table I, corrections are generally insensitive to the nuclear model assumed. A small ambiguity remains, however, due to different values of the partial muon capture rate $\Lambda^{\exp}(1^-)$. $[\Lambda^{\exp}(1^-)$ in Ref. 7] is less than half of that in Ref. 8. See Table I.]

We have

$$P_{\mu}^{\exp}(1^+) = 0.458 ± 0.047,$$

$$R^{\exp}(1^+) = −0.499 ± 0.044,$$  \(17\)

by using the experimental data on $\Lambda^{\exp}(J^\pi)$ by Roesch et al.$^7$, which are qualitatively in agreement with our theoretical values of $A(J^\pi)$ in Table II. Finally we have

$$g_\mu/g_A=10.3_{-4.5}^{+1.5} \quad (10.3_{-4.5}^{+1.5})$$ \(18\)

from $P_{\mu}(1^+)$ in Fig. 3, or

$$g_\mu/g_A=9.1±1.8 \quad (9.0±1.8)$$ \(19\)

from $R(1^+)$ in Fig. 4. The numerical values are given for HM, and those in the parentheses for CK. The numerical values of $g_\mu/g_A$ in Eqs. (18) and (19) are generally consistent with the prediction by PCAC.

By using the experimental data on $\Lambda^{\exp}(J^\pi)$ by Giffen et al.$^8$, we have

$$P_{\mu}^{\exp}(1^+) = 0.476±0.050 \quad \text{and} \quad g_\mu/g_A=9.5_{-2.6}^{+2.6}$$

and

$$R^{\exp}(1^+) = −0.524±0.048 \quad \text{and} \quad g_\mu/g_A=8.1±1.9,$$ \(20\)

for HM.

The results of our calculations of $P_{\mu}(1^+)$ and $R(1^+)$ include both effects of core
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Table III. Effects of core polarization and exchange currents on $P_{\alpha}(1^+)$, $R(1^+)$ and $\Lambda(1^+)$.  

<table>
<thead>
<tr>
<th></th>
<th>I.A.</th>
<th>With C.P.</th>
<th>With E.C.</th>
<th>With C.P. and E.C.</th>
<th>Exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{\alpha}(1^+)$</td>
<td>0.551</td>
<td>0.560</td>
<td>0.517</td>
<td>0.525</td>
<td>0.458±0.047</td>
</tr>
<tr>
<td></td>
<td>(0.542)</td>
<td>(0.565)</td>
<td>(0.501)</td>
<td>(0.524)</td>
<td></td>
</tr>
<tr>
<td>$R(1^+)$</td>
<td>-0.583</td>
<td>-0.697</td>
<td>-0.543</td>
<td>-0.553</td>
<td>-0.499±0.044</td>
</tr>
<tr>
<td></td>
<td>(-0.579)</td>
<td>(-0.615)</td>
<td>(-0.521)</td>
<td>(-0.552)</td>
<td></td>
</tr>
<tr>
<td>$\Lambda(1^+)(10^3\times)$</td>
<td>7.61</td>
<td>6.77</td>
<td>6.96</td>
<td>6.16</td>
<td>6.28±0.29</td>
</tr>
<tr>
<td></td>
<td>(6.16)</td>
<td>(5.00)</td>
<td>(5.66)</td>
<td>(4.54)</td>
<td></td>
</tr>
</tbody>
</table>

Here $g_\mu/g_\chi=7$. Numerical values are given for HM and those for CK in parentheses. I.A., C.P. and E.C. are abbreviations of impulse approximation, core polarization and exchange currents, respectively.

polarization and exchange currents. In the case of beta decays in $^{12}\text{B}$ and $^{12}\text{N}$, these two effects mostly cancel each other in the time component of axial vector current.\textsuperscript{109} It is therefore informative to describe separately individual contributions of these two effects in muon capture. To do this, we first summarize the calculated values of $P_{\alpha}(1^+)$, $R(1^+)$ and $\Lambda(1^+)$ in Table III, where $g_\mu/g_\chi=7$ for simplicity. The effect of the core polarization increases $P_{\alpha}(1^+)$ and $|R(1^+)|$. It is a few percent larger in the case of CK than in the case of HM. In general, $P_{\alpha}(1^+)$ and $|R(1^+)|$ increase with the core polarization effect while they decrease with the exchange current effect. The net result of these two effects is to decrease $P_{\alpha}(1^+)$ and $|R(1^+)|$. This means that the result of $g_\mu/g_\chi$ from the experimental data on $P_{\alpha}$ and $R$ becomes smaller as is seen in Figs. 3 and 4, a result in favor of PCAC. This shows the importance of the exchange current effect. In fact, the magnitude of $g_\mu/g_\chi$ becomes smaller by about 0.6 with the corrections due to the full exchange currents in Figs. 1(a)−(g), compared with the case where the exchange currents in Figs. 1(d) and (e) are taken into consideration only for the time component.] It is noticed here that $P_{\alpha}(1^+)$ and $R(1^+)$ are almost independent of the assumed nuclear models if the two effects of core polarization and exchange currents are properly taken into account. (In the impulse approximation, they have about two percent difference between the HM and CK models.)

Contrary to these, both the core polarization and exchange currents reduce the muon capture rate $\Lambda(1^+)$ as seen in Table III. Apparently, HM has a better fit to the experimental data than CK. We should, however, be careful since the transition rate is much more sensitive to the details of nuclear models than relative quantities, such as polarization of the recoil nuclei and angular distribution.

In the following, the detailed structure of the weak currents is studied for core polarization and exchange currents individually. This is done for the main transition, $0^+ \rightarrow 1^+$, of the muon capture in $^{12}\text{C}$. The core polarization effect is represented by the relative variation of the nuclear matrix element defined as

$$\delta(O) = \langle O \rangle_{\text{C.P.}} - \langle O \rangle / \langle O \rangle = M_{\text{C.P.}} / \langle O \rangle.$$  \hspace{1cm} (21)

Here $M_{\text{C.P.}}$ is given in Eq. (15). The operator $O$ is of the rank 1 and no parity change for exciting the ground state of $^{12}\text{B}$. $\langle O \rangle_{\text{C.P.}}$ and $\langle O \rangle$ are the nuclear matrix elements of the operator $O$ with and without core polarization, respectively. The quantities $\delta(O)$ are summarized in Table IV for three nuclear models, HM, CK and CK'. In the last model CK', the nuclear wave function of CK is used with the core polarization due to the tensor part of residual interaction only. Furthermore, in Table IV, $\tilde{J}_\alpha$ and $\tilde{\rho}_\alpha$ represent,
Table IV. Differences of nuclear matrix elements due to core polarization in the transition, $0^+\rightarrow 1^+$, of the muon capture in $^{14}$C in units of percent.

| Nuclear model | $\delta(|dzL_{\alpha}\hat{J}_p^\alpha|)$ | $\delta(|dzL_{\alpha}\hat{J}_v^\alpha|)$ | $\delta(|dzL_{\alpha}\hat{\rho}_{i}^\alpha|)$ | $\delta(|dzL_{\alpha}\hat{J}_s^\alpha|)$ |
|---------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| HM            | -5                              | -7                              | -13                             | -5                              |
| CK            | -10                             | -12                             | -35                             | -11                             |
| CK'           | -2                              | -2                              | -34                             | -3                              |

For notation, see the text. Here we assume $g_\nu/g_\lambda=7$.

Table V. Ratios of nuclear matrix elements for exchange and impulse currents in the transition $0^+\rightarrow 1^+$ of the muon capture in $^{14}$C in units of percent.

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<th>HM</th>
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Numerical values are given for $g_\nu/g_\lambda=7$ with no core polarization. $\hat{J}_s^\alpha$ represents the sum of $J_{\mu\mu}^\alpha$, $J_{\nu\nu}^\alpha$ and $J_{\pi}^\alpha$. Contributions of these terms are, respectively, given in the same sequence in the brackets. For other symbols, see the text.

respectively, the space and time components of the one-body axial vector current which has no $g_\nu$ term, while $J_{\alpha,\alpha'}^\alpha$ represents space or time component of the $g_\nu$ term in the one-body axial vector current. As is seen in Table IV, the core polarization reduces nuclear matrix elements. (Notice that the numerical values in Table IV are given for the case, $\lambda=-1$, of the emitted neutrino, and this is good enough for a qualitative discussion. This is the same for Table V.) This effect is large in the $\hat{\rho}_{i}^\alpha$ term, the time component of axial vector, particularly in CK, and this does not change by inclusion of the central part of the residual interaction. This fact was noticed in the case of the beta decay and explained by taking a single $j\cdot j$ coupling shell model. On the other hand, the reduction of the space components of vector and axial vector and of the pion-pole term is dependent on the inclusion of the central part, and it reaches up to about 10 percent. As is seen later, the space component of axial vector is also reduced by the exchange currents. This is the reason why the muon capture rate $A(1^+)$ is reduced as is given in Table III.

The ratios of the nuclear matrix elements for exchange and impulse currents are given in Table V, for various diagrams in Fig. 1. The matrix element of the $\hat{\rho}_{i}^\alpha$ term increases drastically by exchange currents. (This can be understood by the soft pion theorem with current algebra.) Contrary to this it decreases by core polarization. A similar effect was seen in the case of beta decays of the $A=12$ system. For the space component of the vector current, the pair current enhances the role of the impulse current, while the pionic and delta currents reduce it. Due to this cancellation, enhancement for the space component of vector current becomes a few percent. This is consistent with the case of M1 form factor of $^{14}$C in inelastic electron scattering. The exchange current effect for the space component of the axial vector is about 5% reduction.

If we neglect the core polarization and exchange current effects, and use the experimental data, $R^{exp}(1^+)$ in Eq. (17), we have

$$g_{\nu}/g_{\lambda}=10.8\pm 1.8. \quad (10.2\pm 1.8) \quad (22)$$
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By comparing these values with those in Eq. (19), we notice that the effects of the core polarization and exchange currents are really important and should be taken into account simultaneously. We also notice that all the exchange currents in Figs. 1(a)–(g) have to be considered in nuclear polarizations, and they have sizable effects to change the ratio $g_F/g_A$.

Finally we describe relations of our work with that of Guichon and Samour, who obtained an overall consistency of the beta decays and muon captures in the $A=12$ system with the canonical value, $g_F/g_A = 7.29$, by including the exchange current effect in the time component of axial vector. In their work, the muon wave function is factorized out of the integrand of nuclear matrix elements and its small component is omitted so that nuclear polarizations become those for the plane wave approximation. The resulting value of $|R(1^+)|$ is 2% less than ours. (This brings by about 7% decrease of $g_F/g_A$.) The percentage decrease of core polarization is almost the same as ours, although they adopted a simple $j-j$ coupling of the $0^+_2$-shell with $2\hbar\omega$ excitation only. For exchange currents, they studied the pionic, pair, $\omega\pi$, $\rho$ and $\Delta$ currents for vector, and $\rho\pi$, $\Delta$ and $\rho$ currents for axial vector. In their final result, they take into account the effects of the pionic and pair currents for vector, and those of $\rho\pi$ for axial vector. On the other hand, in the present work we take into account the diagrams in Fig. 1. This corresponds to the inclusion of the pionic, pair and $\Delta$ currents for vector, and $\rho\pi$ and $\Delta$ currents for axial vector, where the $\rho\pi$ term corresponds to our diagrams in Figs. 1(d) and (e). The space component of the $\rho\pi$ current is not included, since no appreciable changes of nuclear polarizations are observed in agreement with the result of Guichon and Samour.

The authors would like to thank Professor V. L. Telegdi for sending them experimental data before publication. Numerical calculations were performed with the aid of NEAC ACOS 1000, Computer Center, Osaka University. This work is partly supported by Grant-in-Aid for Scientific Research, The Ministry of Education, Science and Culture.

Appendix

— Exchange-Current Density Operator —

Explicit forms for the exchange-current densities are shown in this appendix as the operators acting on the nuclear wave functions. The formulas in Eqs. (a)–(g) represent contributions of the same labels with the same labels in Fig. 1.

(a) $J_{\text{pair}}^{\nu}(x) = -i \left( \frac{f}{m_\pi} \right)^2 \delta(x-r_1) \left[ \sigma_1 \times \sigma_2 \right] \exp \left( -i \mathbf{a} \cdot \mathbf{r} \right) \frac{\exp \left( -i \mathbf{a} \cdot \mathbf{r} \right)}{\omega^2} + (1 \leftrightarrow 2)$.

(b) $J_{\text{pionic}}^{\nu}(x) = i \left( \frac{f}{m_\pi} \right)^2 \left[ \sigma_1 \times \sigma_2 \right] \left[ \sigma_1 \cdot \mathbf{q}_1 \right] \exp \left( -i \mathbf{q}_1 \cdot (r_1 - x) \right) \frac{\exp \left( -i \mathbf{q}_2 \cdot (x - r_3) \right)}{\omega_1^2} \frac{\exp \left( -i \mathbf{q}_2 \cdot (x - r_3) \right)}{\omega_2^2}$

$\times \left( \sigma_1 \cdot \mathbf{q}_1 \right) \left( \sigma_2 \cdot \mathbf{q}_2 \right) \left( \mathbf{q}_1 + \mathbf{q}_2 \right)$.

(c) $J_{\Delta}^{\nu}(x) = i \left( \frac{1+g_\Delta}{18M(M_\Delta-M)} \right) \left( \frac{f^*}{m_\pi} \right)^2 \delta(x-r_1) \int \frac{d^3 \mathbf{q}}{(2\pi)^3} \frac{\exp \left( -i \mathbf{q} \cdot \mathbf{r} \right)}{\omega^2}$

$\times \left[ \mathbf{q} \tau_3^{\nu-1} - \left( \sigma_1 \times \mathbf{q} \right) \left( \sigma_1 \times \sigma_2 \right)^{\nu-1} \right] \times \left( \sigma_3 \cdot \mathbf{q} \right) \left( 1 \leftrightarrow 2 \right)$. 

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\[
\rho_{\text{part}}(x) = i g_s \left( \frac{f}{m^*} \right)^2 \left( r_1 \times r_2 \right) (-i) \delta(x-r_1) \int \frac{dq}{(2\pi)^3} \frac{\exp(-iq \cdot r)}{\omega^2} \sigma_z \cdot q + (1 \leftrightarrow 2). \tag{d}
\]
\[
\rho_{\text{NNB}}(x) = i g_s (g_\mu - 1) \left( \frac{f}{m_\pi} \right)^2 \delta(x-r_1) \left( r_1 \times r_2 \right) (-i) \int \frac{dq}{(2\pi)^3} \frac{\exp(-iq \cdot r)}{\omega^2} \sigma_z \cdot q + (1 \leftrightarrow 2). \tag{e}
\]
\[
J^A_d(x) = -\frac{g_s}{9(M_\pi - M)} \left( \frac{f}{m_\pi} \right)^2 \delta(x-r_1) \int \frac{dq}{(2\pi)^3} \frac{\exp(-iq \cdot r)}{\omega^2} \times [4q \tau_2 (-i) (\sigma \cdot q)(r_1 \times r_2) (-i)] \sigma_z \cdot q + (1 \leftrightarrow 2). \tag{f}
\]
\[
J_{\text{NNB}}(x) = -\frac{k_\sigma}{k^2 + m^2} \cdot \mathbf{k} \cdot J^A_d(x). \tag{g}
\]

Here \( r, \omega, r_1, r_2 \) and \( k_\sigma \) represent \( r_1 - r_2, \sqrt{q^2 + m^2}, (\tau_2)_z - i(\tau_3)_z \) and \( (p_1 - p_2 - p_3 - p_4)_z \), respectively. In Eq. (g), \( k_\sigma \) is contracted with the lepton current \( L_\tau \) to use the Dirac equations for leptons. On the other hand, the space components \( \mathbf{k} \) operate on the nuclear wave functions. The above operators are expanded into multipoles with Eq. (6) in Ref. 3) in terms of the nucleons 1 and 2. Equations (b), (c), (f) and (g) contain the terms proportional to \( \delta(r_1 - r_2) \). These are neglected.

References

19) Particle Data Group, Rev. Mod. Phys. **52** (1980), S1.
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