Weak Processes in the $A=12$ Nuclei with Finite Momentum Transfer

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The elementary particle treatment of the process $\nu^+ \mu^{-}^{12}C \rightarrow \nu^{+}^{12}B$ is re-examined. Specifically, we check the basic assumption that the axial and weak-magnetism nuclear form factors scale in the same manner. The relevance of neutrino reactions in a model-independent test of CVC for finite momentum transfer is also discussed.

The triad of the $A=12$ nuclei has been used extensively for studying fundamental properties of weak interactions.\(^{13-14\text{a-14\text{d}}}\) The processes involving zero momentum transfer ($\beta$-decay, $\gamma$-decay) have been successfully analyzed with the elementary particle treatment (EPT),\(^{13}\) the results providing model-independent tests of CVC\(^{13}\) and second-class currents (SCC).\(^{14\text{a,b}}\) The use of EPT for finite-momentum transfer processes\(^{13\text{a-b}}\) ($\mu$-capture, electron scattering) has not met such success because the number of available experimental quantities is not large enough to determine all the relevant form factors empirically. On the other hand, the impulse approximation (IA)\(^{13}\) offers a definite, albeit model-dependent, prescription for calculating necessary form factors, once the model nuclear wave functions are given. In all analyses done so far with EPT,\(^{13-14\text{a-b}}\) the number of independent form factors is reduced by assuming certain specific relations between them, and the IA results\(^{13}\) and/or model calculations\(^{14\text{a,b}}\) based on PCA are cited to argue for the plausibility of these assumptions. Thus, a completely model-independent EPT has not been feasible up to now. Even with the recent addition\(^{13\text{b-d}}\) of the polarization data for $\nu^+ \mu^- \rightarrow \nu^+ \mu^- B$, the situation remains essentially the same. Therefore, a model calculation based on IA (and supplemented by the inclusion of exchange currents), as was done recently by the Osaka group,\(^{14\text{a,b}}\) is useful in order to obtain numbers that can be confronted with experiment. In this note, however, we wish to take another look at EPT to discuss some points which have not been considered in the literature. Specifically, we mention that the existing data, though insufficient for the individual determination of each form factor, can still be used to check whether the axial and weak-magnetism nuclear form factors scale in the same manner. The identical scaling is a key assumption in the EPT analyses done so far.\(^{13\text{a-b}}\) Additionally, we discuss the relevance of neutrino reactions in testing CVC for finite momentum transfer in the EPT context.\(^{13\text{b-d}}\)

The following is the list of processes relevant to our present discussion (the abbreviation “gnd” below stands for “ground state”).

\begin{align*}
\langle ^{12}\text{C}; 0^+ | V^\nu_{\gamma}(0)| ^{12}\text{B}; 1^+ \rangle &= \epsilon_{\mu\nu\rho\sigma} q_\rho Q_{\mu} F_{\nu}^+(q^2) \frac{2m_\mu}{2m_\nu}, \\
\langle ^{12}\text{C}; 0^+ | V^\nu_{\gamma}(0)| ^{12}\text{N}; 1^+ \rangle &= \epsilon_{\mu\nu\rho\sigma} q_\rho Q_{\mu} F_{\nu}^-(q^2) \frac{2m_\mu}{2m_\nu}, \\
\langle ^{12}\text{C}; 0^+ | A^\nu_{\gamma}(0)| ^{12}\text{B}; 1^+ \rangle &= \xi_{\mu} F_{\mu}^+(q^2) + q_\rho Q_{\mu} \frac{F_{\nu}^+(q^2)}{2m_\mu} + q_\rho Q_{\mu} \frac{F_{\nu}^-(q^2)}{2m_\mu}, \\
\langle ^{12}\text{C}; 0^+ | A^\nu_{\gamma}(0)| ^{12}\text{N}; 1^+ \rangle &= \xi_{\mu} F_{\mu}^-(q^2) + q_\rho Q_{\mu} \frac{F_{\nu}^+(q^2)}{2m_\mu} + q_\rho Q_{\mu} \frac{F_{\nu}^-(q^2)}{2m_\mu}.
\end{align*}
\[
\langle ^{13}\text{C}; 0^+ | j_\nu(0) | ^{12}\text{C}; 1^+ \rangle = \epsilon_{\text{isospin}} \epsilon_{\nu} Q_{\nu} \frac{\mu(q^2)}{2m_\nu M},
\]

where \( \xi \) is the polarization vector of the spin-1 nucleus, and \( m_\nu \) and \( M \) are the nucleon and the nuclear mass, respectively. The four-vector \( q \) and \( Q \) are defined as \( q = p_\nu - p_\tau, \quad Q = p_\nu + p_\tau, \quad p_\nu \) and \( p_\tau \) being the initial and final momentum, respectively. Since there are by now rather stringent upper limits to the strength of SCC, we assume that there is strictly no SCC. Then, \( F_{\mu}^{-}(q^2) = F_{\mu}^{+}(q^2) = F_{\mu}(q^2) \), \( F_{\tau}^{-}(q^2) = F_{\tau}^{+}(q^2) = F_{\tau}(q^2) \), \( F_{\tau}^{+}(q^2) = F_{\tau}^{+}(q^2) = F_{\tau}(q^2) \). Now, if CVC holds, we should have\(^*\) \( | j_\nu(r, l_{\nu}^{+}) | = \pm V_{\ell}(r) \), where \( l_{\nu}^{+} \) are the isospin operators. This implies

\[
F_{\mu}(q^2) = \sqrt{2} \mu(q^2). \tag{1}
\]

For zero momentum transfer processes, we should have

\[
\mu = \frac{2\pi \ln 2}{m_\nu^2} (G \cos \theta)^2 F_{\mu}(0)^2,
\]

\[
S(E_\nu) = \text{const} + \left\{ \frac{8}{3} R \frac{F_\mu(0)}{F_\tau(0)} \frac{1.3 \times 10^{-2} m_\nu}{\text{MeV}} - \frac{E_\nu}{2m_\nu} \right\},
\]

\[
\alpha_\tau + \alpha_\nu = - \frac{1}{3m_\nu} \frac{F_\tau(0)}{F_\mu(0)}.
\]

Solving for \( F_\mu(0) \), one obtains \( F_\mu(0) = 2.76 \pm 0.50 \). On the other hand, the observed value\(^1\) \( \Gamma = 37.0 \pm 1.1 \text{ eV of the } \gamma \text{-decay gives } | \mu(0) | = 1.97 \pm 0.03 \), via the relation \( \Gamma = \alpha | \mu(0) | E_\nu^4 / 3 m_\nu^2 \). Thus, \( \sqrt{2} | \mu(0) | = 2.79 \pm 0.04 \), proving \( | F_{\mu}(0) | = \sqrt{2} | \mu(0) | \) within experimental errors. To check CVC for finite momentum transfer using Eq. (1) is a very interesting problem, as was emphasized in Ref. 6).

The electron scattering data\(^{10,11}\) gives \( | \mu(q^2) | \) through the relation \( d \sigma / d q^2 = \sigma^0 (1 / 2 m_\nu^2) | \mu(q^2) |^2 \)

\[
\Gamma_\nu = \frac{(G \cos \theta)^2}{2 \pi^2} \frac{E_\nu^2 C}{(1 + m_\nu/M)} \left( \frac{a Z m_\nu}{1 + m_\nu/M} \right) (3G_\nu^2 + 2G_\tau G_\pi + G_\tau^2),
\]

\[
P_\nu = \frac{2}{3} \frac{3G_\nu^2 + 2G_\tau G_\pi + G_\tau^2}{2G_\pi^2 + 2G_\pi G_\tau + G_\tau^2},
\]

\[
R = - \frac{1}{3} \frac{3G_\nu^2 + 2G_\tau G_\pi}{G_\tau^2},
\]

where \( G_\nu = F_\nu(q^2) + E_\nu F_\tau(q^2) / 2m_\nu \) and \( G_\pi = - (F_\nu(q^2) + 2m_\nu m_\tau F_\tau(q^2) / m_\tau^2 - F_\tau(q^2)) / 2m_\nu \). The fact that all the observables can be described in terms of \( G_\nu \) and \( G_\pi \)\(^{11}\) implies that the \( \mu \)-capture data cannot determine the four form factors individually. Thus, to proceed with EPT, one needs to impose certain restrictions on the form factors. The canonical assumptions are:

\[
F_{\mu}(q^2) / F_{\mu}(0) = F_{\pi}(q^2) / F_{\pi}(0), \tag{3}
\]

\[
F_{\tau}(q^2) / F_{\tau}(0) = F_{\tau}(q^2) / F_{\tau}(0). \tag{4}
\]

Equation (3) is rather strongly motivated by IA,\(^{12,13}\) which dictates that, to the leading order, \( F_{\pi}(q^2), F_{\tau}(q^2) \propto \langle ^{13}\text{C} | j_{\pi}(r) | ^{12}\text{C} \rangle, \) whereas Eq. (4) is more model-dependent even

\(^*\) Electromagnetic effects such as Coulomb corrections, isospin mixing are neglected here.
within IA. As for $F_{\pi}(q^2)$, an argument based on PCAC suggests

$$F_{\pi}(q^2)/F_{\pi}(0) = \left(1 + \epsilon(q^2)\right)/(1 + q^2/m^2),$$

where $\epsilon(q^2) = 0$ in the naive PCAC argument or in IA, but a model\(^{16}\) which takes into account nuclear dynamical effects gives $\epsilon(q^2) = 0.15$. Here and hereafter $q^2 = 0.74 m^2$ means the four momentum transfer squared for the $\mu$-capture. All the EPT analyses done so far use more than one assumption out of those listed in Eqs. (3)–(5), and check CVC, Eq. (1), using the $\mu$-capture data; or, conversely, they use CVC to make a prediction on the observables in the $\mu$-capture. Now, as stated before, the breakdown of Eq. (3) implies that IA is in serious doubt. We therefore consider it very informative to test Eq. (3) without getting compounded with the assumptions of Eqs. (4) and (5).\(^{16}\) This test is possible even if the $\mu$-capture data cannot determine each form factor separately. The fact that $G_A$ involves only $F_{\pi}(q^2)$ and $F_{\omega}(q^2)$ means that, if we treat $G_A$ as a whole without decomposing it into the form factors, we need to deal with only three (instead of four) unknowns, i.e., $F_{\pi}(q^2)$, $F_{\omega}(q^2)$ and $G_A$, in checking Eq. (3). The two observables in the $\mu$-capture plus CVC, Eq. (1), will then determine these three, allowing to test the scaling, Eq. (3). Of course, one can turn around the argument and assume Eq. (3) to test CVC. The present authors however consider the first procedure somewhat more natural because the meaning of the CVC test seems rather obscure once one introduces an assumption, Eq. (3), on nuclear dynamics. In the following we therefore limit ourselves to the first procedure. To check Eq. (3) is of great interest also because the effects of exchange currents for vector and axial vector currents can be quite different, a point emphasized by Chemo and Rho.\(^{21}\) Now, possible deviations from the scaling equation (3) may be parametrized as

$$[F_{\pi}(q^2)/F_{\pi}(0)]/[F_{\omega}(q^2)/F_{\omega}(0)] = 1 + \eta.$$ 

As for the $\mu$-capture observables to be used in analyses, we can in principle choose any two of the available three data. The sharpness of the conclusion however might depend on the choice to a certain degree because of the non-linear relations between the observables and the form factors and also because of the slightly different size of the error bar of each observable. We here choose two cases as representatives. In case A, $I_{\pi}$ and $P_{\pi}$ are used as input and predictions are made for $R$. In case B, $I_{\omega}$ and $R$ are treated as input and predictions are made for $P_{\omega}$. In both cases, we have evaluated the allowable range of $\eta$. Table I gives the results of these analyses. Because there are three different sets of values for $I_{\pi}$ and $P_{\pi}$, the results are given for various possible choices. Table I shows that $|\eta| \lesssim 0.08$ for all the cases studied. The violation of the exact scaling, Eq. (3), by this amount is not alarmingly large, because we are here neglecting "trivial" charge...

| Table 1. Results of EPT analyses of the $\mu$-capture data. |
|-----------------|-------------|-----------|-----------|
| Input | $P_{\pi}$ | $\eta$ | $R$ |
| $I_{\pi}(10^{3} \text{sec}^{-1})$ | $P_{\pi}$ | $\eta$ | $R$ |
| CASE A | | | |
| $I_{\pi} = 6.28 \pm 0.29^{a}$ | $0.458 \pm 0.047^{c}$ | $0 \leq \eta \leq 0.08$ | $0.470 \pm 0.056$ |
| $I_{\pi} = 5.974 \pm 0.350^{a}$ | $0.476 \pm 0.050^{c}$ | $-0.01 \leq \eta \leq 0.06$ | $0.493 \pm 0.062$ |
| $I_{\pi} = 6.28 \pm 0.29^{a}$ | $0.467 \pm 0.045^{c}$ | $0 \leq \eta \leq 0.08$ | $0.481 \pm 0.055$ |
| CASE B | | | |
| $I_{\omega}(10^{3} \text{sec}^{-1})$ | | | |
| $I_{\omega} = 6.28 \pm 0.29^{a}$ | $-0.506 \pm 0.041^{d}$ | $-0.01 \leq \eta \leq 0.07$ | $0.488 \pm 0.032$ |
| $I_{\omega} = 5.974 \pm 0.350^{a}$ | $-0.506 \pm 0.041^{d}$ | $-0.04 \leq \eta \leq 0.05$ | $0.488 \pm 0.032$ |

a) Ref. 19.
b) Ref. 20.
c) Derived in Ref. 14) from the observed value\(^{11}\) of $P_{\pi}$, by subtracting the contributions from the excited final states. Depending on which value of $I_{\omega}$ is used, two different values of $P_{\omega}$ are obtained.
d) Ref. 13.
e) Ref. 12.

\(^{16}\) Our approach here is a generalization of what Leroy and Palffy\(^{21}\) did within the framework of IA.
symmetry breaking effects of order $aZ$ anyway. Thus, one may conclude that the identical scaling of $F_M(q^2)$ and $F_S(q^2)$ seems to hold within the experimental errors. We add also that any model calculations which give $y$ much larger than 0.08 are in direct contradiction with the data.

As we have seen above, if the $\mu$-capture is the only source of information, EPT can be used to put constraints on a viable model on nuclear dynamics but cannot be used to make a model-independent test of the symmetry properties of

$$
\frac{\partial^2}{\partial \theta_e \partial \phi_e} \propto 1 - \frac{1}{3} \cos \theta_e \pm \frac{2}{3} (p_e + p_\mu_1)(1 - \cos \theta_e) \cdot a_M
$$

$$
+ \frac{2}{3} (p_e - p_\mu_1)(1 + \cos \theta_e) \cdot a_T + \frac{1}{3} q^2 (1 + \cos \theta_e) \cdot a_T^2,
$$

where $a_M = F_M(q^2)/[F_S(q^2) - 2m_\mu]$, and $a_T = F_T(q^2)/[F_S(q^2) - 2m_\mu]$. Here again, $F_S(q^2)$ do not appear in the final expression provided the lepton mass can be ignored. The above results indicate that we can directly obtain $a_M$ by looking at the term that changes the sign in going from the $\nu$-to-$\bar{\nu}$-reactions. The value of $a_M$ combined with the two observables in the $\mu$-capture will give $F_M(q^2)$ in a totally model-independent way. Then, by comparing this $F_M(q^2)$ with $\mu(q^2)$ obtained from electron scattering, one can make a direct test of CVC for finite momentum transfer. These experiments may not be too easy but, we think, worth contemplating.

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