Axions at the Intermediate Mass Scale in SUSY SU(5) Models

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On the basis of supersymmetric SU(5) scheme, axion models are investigated by taking into account constraints extracted from cosmology (domain wall problem) and astrophysics (invisibility of axion). A global U(1) axial symmetry is assumed to break down spontaneously at an intermediate mass scale in the tree approximation.

Two minimal cases are examined by working the Peccei-Quinn mechanism in R-symmetry peculiar to SUSY theories and in the ordinary chiral phase transformations. The former case suggests the number of generations to be four, while the latter yields a certain relation among P-Q charges, in order to get rid of the domain structure.

§ 1. Introduction

While quantum chromodynamics has turned out to be a valid theory in perturbative aspects of the strong interaction, it has profound problems yet to be settled for nonperturbative phenomena. One of them is the strong CP non-invariance manifesting itself in the gauge invariant \( \theta \)-vacuum.\(^3\) Peccei and Quinn presented a device to solve it by adding an axial \( U(1) \) symmetry to the strong and electroweak dynamics.\(^2\) A pseudo Nambu-Goldstone boson, axion, \(^3,4\) produced by the explicit breakdown of the symmetry due to anomaly has not been discovered in the region of several tens of kiloelectron volt.\(^5\) Being motivated by this undiscovery, various models of invisible axion\(^6\) have been offered in which the mass of axion and the coupling of it to matter fermions are extraordinarily small.

Recently from the standpoints of cosmology and astrophysics, a couple of conditions of constraint have been pointed out on making realistic models of axion:

1. Energy scale \( A_{\text{PQ}} \), which determines uniquely both the axion mass and the coupling strength, should be bounded from below by \( 10^7 \) GeV due to the energy loss rate of red giants\(^7\) and loosely from above by \( 10^{12} \) GeV due to the energy density of our Universe.\(^8\) These bounds enforce us to envisage an intermediate mass scale between the electroweak \( A_{\text{WS}} \) and the grand unification scale \( A_{\text{GUT}} \).

2. The axial \( U(1) \) symmetry imposed breaks spontaneously to leave occasionally a discrete subgroup \( Z_n \), under which transformations \( \theta_{\text{QCD}} \) stands unchanged modulo \( 2\pi \).\(^9\) Generally speaking, such a discrete symmetry connects degenerate vacua so that it necessarily leads to the formation of domain walls in the early universe. If we would like to get rid of this domain structure in the framework of particle physics without changing the standard model of Friedmann's expanding universe, only a trivial discrete group is permitted to remain unbroken in axion models.

In this paper by taking into account the above two constraints we shall discuss axions on the basis of the supersymmetric SU(5) grand unified scheme. Here are in order guiding principles for us to make models:

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(A) The primary symmetry is SUSY SU(5)_gauge × U(1)_global, where U(1) group is anomalous corresponding to Peccei and Quinn’s axial symmetry. Technical naturalness is enjoyed by virtue of supersymmetry.

(B) By setting aside the breakdown of SUSY, breaking series of other symmetries are realized in the following way:

\[ SU(5) \times U(1)_{RQ} \xrightarrow{\Lambda_{RQ}} SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{RQ} \]

\[ \xrightarrow{\Lambda_{SS}} SU(3)_C \times SU(2)_L \times U(1)_Y \xrightarrow{\Lambda_{ES}} SU(3)_C \times U(1)_{EM}. \] (1.1)

For simplicity, we try to find out an intermediate mass scale \( \Lambda_{RQ} \) in the spontaneously broken potential in the tree approximation. As open problems are left how to break supersymmetry and at what scale \( \Lambda_{SS} \), especially what relation there is between \( \Lambda_{RQ} \) and \( \Lambda_{SS} \).

(C) Renormalizability; hence the gauge symmetry is free from anomaly.

(D) Asymptotic freedom of SU(3) color sector. Weak angle \( \beta \) and the life time of proton are consistent with experimental values, respectively.

In § 2 we shall use R-symmetry characteristic of the SUSY theory as an anomalous U(1) group. In order to obey two conditions (1) and (2) mentioned above it is shown that the number of generations prefers to be four or six. Six-generation, however, is unfavorable because of asymptotic non-freedom in SUSY theories. In § 3 by providing an axial U(1) symmetry based on the ordinary phase freedom of particles, we shall find that a certain relation among the U(1) charges is required. The last section is devoted to several discussions.

§ 2. Axions based on R-symmetry

As is well known, \( R \)-symmetry\(^{10} \) arises from the freedom of chiral rotations with respect to Majorana spinor coordinates. It may safely be said, however, that the physical significance of the symmetry has not fully been elucidated. It is interesting to see what will result when the global symmetry is applied to axion models.

Provided that the global U(1) group should remain unbroken over the GUT threshold \( \Lambda_{GUT} \) as assumed in (B) in the preceding section, various axion models\(^{9,11,12} \) presented hitherto are seen to be insufficient and should be extended.\(^{13} \) For that purpose are required at least two extra fields \( \Phi_{24} \) and \( \Phi_0 \) besides the set of fields in the standard SUSY SU(5) model.\(^{14} \) Namely, except for gauge fields we have seven left-handed superfields as follows:

\[ \Phi_{24}, \Phi_0(1), \Sigma(24), H(5), H'(5^*), T_i(10) \text{ and } \bar{F}_i(5^*), \] (2.1)

where dimensions of the representation with respect to SU(5) are denoted in the parentheses and the freedom of generation is written by index \( i \) running from 1 to \( N_g \). The first two fields should serve mainly to produce an intermediate mass scale at which the global U(1) symmetry would break spontaneously. Each of the remainders will play the same

\(^{13} \) Recently, Holman et al.\(^{13} \) have presented Spin(10) and non-supersymmetric axion models compatible with cosmological constraints.
role as in the standard model. It is noted here that R-charge of the Higgs field $\Sigma$ should vanish. Thus we shall study how this minimally extended model may meet requirements from cosmology and astrophysics.

The most general superpotential $W$ is composed of the following interaction terms, of the same chirality, gauge invariant, renormalizable and invariant under the transformation of matter fields $(T_i, \bar{T}_i)$ to $(-T_i, -\bar{T}_i)$:

$$
\begin{align*}
&\text{Tr } \Phi_{24}^2, \text{ Tr } \Phi_{24}^3, \Phi_0, \Phi_0^2, \Phi_0^3, \text{ Tr } \Phi_{24}^2 \Phi_0, \\
&\text{Tr } \Phi_{24} \Sigma, \text{ Tr } \Phi_{24}^2 \Sigma, \text{ Tr } \Phi_{24}^3 \Sigma, \text{ Tr } \Phi_0 \Sigma^2, \text{ Tr } \Phi_{24} \Phi_0 \Sigma \\
&H' H, \quad H' \Phi_{24} H, \quad H' \Sigma H, \quad \Phi_0 H' H, \quad HTT \quad \text{and } H' \bar{F} T. 
\end{align*}
$$

Since the superpotential of our model is required to be R-symmetric, every interaction term should have R-charge 2. It is immediately read off that all of the terms cannot take part in. Such R-symmetric potentials as are able to produce two different scales $\Lambda_{\text{GUT}}$ and $\Lambda_{\text{EQ}}$ are given by the following three combinations:

$$
\begin{align*}
W_a &= a_m \Phi_{24}^2 + (b_m)^2 \Phi_0 + c \Phi_{24}^2 \Sigma + d \Phi_0 \Sigma^2 + W_a' \quad (2.3a) \\
&\text{for } \phi_{24}=1, \quad \phi_0=2 \quad \text{and } \sigma=0, \\
W_b &= a_m \Phi_{24}^2 + (b_m) \Phi_0^2 + c \Phi_{24}^2 \Sigma + d \Phi_{24} \Phi_0 \Sigma + W_b' \quad (2.3b) \\
&\text{for } \phi_{24}=1, \quad \phi_0=1 \quad \text{and } \sigma=0, \\
W_c &= (a_m) \Phi_0 + b_m \Phi_{24} \Sigma + c \Phi_{24} \Sigma^2 + d \Phi_0 \Sigma^2 + W_c' \quad (2.3c) \\
&\text{for } \phi_{24}=2, \quad \phi_0=2 \quad \text{and } \sigma=0,
\end{align*}
$$

where interactions involving fields $\Phi_{24}$, $\Phi_0$ and $\Sigma$ only are explicitly written down. Contractions are understood and coupling constants tacked with the suffix $m$ have mass dimensions. Small letters stand for $R$-charges $Q_R$ of the scalar partners of superfields, respectively. It may be confirmed that we would have two mass scales independent of each other, say, of $\Lambda_{\text{GUT}}=O(10^{14}\text{GeV})$ and $\Lambda_{\text{EQ}}=O(10^{10}\text{GeV})$, provided that coupling constants with mass dimensions are of $O(1)$ and those without dimension are of $O(1)$ and that vacuum expectation values are taken to be $<\Phi_{24}^2> = (\Phi_0) = 0$ (1) and $<\Sigma^2> = O(\Lambda_{\text{GUT}})$.

It is left to determine the $R$-charges of superfields and the superpotentials belonging to the Weinberg-Salam sector. After simple manipulations we arrive at results as

| Table I. The $R$-charges $Q_R$, superpotentials and anomalies of $U(1)_R$. |
|---|---|---|---|---|---|---|---|
| $\phi_{24}$ | $\phi_0$ | $\Sigma$ | $H$ | $H'$ | $T_i$ | $F_i$ | $A$ |
| $W_a$ | 1 | 2 | 0 | $h$ | $-h$ | 1 | $-h/2$ | $3h/2-1$ | $-2N_a$ | $H' H H' \Sigma H, H'TT, H' \bar{F} T$ | $W_a$ |
| $W_b$ | 1 | 1 | 0 | $h$ | $-h$ | 1 | $-h/2$ | $3h/2-1$ | $-2N_a$ | $H' H H' \Sigma H, H'TT, H' \bar{F} T$ | $W_b$ |
| $W_c$ | 1 | 1 | 0 | $h$ | $-h$ | 1 | $-h/2$ | $3h/2-1$ | $-2N_a$ | $H' H H' \Sigma H, H'TT, H' \bar{F} T$ | $W_c$ |

*) In our models $SU(5)$ is able to break down not only to $SU(3) \times SU(2) \times U(1)$ but also to $SU(4) \times U(1)$ as is the case in other models. We restrain ourselves, however, from discussing this problem here.
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Colored particles within $H$ and $H'$ obtain masses of $O(A_{\text{GUT}})$ in the case $h + h' = 2$ (i.e., in models of $W_a$, $W_b$, and $W_c$), otherwise (in $W_d$ and $W_e$) of $O(A_{\text{GUT}})$.

Next we proceed to evaluate the anomaly $A$ of the $R$-symmetric current defined by

$$A = 2 \text{Tr} \ T_a^2 Q_R,$$

in terms of all colored fermions including gauginos. $T_a$'s denote the generators of $SU(5)$ with the conventional normalization. Then vacuum is rotated from $\theta_{\text{QCD}}$ to $\theta_{\text{QCD}} - A \alpha$ with chiral angle $\alpha$. They are also shown in Table I with $N_\alpha$ the number of generations.

By examining the anomaly in connection with the domain wall problem, we can find that a discrete symmetry survives as $Z_A$. Therefore in order to prevent the domain structure from appearing, we are led to the conclusion that the model potential $W_{c1}$ is most preferable with $N_\alpha = 4$ or 6 and $h = 2k$ ($k = \text{integer}$), in the cases of which the discrete subgroups reduce to superficial $Z_2$. It is well known, however, that more than 5-generation destroys asymptotic freedom in color sector in SUSY theories. Thus we can conclude that four-generation is favorable.

It is worthwhile to mention that each of these potentials has one more global $U(1)$ symmetry. This turns out to be connected with $B-L$ conservation. Since the charges $Q$ of the symmetry are given in Table II, we can verify the following relation:

$$B - L = 2/5 \times (2/5) Y - Q/h_0).$$

It may be easily confirmed that if the global $U(1)_Q$ has no $SU(3)_c$ anomaly.

We shall here present several parameters evaluated by means of the renormalization group. They read as, in the case of $W_{c1}$,

$$\sin^2 \theta_W = 0.230, \quad A_{\text{GUT}} = 2.2 \times 10^{16} \text{ GeV} \quad \text{and} \quad a_{\text{GUT}} = 17.2$$

for $N_\alpha = 4$, $\alpha_{\text{GUT}}(A_{\text{GUT}}) = 0.12$ and $a_{\text{GUT}}(A_{\text{GUT}}) = 1/129$.

By the absence of contribution from new particles they are identical with those of the standard SUSY SU(5) model.

Table II. The charges $Q$ of global $U(1)_Q$ free from anomaly.

<table>
<thead>
<tr>
<th>$\Phi_0$</th>
<th>$\Phi_0$</th>
<th>$\Sigma$</th>
<th>$H$</th>
<th>$H'$</th>
<th>$T_1$</th>
<th>$F_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q$</td>
<td>0</td>
<td>0</td>
<td>$h_0$</td>
<td>$-h_0$</td>
<td>$-h_0/2$</td>
<td>$3h_0/2$</td>
</tr>
</tbody>
</table>

§ 3. Axions based on the ordinary chiral phase transformation

In this section by making use of the freedom of ordinary chiral phase transformations on fields, we shall look for axion models obeying two conditions stated in the Introduction. In order that the provided global $U(1)$ symmetry not only be anomalous but also remain unbroken at the scale $A_{\text{GUT}}$, it is found out that three fields $\Phi_0(1), N(5)$ and $N'(5^*)$, added to five fields in the standard SUSY SU(5) model, compose the minimal set. Then sixteen interactions arise newly in the superpotential $W$ through the same procedure as used in (2·2) as follows:

$$N'N, \quad N'N\Phi_0, \quad N'\Sigma N, \quad N'\Sigma H, \quad H'\Sigma N,$$
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\[ N'H\Phi_e, \ NH'\Phi_e, \ N'H, \ H'N, \ NTT, \ N'\bar{F}T, \Phi_e, \ \Phi_e^2, \ \Phi_e^3, \ \Phi_e\Sigma^2 \text{ and } \Phi_eH'H. \quad (3\cdot1) \]

All of the terms obviously cannot take part in the potential symmetric in global \( U(1) \) transformations. It is also clear that the field \( \Phi_e \) and neither \( N \) nor \( N' \), should serve for the \( U(1) \) symmetry to break down without spoiling gauge symmetries. Therefore the chiral charge \( \phi \) of \( \Phi_e \) cannot vanish. After a short manipulation the following two superpotentials arise as candidates with \( W_s \) the potential of the standard model:

\[ W_a = aN'N\Phi_e + bN'\Sigma H + cH'N\Phi_e + dN'H' + eN'\bar{F}T + W_s, \quad (3\cdot2a) \]
\[ W_s = aN'N\Phi_e + bH'\Sigma N + cN'\Phi_e H + dN'H'N + eNTT + W_s. \quad (3\cdot2b) \]

The \( U(1) \) charges \( Q_{PQ} \) denoted by small letters of fields are, respectively, given in Table III. An intermediate mass scale \( \Lambda_{PQ} \) is realized by taking vacuum expectation values as \( \langle \Sigma \rangle_0 = O(\Lambda_{PQ}) \) and \( \langle f'\rangle_0 = O(\Lambda_{PQ}) \).

<table>
<thead>
<tr>
<th>Table III. The ( P-Q ) charges and anomalies of ( U(1)_{PQ} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N )</td>
</tr>
<tr>
<td>( W_a )</td>
</tr>
<tr>
<td>( W_s )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table IV. The charges ( Q ) of global ( U(1)_Q ) free from anomaly.</th>
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</thead>
<tbody>
<tr>
<td>( N )</td>
</tr>
<tr>
<td>( Q )</td>
</tr>
</tbody>
</table>

Next the anomaly of this group \( U(1)_{PQ} \) is evaluated as

\[ A = 2 \text{Tr} \ T_a^2 Q_{PQ} = -\phi = 0, \quad (3\cdot3) \]

irrespective of \( W_a \) or \( W_s \). Note that gauge superfields have no \( U(1) \) charge \( Q_{PQ} \). In order to avoid the domain structure in the early Universe, it is imperative that the following condition should be satisfied between two charges \( h \) and \( \phi \) of \( H(5) \) and \( \Phi_e(1) \), respectively:

\[ h = k \times \phi \quad \text{for arbitrary } \phi, \ k; \text{integer}. \quad (3\cdot4) \]

With this condition no discrete symmetry remains unbroken.

Now these models contain also one more \( U(1) \) symmetry which is shown to be connected with \( B-L \) conservation as in Eq. (2\cdot5). Namely, by using the charges given in Table IV, we have the following relation:

\[ B-L = 2/5 \times (2\sqrt{5}/3 Y - Q/h_0). \quad (3\cdot5) \]

It is straightforward to see that the global \( U(1) \) has no \( SU(3)_c \) anomaly.

Finally, various parameters are evaluated again by the method of the renormalization group. The new particles \( N \) acquire masses of \( O(\Lambda_{PQ}) \), \( N' \) of \( O(\Lambda_{PQ}) \) and vice versa through couplings to \( \Sigma \) and \( \Phi_e \), while the masses of the particles \( H \) and \( H' \) are largely splitted with respect to \( SU(2) \) doublet- and colored parts, respectively, as in models discussed in § 2. In the one-loop approximation we have...
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\[
\sin^2 \theta_w = 0.223, \quad A_{\text{GUT}} = 0.92 \times 10^{14} A_{\text{WS}} \quad \text{and} \quad a_{\text{GUT}} = 30.3
\]

for \( N_\sigma = 3, \quad a_\text{WS}(A_{\text{WS}}) = 0.12, \quad a_\text{EM}(A_{\text{WS}}) = 1/129 \) and \( A_{\text{WS}} = 10^{10} \text{GeV} \), (3.6)

which are not so much perturbed from the values obtained in the standard SUSY SU(5) model.\(^{15}\)

\§ 4. Discussion and remaining problems

We have studied what axion models emerge as minimal ones obeying the constraints pointed out recently by cosmology and astrophysics. They seem to suggest an intermediate mass scale requisite and to restrict field contents to a large extent, embodying the global \( U(1) \) symmetry after the Peccei-Quinn mechanism. Two alternative cases have been examined; one is axion models based on \( R \)-symmetry only the SUSY theory have and the other is based on the ordinary chiral phase transformation. It was concluded that, in the former case, the number of generations is preferred to be four by considering also asymptotic freedom of color sector. Most favorable superpotential has also been given explicitly. In the latter case, a certain relation between the \( P-Q \) charges was derived in order to bypass the domain wall problem.

Although we have set aside the breakdown of supersymmetry, our potential models have produced parameters such as weak angle, grand unification mass and its coupling constant not so much different from values evaluated in the standard SUSY SU(5) model.\(^{15}\)

Several models presented in this paper are minimal in double senses that minimal set of fields is assumed in order to bring about the mass scale different from the grand unification scale and that the intermediate scale is supposed to arise in the potential in the tree approximation. Moreover as open problems are left how to break supersymmetry and at what scale \( A_{\text{WS}} \). It will be very interesting also to investigate what relation there is between two scales \( A_{\text{WS}} \) and \( A_{\text{SS}} \). Although unless these problems have been settled, our models are not yet realistic, the results obtained here are instructive as for such simple and minimal assumptions. Our models deserve to be refined henceforth.

Acknowledgements

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