berg's *method in obtaining the existence criteria* of the 5R mechanism and certain other overconstrained mechanisms with five links and consisting of two or three prismatic pairs. The authors were perhaps guilty of not making this point very clear in the paper. The authors, however, do wish to emphasize one significant point here. It should be noted that the results in [21] and [22] were obtained by considering the five-link H-H-H-H-H mechanism proposed by Voinea and Atanasiu (see authors' additional reference [28]) which is itself an overconstrained mechanism. The results in the present paper have, on the other hand, been obtained by considering the more general zero family mechanisms. The present results, therefore, go beyond those of [21] and [22] and show that there are no mechanisms with two passive couplings and consisting of two or three prismatic pairs other than those obtained in [21] and [22] and confirmed in this study.

The 19 mechanisms in Table 1 that have C-pairs do indeed have, as Professor Hunt has rightly pointed out, a mobility of two and represent essentially trivial cases. The authors feel that these mechanisms should not have been included in the table as they detract from its value. Further, the statement in the latter part of the paper that the results remain unaffected even if one of the helical pairs is replaced by a cylinder pair is certainly not correct. Clearly, the replacement of a helical pair by a cylinder pair will result in an increase in mobility from one to two.

The discusser states that the property of parallelism of the helical pairs of finite pitch (including zero) is the only criterion for the existence of the 16 mechanisms with mobility one listed in Table 1 and that the equations in the entries 13 to 35 in the table appear to add restrictive conditions that are unnecessary. This is, however, not correct. A closer examination of these equations shows that they are not additional restrictions but are merely loop-closure conditions that must be satisfied by the constant kinematic angles of the mechanisms in order that the parallelism of the helical pairs may be preserved. Thus, for example, take the case of the five-link H-H-P-P-H mechanism shown in Fig. 7. If the three helical pairs in this mechanism are parallel to one another, the three twist angles β, γ, and δ and the two constant displacement angles χ_k and ξ_k at the two prismatic pairs cannot all have arbitrary values. A closed configuration with parallel helical pairs will result only when these five kinetic angles satisfy equation (31). Similar considerations apply to equations (35) through (38), equation (41), equation (43), as well as to the latter portions of equations (25) through (27).

The discusser remarks that the way in which the authors reduce the freedom of a cylinder pair from two to one is restrictive. The authors disagree. Whether one regards a cylinder pair as a combination of two helical pairs of different pitches (as the discusser suggests) or as a combination of a prismatic pair and a helical pair of finite pitch (including zero) as the authors have done is really not important. Both are equally valid concepts. The actual concept employed in any given case is decided entirely by the context of the development. The only important point to note in the present context is that a cylinder pair is a joint with two degrees of freedom in which the rotation is independent of the translation. The pair reduces to a joint with one degree of freedom if either the rotation or the translation is suppressed or if the rotation and the translation are forced to have a constant ratio between them. The cylinder pair reduces in these cases to a prismatic, revolute, or helical pair, respectively.

The discusser's statement that the derived criteria are neither necessary nor sufficient is not correct. In the general context of Dimentberg's method, it should be pointed out that the criteria are always necessary. This follows as a direct consequence of the method since the criteria are derived assuming that a mechanism of the desired type exists. This, however, does not guarantee that the derived criteria will always yield a nontrivial mechanism of the desired type. It should, therefore, be emphasized that the existence criteria obtained by Dimentberg's method are always necessary, but not always sufficient.

The discusser complains that the process of revelation is laborious. This is admittedly a drawback which could be quite significant in certain cases. One should, however, not forget that the Dimentberg approach has other compensatory features. First, as already mentioned, the method is capable of yielding the necessary conditions for existence. These include all possible solutions since any and every mechanism of the type under consideration must satisfy these conditions. Second, and this is more important, there is the assurance of finite mobility. This follows from the nature of the method. Since one starts with a parent mechanism of assured finite mobility, the finite mobility of the derived mechanisms is assured. The same cannot be said for other methods including screw theory. These other methods are always concerned essentially with only transitory or instantaneous mobility. Finite mobility results, as the discusser himself has pointed out, only when it can be shown that instantaneous mobility exists in all positions of the mechanism. Further, as mentioned in the paper, Dimentberg's method is particularly well-suited for obtaining the existence criteria of mechanisms in which there are conditions imposed not only on the twist angles, but also on the link lengths. The 5R mechanism considered in this paper and the R-C-R-C mechanism considered in Appendix D of [20] are examples of such mechanisms. Screw theory is certainly not capable of handling such cases.

The authors are sorry to note that the last paragraph of Professor Hunt's comments does not contribute to the discussion of the subject on hand. The discusser is entitled to his opinion on the mobility of space mechanisms, but the authors do not share his view. The authors know that the mobility of mechanisms has been an important and interesting enough topic to have engaged the attention of several classical kinematicians as well as leading modern investigators like Dimentberg, Moroshkin, Sharikov, Voinea, and Atanasiu, and others. The authors are also aware of Professor Waldron's and the discusser's own interest in this area.

The authors also do not agree with the last sentence of the discusser's comments. Surely the application of new and interesting methods of study to known problems has been attempted before in the investigation of mechanism mobilities. Such studies reveal the versatility and usefulness of the methods employed. The authors like to cite references [21] and [22] themselves as examples in this regard. The authors recognize the contribution made by these papers to mobility study, but they do not think that the mechanisms obtained and discussed in these references are as exceptional and unique as the discusser would have us believe.

Additional Reference

28 Voinea, R. P., and Atanasiu, M. C., "Geometrical Theory of Screws and Some Applications to the Theory of Mechanisms," *Revue de Mécanique Appliquée*, Vol. 7, No. 4, 1962, pp. 845–860.

Analysis and Synthesis of Mechanical Error in Linkages—A Stochastic Approach[1]

C. Amarnath[2] and B. K. Gupta.[3] The authors have presented a simple and elegant tool for the analysis and synthesis of errors in linkages. A few comments on certain assumptions are in order. The authors have assumed that the pin may randomly lie anywhere inside the clearance circle. It has been shown [9] that the pin center has a predictable locus, and is restricted to a very small portion of the clearance circle; in other words the probabili-

[1] By S. G. Dhande and J. Chakraborty, published in the Aug. 1973 issue of the JOURNAL OF ENGINEERING FOR INDUSTRY, TRANS. ASME, Series B, Vol. 95, No. 3, pp. 672–676.

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ty is not uniform throughout the clearance circle. Further, in assuming equation (2) and deriving equation (6) the authors have considered r_{ij} to be an arbitrary constant. In practice, r_{ij} is a random variable and is normally distributed. Thus, the probability density function $f(x_{ij}, y_{ij})$ would be considerably different from that given in equation (2) of the authors.

An examination of equation (10) reveals that when the tolerances on links and the clearances are numerically close (as in Table 2), the total contribution of the clearances to the variance of Φ is higher, and as such it would be desirable to consider the effect of variance in the clearance also.

It would be of great help if the authors could elaborate their reasons for assuming Φ to be normally distributed, particularly since the right hand side of equation (9) involves two sets of variables of different distributions.

Author's Closure

In the paper referred to by the discussers, attempts were made to find the locus of the journal center when the relative speed of the journal, and the race were sinusoidal and the load is either sinusoidal or constant under a number of assumptions. The results obtained in the referred paper, obviously, are irrelevant in our case because:

i) the relative speed is not sinusoidal;
ii) the load is neither constant nor sinusoidal;
iii) we assume the random presence of foreign particles in the bearing and random vibrating environment;
iv) our assumption is more general as we do not consider journal, roller, or ball bearings, but are interested only in the probability.

Under these circumstances, we feel that the locus of the pin center for a linkage mechanism cannot be "predictable," as has been claimed by the discussers.

It is an extremely difficult task to find the exact probability density for the location of the pin axis. The position of the pin axis depends on many factors, some of which are given next:

i) exact value of the clearance,
ii) viscosity of the lubricant,
iii) type of bearing,
iv) relative angular velocity of the links joined by the bearing,
v) nature and value of the load,
vi) nature of environment—vibrating or still
vii) existence of foreign particle in the bearing, and so on.

In the absence of enough data about the preceding, we made the assumptions that the probability density is uniform. After the publication of this paper, we made some more studies assuming normal distribution, and we hope to publish the results soon.

r_{ij}—the radial clearance of the hinge between the ith and jth links—is considered to be constant. For precision bearings, either journal or antifriction, the standard deviation of r_{ij} is extremely small. We assumed that for bearings used in linkages for function generating purposes, the amount of error by taking r_{ij} itself to be constant will be significantly smaller than the errors due to the play of the pin axis inside the clearance circle and variation of link lengths due to their tolerances.

Allocation of tolerances and clearances using dynamic programming technique has been done only for a particular position of the input link, and the closeness of tolerances and clearances is coincidental. Because of the complicated nature of equation (9), it is not justified to claim that the total contribution of the clearances to the variance of Φ is higher. The variance of Φ depends not only on the variances of the random variables but also on the coefficients given by equation (10).

In our paper, we gave reasons as to why the probability distribution of Φ can be assumed to be normal and we feel that this point does not require further elaboration.

References

9 Gupta, B. K., and Phelan, R. M., "The Load Capacity of Short Journal Bearings With Oscillating Effective Speeds," *Journal of Basic Engineering*, TRANS. ASME, Series D, Vol. 86, No. 2, June 1964, pp. 348–354.

Closed Form Displacement Relationships of Single and Multi-Loop Six-Link Spatial Mechanisms[1]

J. Duffy[2] and J. Rooney[3] 1 *Introduction.* The analysis of single-loop spatial mechanisms with more than four links has proven to be of paramount importance. All future design and optimization procedures associated with these mechanisms will be highly dependent on an efficient analysis procedure. It is not surprising that this most important subject has attracted the attention of many eminent research workers, including Freudenstein, Dimentberg, Yang, Woo, Yuan, Wallace, et al. The major objective in the analysis of spatial mechanisms surely must be to obtain input-output displacement equations free of extraneous roots. This becomes the central problem which has been clearly delineated by Wallace [18][4].

It is unfortunate that the conclusions presented by the authors in Section 6 entitled, "Extension of Results," are incorrect. This is because the methods for deriving closed form, input-output displacement equations for the spatial six link RCRRPR and 5R-C mechanisms presented in Sections 4 and 5 are erroneous in that they lead to displacement equations which contain a large number of extraneous or unwanted roots. Clearly it is undesirable to make use of displacement equations which contain extraneous roots. In addition to the excessive labor involved in algebra, programming, and computer time, it is necessary to determine, for example, from 128 real roots which ones lead to closure of the real mechanism. Admittedly, extraneous roots appear to occur in pairs [19, 20]; however, this is also the case for meaningful roots at turning and change points.

2 *Closed-Form Displacement Analysis of the RCRRPR Mechanism (Section 4).* The correct derivation of the input-output equation is given at the same conference [21] where the problem is reduced to eliminating the half-tangent of O_3 between a cubic equation and a quadratic equation. Careful study of these equations yields the fact that the coefficient of the cubic term is proportional to the coefficient of the quadratic term in the quadratic equation. The resulting single elimination gives an eighth degree polynomial for a six-link RCRRPR mechanism with general proportions. (For the very special case $S_3 = S_4 = 0$, considered by the authors, the coefficient of the cubic term is zero.) Furthermore, this algebraic result is in agreement with the eight assembly configurations obtained independently using directed line vector geometry [22] for the basic five-link RRCRP structure from which the RCRRPR mechanism is derived.

In Section 4 the authors have performed two eliminations. Firstly they eliminate θ_4 to obtain equation (43). Following this they eliminate θ_3 between equations (43) and (45). For a mechanism with general proportions this procedure gives a sixteenth degree polynomial (Section 6, (2)) which contains eight extraneous roots. Yuan [3] has already noted extraneous roots in the sixteenth degree polynomial he derived for the RPRCRR mechanism.

3 *Closed-Form Displacement Analysis of the RCRRRR and RRRCRR Mechanisms (Section 5).* The correct sixteenth degree polynomial input-output displacement equations have been derived in [23] for the more complex RCRRRR and RRRCRR mechanisms with general proportions by eliminating two extraneous variables in a single operation. This is in agreement with the sixteen assembly configurations obtained independently using directed line vector geometry [22] for the five-link RRCRR structure from which the RCRRRR and RRRCRR mechanisms are derived. Furthermore, mechanism proportions were selected using a physi-

[1] By A. H. Soni, R. V. Dukkipati, and M. Huang, published in the Aug. 1973 issue of the JOURNAL OF ENGINEERING FOR INDUSTRY, TRANS. ASME, Series B, Vol. 95, No. 3, pp. 709–716.

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[4] Numbers in brackets designate References at end of discussion.