ty is not uniform throughout the clearance circle. Further, in assuming equation (2) and deriving equation (6) the authors have considered r_{ij} to be an arbitrary constant. In practice, r_{ij} is a random variable and is normally distributed. Thus, the probability density function $f(x_{ij}, y_{ij})$ would be considerably different from that given in equation (2) of the authors.

An examination of equation (10) reveals that when the tolerances on links and the clearances are numerically close (as in Table 2), the total contribution of the clearances to the variance of Φ is higher, and as such it would be desirable to consider the effect of variance in the clearance also.

It would be of great help if the authors could elaborate their reasons for assuming Φ to be normally distributed, particularly since the right hand side of equation (9) involves two sets of variables of different distributions.

Author's Closure

In the paper referred to by the discussers, attempts were made to find the locus of the journal center when the relative speed of the journal, and the race were sinusoidal and the load is either sinusoidal or constant under a number of assumptions. The results obtained in the referred paper, obviously, are irrelevant in our case because:

i) the relative speed is not sinusoidal;
ii) the load is neither constant nor sinusoidal;
iii) we assume the random presence of foreign particles in the bearing and random vibrating environment;
iv) our assumption is more general as we do not consider journal, roller, or ball bearings, but are interested only in the probability.

Under these circumstances, we feel that the locus of the pin center for a linkage mechanism cannot be "predictable," as has been claimed by the discussers.

It is an extremely difficult task to find the exact probability density for the location of the pin axis. The position of the pin axis depends on many factors, some of which are given next:

i) exact value of the clearance,
ii) viscosity of the lubricant,
iii) type of bearing,
iv) relative angular velocity of the links joined by the bearing,
v) nature and value of the load,
vi) nature of environment—vibrating or still
vii) existence of foreign particle in the bearing, and so on.

In the absence of enough data about the preceding, we made the assumptions that the probability density is uniform. After the publication of this paper, we made some more studies assuming normal distribution, and we hope to publish the results soon.

r_{ij}—the radial clearance of the hinge between the ith and jth links—is considered to be constant. For precision bearings, either journal or antifriction, the standard deviation of r_{ij} is extremely small. We assumed that for bearings used in linkages for function generating purposes, the amount of error by taking r_{ij} itself to be constant will be significantly smaller than the errors due to the play of the pin axis inside the clearance circle and variation of link lengths due to their tolerances.

Allocation of tolerances and clearances using dynamic programming technique has been done only for a particular position of the input link, and the closeness of tolerances and clearances is coincidental. Because of the complicated nature of equation (9), it is not justified to claim that the total contribution of the clearances to the variance of Φ is higher. The variance of Φ depends not only on the variances of the random variables but also on the coefficients given by equation (10).

In our paper, we gave reasons as to why the probability distribution of Φ can be assumed to be normal and we feel that this point does not require further elaboration.

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Closed Form Displacement Relationships of Single and Multi-Loop Six-Link Spatial Mechanisms[1]

J. Duffy[2] and J. Rooney[3] 1 *Introduction.* The analysis of single-loop spatial mechanisms with more than four links has proven to be of paramount importance. All future design and optimization procedures associated with these mechanisms will be highly dependent on an efficient analysis procedure. It is not surprising that this most important subject has attracted the attention of many eminent research workers, including Freudenstein, Dimentberg, Yang, Woo, Yuan, Wallace, et al. The major objective in the analysis of spatial mechanisms surely must be to obtain input-output displacement equations free of extraneous roots. This becomes the central problem which has been clearly delineated by Wallace [18][4].

It is unfortunate that the conclusions presented by the authors in Section 6 entitled, "Extension of Results," are incorrect. This is because the methods for deriving closed form, input-output displacement equations for the spatial six link RCRRPR and 5R-C mechanisms presented in Sections 4 and 5 are erroneous in that they lead to displacement equations which contain a large number of extraneous or unwanted roots. Clearly it is undesirable to make use of displacement equations which contain extraneous roots. In addition to the excessive labor involved in algebra, programming, and computer time, it is necessary to determine, for example, from 128 real roots which ones lead to closure of the real mechanism. Admittedly, extraneous roots appear to occur in pairs [19, 20]; however, this is also the case for meaningful roots at turning and change points.

2 *Closed-Form Displacement Analysis of the RCRRPR Mechanism (Section 4).* The correct derivation of the input-output equation is given at the same conference [21] where the problem is reduced to eliminating the half-tangent of O_3 between a cubic equation and a quadratic equation. Careful study of these equations yields the fact that the coefficient of the cubic term is proportional to the coefficient of the quadratic term in the quadratic equation. The resulting single elimination gives an eighth degree polynomial for a six-link RCRRPR mechanism with general proportions. (For the very special case $S_3 = S_4 = 0$, considered by the authors, the coefficient of the cubic term is zero.) Furthermore, this algebraic result is in agreement with the eight assembly configurations obtained independently using directed line vector geometry [22] for the basic five-link RRCRP structure from which the RCRRPR mechanism is derived.

In Section 4 the authors have performed two eliminations. Firstly they eliminate θ_4 to obtain equation (43). Following this they eliminate θ_3 between equations (43) and (45). For a mechanism with general proportions this procedure gives a sixteenth degree polynomial (Section 6, (2)) which contains eight extraneous roots. Yuan [3] has already noted extraneous roots in the sixteenth degree polynomial he derived for the RPRCRR mechanism.

3 *Closed-Form Displacement Analysis of the RCRRRR and RRRCRR Mechanisms (Section 5).* The correct sixteenth degree polynomial input-output displacement equations have been derived in [23] for the more complex RCRRRR and RRRCRR mechanisms with general proportions by eliminating two extraneous variables in a single operation. This is in agreement with the sixteen assembly configurations obtained independently using directed line vector geometry [22] for the five-link RRCRR structure from which the RCRRRR and RRRCRR mechanisms are derived. Furthermore, mechanism proportions were selected using a physi-

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[1] By A. H. Soni, R. V. Dukkipati, and M. Huang, published in the Aug. 1973 issue of the JOURNAL OF ENGINEERING FOR INDUSTRY, TRANS. ASME, Series B, Vol. 95, No. 3, pp. 709–716.

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[4] Numbers in brackets designate References at end of discussion.

cal model as suggested in [22] in order to obtain sixteen real roots for the input-output equation. The numerical values of the roots were validated since there was exact agreement with the physical model. In addition, numerical values were validated using the closure equations.

In Section 5 the authors have performed three eliminations. They eliminate ξ and k to obtain equations (51), (52), and they finally eliminate χ to obtain equation (53). The equations suggested by the authors in Section 6, (3), (4), and (5) contain, respectively 48, 112, and 48 extraneous roots. Also input-output equations for the two inversions RCRRRR and RRRCRR must be of the same degree since they can be derived from the same basic structure (5). Conclusions (4) and (5) are therefore incorrect.

4 *Discusser's Conclusions.* The paper does not recognize the fact that in general performing more than one algebraic elimination introduces extraneous or unwanted roots [24].

In addition, it does not take advantage of the following critical problem formulations for deriving input-output equations for the single loop mechanisms under consideration:

(a) For the 4R-P-C mechanisms it is necessary to eliminate a single extraneous angular displacement in one operation.

(b) For 5R-C mechanisms it is necessary to eliminate two extraneous angular displacements in one operation.

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Authors' Closure

It is unfortunate that the discussers' remarks and conclusions are incorrect. We suggest our readers examine the following:

1 The fact that Sylvester Dyaltic eliminant method introduces extraneous roots is a well-known fact [25, 9, 14, 29, 26]. It is a pity that the discussers realized this fact only recently.

2 Establishing the order of input-output polynomials from basic structures (which the discussers call line vector geometry method) still remains to be proven.

3 The discussers conclusions are based on the projections of the input-output polynomial orders from the basic five-link structures (kinematic chains whose mobility is zero [27, 28]), to a mechanism by inversion technique which still remains to be proven for mechanisms having more than five-links.

4 The discussers in reference [21] oversimplified the input-output polynomial for RCRRPR six-link mechanism by taking dependent equations. The discussers also did not realize the fact that their elimination process does not guarantee the absolute value of the variable angle eliminated is ≤ 1.

The same is also true for their conclusion (b) in which they use dependent equations in eliminating two extraneous displacements in one operation. Their conclusions are based on reference [24], which is an unpublished work.

The present state-of-the-art is inadequate in some areas and the understanding and acceptance of the available technology is limited. Although Sylvester Dialytic method introduces extraneous roots, systematic procedure for obtaining the input-output order of polynomials for space mechanisms—for instance, the 3×3 matrices with dual elements as originally proposed by Dimentberg [5, 11], Yang and Freudenstein [2, 8], Soni and Harrisberger [10], Soni and Pamidi [9], or the line geometry method by Yuan [14, 26]—nevertheless establishes the upper limit for the input-output polynomial order for the space mechanisms. However, the exact order of the input-output polynomials should be confirmed by the sophisticated analog techniques developed by Crossely and Torfason [31, 32], and Timm [30].

It is unfortunate that a lot of claims by the discussers are based on an unpublished work. We wished that the discussers would have given us the opportunity to examine their work.

Looking objectively at the discussion and author's closure, the reader will observe that the matrix approach and other similar approaches permit one to establish an upper limit of the input-output polynomial. The researching kinematicians have yet to develop theories that will predict theoretically the lower limit of input-output of the polynomials. Such theories have to deal mathematically with the generation and intersection of the surfaces.

Unlike the discussers, we hope that our readers will recognize the contribution this paper is making in promotive research for the first time in the kinematic analysis of multiloop mechanisms.

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Stability and Kinematic Accuracy of Hydraulic Copying Mechanisms in Metal Cutting[1]

M. Massoud and W. A. Morcos.[2] The authors are to be congratulated for presenting a comprehensive study on the accuracy of the copying mechanisms. From a practical standpoint, the most valuable aspect of the paper is the stability investigation of the operation. The writers wish to point out that the authors' equation (9) indicates that the frequency ω_c is a function of the radius of workpiece r which is a variable parameter, $r = r(y)$; consequently, $\omega_c = \omega_c(y)$. As ω_f is closely related to the frequency ω_c, it is then safe to assume that $\omega_f = \omega_f(y)$. However, subsequent analysis of the equation of motion is based upon the assumption that ω_f is an independent parameter. Further, the authors' equation (10) assumes that F_s is composed of the components of F_q and F_r along the axis of power cylinder. Unless otherwise assumed, a bearing friction component should have been

[1] By W. M. Mansour, M. O. M. Osman, and G. M. L. Gladwell, published in the Aug. 1973 issue of the JOURNAL OF ENGINEERING FOR INDUSTRY, TRANS. ASME, Series B, Vol. 95, No. 3, pp. 787–793.

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