cal model as suggested in [22] in order to obtain sixteen real roots for the input-output equation. The numerical values of the roots were validated since there was exact agreement with the physical model. In addition, numerical values were validated using the closure equations.

In Section 5 the authors have performed three eliminations. They eliminate ξ and k to obtain equations (51), (52), and they finally eliminate χ to obtain equation (53). The equations suggested by the authors in Section 6, (3), (4), and (5) contain, respectively 48, 112, and 48 extraneous roots. Also input-output equations for the two inversions RCRRRR and RRRCRR must be of the same degree since they can be derived from the same basic structure (5). Conclusions (4) and (5) are therefore incorrect.

4 *Discusser's Conclusions.* The paper does not recognize the fact that in general performing more than one algebraic elimination introduces extraneous or unwanted roots [24].

In addition, it does not take advantage of the following critical problem formulations for deriving input-output equations for the single loop mechanisms under consideration:

(a) For the 4R-P-C mechanisms it is necessary to eliminate a single extraneous angular displacement in one operation.

(b) For 5R-C mechanisms it is necessary to eliminate two extraneous angular displacements in one operation.

Additional References

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19 Duffy, J., and Habib-Olahi, H. Y., "A Displacement Analysis of Spatial Five-Link 3R-2C Mechanisms. Part 2: Analysis of RRCRC Mechanism," *Journal Mechanisms,* Vol. 6, 1971, pp. 463–473.

20 Yuan, M. S. C., "Displacement Analysis of the RPRCRR Mechanism," *Journal of Applied Mechanics,* TRANS. ASME, Series E, Vol. 38, 1971.

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22 Rooney, J., and Duffy, J. "On the Closures of Spatial Mechanisms," ASME Paper No. 72-Mech-77.

23 Salmon, G., *Lessons Introductory to the Modern Higher Algebra,* Dublin, Hodges, Foster and Co., 1876.

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Authors' Closure

It is unfortunate that the discussers' remarks and conclusions are incorrect. We suggest our readers examine the following:

1 The fact that Sylvester Dyaltic eliminant method introduces extraneous roots is a well-known fact [25, 9, 14, 29, 26]. It is a pity that the discussers realized this fact only recently.

2 Establishing the order of input-output polynomials from basic structures (which the discussers call line vector geometry method) still remains to be proven.

3 The discussers conclusions are based on the projections of the input-output polynomial orders from the basic five-link structures (kinematic chains whose mobility is zero [27, 28]), to a mechanism by inversion technique which still remains to be proven for mechanisms having more than five-links.

4 The discussers in reference [21] oversimplified the input-output polynomial for RCRRPR six-link mechanism by taking dependent equations. The discussers also did not realize the fact that their elimination process does not guarantee the absolute value of the variable angle eliminated is ≤1.

The same is also true for their conclusion (b) in which they use dependent equations in eliminating two extraneous displacements in one operation. Their conclusions are based on reference [24], which is an unpublished work.

The present state-of-the-art is inadequate in some areas and the understanding and acceptance of the available technology is limited. Although Sylvester Dialytic method introduces extraneous roots, systematic procedure for obtaining the input-output order of polynomials for space mechanisms—for instance, the 3 × 3 matrices with dual elements as originally proposed by Dimentberg [5, 11], Yang and Freudenstein [2, 8], Soni and Harrisberger [10], Soni and Pamidi [9], or the line geometry method by Yuan [14, 26]—nevertheless establishes the upper limit for the input-output polynomial order for the space mechanisms. However, the exact order of the input-output polynomials should be confirmed by the sophisticated analog techniques developed by Crossely and Torfason [31, 32], and Timm [30].

It is unfortunate that a lot of claims by the discussers are based on an unpublished work. We wished that the discussers would have given us the opportunity to examine their work.

Looking objectively at the discussion and author's closure, the reader will observe that the matrix approach and other similar approaches permit one to establish an upper limit of the input-output polynomial. The researching kinematicians have yet to develop theories that will predict theoretically the lower limit of input-output of the polynomials. Such theories have to deal mathematically with the generation and intersection of the surfaces.

Unlike the discussers, we hope that our readers will recognize the contribution this paper is making in promotive research for the first time in the kinematic analysis of multiloop mechanisms.

Additional References

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31 Crossley, F. R. E., "Introducing the Electronic Analogue Computer by Way of Kinematic Geometry," *Bull. Mech. Eng. Educ.,* Vol. 4, 1965, pp. 145–149.

32 Torfason, L. E., and Crossley, F. R. E., "The Intersections of Solids Shown by Electronic Analog for Mechanism Simulation," TRANS. ASME, JOURNAL OF ENGINEERING FOR INDUSTRY, Series B, Vol. 93, No. 1, Feb. 1961, pp. 17–26.

Stability and Kinematic Accuracy of Hydraulic Copying Mechanisms in Metal Cutting[1]

M. Massoud and W. A. Morcos.[2] The authors are to be congratulated for presenting a comprehensive study on the accuracy of the copying mechanisms. From a practical standpoint, the most valuable aspect of the paper is the stability investigation of the operation. The writers wish to point out that the authors' equation (9) indicates that the frequency ω_c is a function of the radius of workpiece r which is a variable parameter, $r = r(y)$; consequently, $\omega_c = \omega_c(y)$. As ω_f is closely related to the frequency ω_c, it is then safe to assume that $\omega_f = \omega_f(y)$. However, subsequent analysis of the equation of motion is based upon the assumption that ω_f is an independent parameter. Further, the authors' equation (10) assumes that F_s is composed of the components of F_q and F_r along the axis of power cylinder. Unless otherwise assumed, a bearing friction component should have been

[1] By W. M. Mansour, M. O. M. Osman, and G. M. L. Gladwell, published in the Aug. 1973 issue of the JOURNAL OF ENGINEERING FOR INDUSTRY, TRANS. ASME, Series B, Vol. 95, No. 3, pp. 787–793.

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added; equation (10) should then assume the form

$$F_s = F_q \cos(\gamma - C_s) - F_r \sin(\gamma - C_s) \pm \mu\sqrt{F_p^2 + \{F_q \sin(\gamma - C_s) + F_r \cos(\gamma - C_s)\}^2}$$

This, however, would not affect the method of analysis. The writers also question whether, in equation (15), the number ¼ in the second term is accurate.

The stability analysis boils down to examining the eigenvalues of the approximate fundamental solution $Z(2\pi)$ of the matricial equation $Y' = \mathbf{A}\mathbf{Y}$ with initial values given by a unit matrix. Slightly reworded, we want to find the state transition matrix of the state equation $\mathbf{y}' = \mathbf{A}\mathbf{y}$. A simpler approach to the numerical solution would be to obtain $Z(2\pi)$ using a Picard iteration solution:

$$Z(2\pi) = \sum_{k=0}^{\infty} Z_k(2\pi)$$

where

$$Z_0 = \mathbf{I}$$

$$Z_n(2\pi) = \int_0 A(\tau)Z_{n-1}(\tau)d\tau \text{ for } n = 2, \ldots k$$

The advantage of this method is that the solution is obtained directly from the state formulation equation (21) instead of solving the static differential equation (18). The algorithm of the new method depends mainly on one recursive formula. However, the main advantage lies in the fact that sufficient components Z_n (2π) can be used to reduce the error in the solution to any required degree. A constraint equation can also be included in the digital solution to check the eigenvalues. Having $tr(A) = -a_1\tau + a_2 \sin \tau_1$ then,

$$|Y| = |Y_0| \exp\left(\int_0^\tau tr(A)d\tau\right)$$

$$= |Y_0| \exp(-a_1\tau + a_2 \cos \tau - a_2),$$

and $|Y_{2\pi}| = |Y||G|$

$$= |Y_0||G| \exp\{-a_2 - a_1\tau + a_2 \cos \tau\}$$

$$= |Y_0| \exp\{-a_2 - a_1(\tau + 2\pi) + a_2 \cos \tau\}$$

thus, $|G| = \exp(-2\pi a_1)$

$$= \lambda_1\lambda_2\lambda_3$$

where $\lambda_m = e^{j2\pi\sigma m}$, the eigenvalues of G.

Accordingly, $\sigma_1 + \sigma_2 + \sigma_3 = -ja_1$. This constraint condition can be used to check the accuracy of the eigenvalues or, at least, to estimate the third value.

Finally, the writers would like to point out that they enjoyed reading and discussing the analysis of a fine applied metal cutting problem.

Author's Closure

The authors wish to thank Professors Massoud and Morcos for their comments. It is obvious that ω_f is a function of y, but it is also obvious that the stability analysis is based on a linearized model with small variations from the mean as given by equation (18). Under these assumptions it is quite legitimate to visualize ω_f as a parameter reflecting the effect of n only.

The authors were also aware of the possibility of including dry friction as well as other recent refinements to the mathematical modeling of the cutting forces. However, it was decided that their inclusion in the analysis at this stage might further complicate an already complicated investigation. The effect of dry friction as well as thin-layer lubrication is currently being investigated by the second author.

The reviewers were hasty to conclude that the second term in equation (15) is an error. V is the total volume of oil as defined in the nomenclature and not half volume as used in current literature. The coefficient (¼) is correct.

Regarding the final remark, the authors are grateful to be shown another way of achieving the same results.

Investigation of Pulse Tube Refrigeration[1]

R. C. Longsworth.[2] The theoretical relation for no load cold end temperature based on an isothermal process in the hot end predicts temperatures that are slightly warmer than those proposed previously based on the assumption of an isentropic process in the hot end. Most of the data reported in [12] confirms the isentropic assumption; however, the spread in data is biased towards the isothermal assumption. It would be most interesting to know the no load cold end temperatures based on the present data and whether or not they agree with the present theory.

The proposed relation for heat pumping rates is oversimplified in that the proportionality constant K is not really a constant but is dependent on pulse rate, tube geometry, pressure ratio, etc. In developing the empirical relation for heat pumping rate given in (4) based on over 250 tests with helium it was found that the data correlated poorly based on the assumption (equation 19) that the effective mass of gas is proportional to $P_H - P_L$. Better correlation was achieved assuming the effective mass of gas to be proportional to P_H; however, no good explanation of the reason for this has been offered.

The present tests with air were all run at a relatively low value of the Fourier number, assuming smooth flow was actually achieved in the tube. Only between 30 and 50 percent of the heat that could have been exchanged between the gas and walls was actually transfered for the present tests. Heat pumping rate is thus not directly proportional to speed in the range of these tests. It seems quite coincidental that reasonably good correlation is achieved in the present tests with the proportionately constant $k = 1$.

The present data are shown plotted in the figure using the correlation given in [4] along with some limited data for air given in [12]. The correlating equation from [4] is

$$qm = \frac{(L_T/L_O)^{\gamma-1}}{5.29} \frac{(T_C - T_{CO})}{T_H - T_{CO}} \frac{L_O N D_P^2}{R} C_P P_H \frac{(P_H)^{0.375}}{345} (\pi^{\frac{\gamma-1}{\gamma}} - 1)$$

The equation for the curve drawn through the data points in this figure is

$$Q_{out} \equiv q = 5.6\, qm(1 - e^{-26.7\, N_F})$$

in which the Fourier number, N_F, is evaluated on the basis of properties at T_H and P_H.

From this figure it is seen that reasonably good correlation is obtained with the preceding relations and also the present data seem to be consistent with previous data.

In error:

1 Equation 9 should be: $Lc = (Lt\text{-}Lo)/\pi^{1/2} + Lo/\pi$

2 Page 3, right column, paragraph 1: If Lt, Lo, Th, and π are known . . . .

[1] By K. G. Narayankhedkar and V. D. Mane, published in the February 1973 issue of the JOURNAL OF ENGINEERING FOR INDUSTRY, TRANS. ASME, Series B, Vol. 95, No. 1, pp. 373–378.

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