added; equation (10) should then assume the form

$$F_s = F_q \cos(\gamma - C_s) - F_r \sin(\gamma - C_s)$$
$$\pm \mu\sqrt{F_p^2 + \{F_q \sin(\gamma - C_s) + F_r \cos(\gamma - C_s)\}^2}$$

This, however, would not affect the method of analysis. The writers also question whether, in equation (15), the number ¼ in the second term is accurate.

The stability analysis boils down to examining the eigenvalues of the approximate fundamental solution $Z(2\pi)$ of the matricial equation $Y' = \mathbf{AY}$ with initial values given by a unit matrix. Slightly reworded, we want to find the state transition matrix of the state equation $\mathbf{y}' = \mathbf{Ay}$. A simpler approach to the numerical solution would be to obtain $Z(2\pi)$ using a Picard iteration solution:

$$Z(2\pi) = \sum_{k=0}^{\infty} Z_k(2\pi)$$

where

$$Z_0 = \mathbf{I}$$

$$Z_n(2\pi) = \int_0 A(\tau)Z_{n-1}(\tau)d\tau \text{ for } n = 2,\ldots k$$

The advantage of this method is that the solution is obtained directly from the state formulation equation (21) instead of solving the static differential equation (18). The algorithm of the new method depends mainly on one recursive formula. However, the main advantage lies in the fact that sufficient components $Z_n(2\pi)$ can be used to reduce the error in the solution to any required degree. A constraint equation can also be included in the digital solution to check the eigenvalues. Having $tr(A) = -a_1\tau + a_2 \sin \tau_1$ then,

$$|Y| = |Y_0| \exp\left(\int_0^\tau tr(A)d\tau\right)$$
$$= |Y_0| \exp(-a_1\tau + a_2 \cos \tau - a_2),$$

$$\text{and } |Y_{2\pi}| = |Y||G|$$
$$= |Y_0||G| \exp\{-a_2 - a_1\tau + a_2 \cos \tau\}$$
$$= |Y_0| \exp\{-a_2 - a_1(\tau + 2\pi) + a_2 \cos \tau\}$$

$$\text{thus, } |G| = \exp(-2\pi a_1)$$
$$= \lambda_1\lambda_2\lambda_3$$

where $\lambda_m = e^{j2\pi\sigma m}$, the eigenvalues of G.

Accordingly, $\sigma_1 + \sigma_2 + \sigma_3 = -ja_1$. This constraint condition can be used to check the accuracy of the eigenvalues or, at least, to estimate the third value.

Finally, the writers would like to point out that they enjoyed reading and discussing the analysis of a fine applied metal cutting problem.

Author's Closure

The authors wish to thank Professors Massoud and Morcos for their comments. It is obvious that ω_f is a function of y, but it is also obvious that the stability analysis is based on a linearized model with small variations from the mean as given by equation (18). Under these assumptions it is quite legitimate to visualize ω_f as a parameter reflecting the effect of n only.

The authors were also aware of the possibility of including dry friction as well as other recent refinements to the mathematical modeling of the cutting forces. However, it was decided that their inclusion in the analysis at this stage might further complicate an already complicated investigation. The effect of dry friction as well as thin-layer lubrication is currently being investigated by the second author.

The reviewers were hasty to conclude that the second term in equation (15) is an error. V is the total volume of oil as defined in the nomenclature and not half volume as used in current literature. The coefficient (¼) is correct.

Regarding the final remark, the authors are grateful to be shown another way of achieving the same results.

Investigation of Pulse Tube Refrigeration[1]

R. C. Longsworth.[2] The theoretical relation for no load cold end temperature based on an isothermal process in the hot end predicts temperatures that are slightly warmer than those proposed previously based on the assumption of an isentropic process in the hot end. Most of the data reported in [12] confirms the isentropic assumption; however, the spread in data is biased towards the isothermal assumption. It would be most interesting to know the no load cold end temperatures based on the present data and whether or not they agree with the present theory.

The proposed relation for heat pumping rates is oversimplified in that the proportionality constant K is not really a constant but is dependent on pulse rate, tube geometry, pressure ratio, etc. In developing the empirical relation for heat pumping rate given in (4) based on over 250 tests with helium it was found that the data correlated poorly based on the assumption (equation 19) that the effective mass of gas is proportional to $P_H - P_L$. Better correlation was achieved assuming the effective mass of gas to be proportional to P_H; however, no good explanation of the reason for this has been offered.

The present tests with air were all run at a relatively low value of the Fourier number, assuming smooth flow was actually achieved in the tube. Only between 30 and 50 percent of the heat that could have been exchanged between the gas and walls was actually transfered for the present tests. Heat pumping rate is thus not directly proportional to speed in the range of these tests. It seems quite coincidental that reasonably good correlation is achieved in the present tests with the proportionately constant $k = 1$.

The present data are shown plotted in the figure using the correlation given in [4] along with some limited data for air given in [12]. The correlating equation from [4] is

$$qm = \frac{(L_T/L_O)^{\gamma-1}}{5.29} \frac{(T_C - T_{CO})}{T_H - T_{CO}} \frac{L_o N D_P^2}{R} C_P P_H \frac{(P_H)^{0.375}}{345} (\pi^{\frac{\gamma-1}{\gamma}} - 1)$$

The equation for the curve drawn through the data points in this figure is

$$Q_{out} \equiv q = 5.6\, qm(1 - e^{-26.7 N_F})$$

in which the Fourier number, N_F, is evaluated on the basis of properties at T_H and P_H.

From this figure it is seen that reasonably good correlation is obtained with the preceding relations and also the present data seem to be consistent with previous data.

In error:

1 Equation 9 should be: $Lc = (Lt\text{-}Lo)/\pi^{1/2} + Lo/\pi$

2 Page 3, right column, paragraph 1: If Lt, Lo, Th, and π are known . . . .

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[1] By K. G. Narayankhedkar and V. D. Mane, published in the February 1973 issue of the JOURNAL OF ENGINEERING FOR INDUSTRY, TRANS. ASME, Series B, Vol. 95, No. 1, pp. 373–378.

[2] Manager Development Engineer, Air Products and Chemical, Inc., Mem. ASME.

3 Fig. 5: a) Omit *Lo* from title block since it is a variable. b) Longsworth relation per reference [4] lies parallel to the curve shown but passes through the first data point, 9.9 watts at 31.5 mm.

4 Fig. 9: a) Omit *Lt* from title block since it is a variable. b) Move data point for 400 mm tube from 40 to 45 mm.

Additional References

[12] Longsworth, R. C., "An Analytical and Experimental Investigation of Pulse Tube Refrigeration," Ph.D. thesis, Syracuse University, Syracuse, New York, 1967.

Author's Closure

We thank Dr. Longsworth for his discussion on this paper. It is quite interesting to note that the spread of the data reported in [12] is biased towards the isothermal assumption. This itself justifies the assumption of isothermal process in the hot end. As the heat pumping rates were determined only at cold temperature $T_c = T_H$, the no load cold end temperatures based on present data were not reported. However, experimentation is under way for the same.

The proposed relation for heat pumping rate is, no doubt, a simplified relation. However, it provides a reasonably good correlation, when proportionality constant K is taken equal to 1, combined with the assumption that the effective mass of gas is proportional to $(P_H - P_L)$.

Discussion about the correlation from [4] and its extension for the present data forms a valuable supplement to our paper. We may mention here that the correlating equation from [4] does not hold good for pulse rates above optimum pulse rate. As per this correlation, the heat pumping rate increases with increase in the pulse rate, whereas the heat pumping rate should decrease at pulse rates above optimum pulse rate as per experimental investigation.

Corrections in equation (9) and Fig. 9 may be made as suggested by Dr. Longsworth. However, there are no mistakes in page 3, right column, paragraph 1.

In Fig. 5, Dr. Longsworth's relation does not pass through the first data point.