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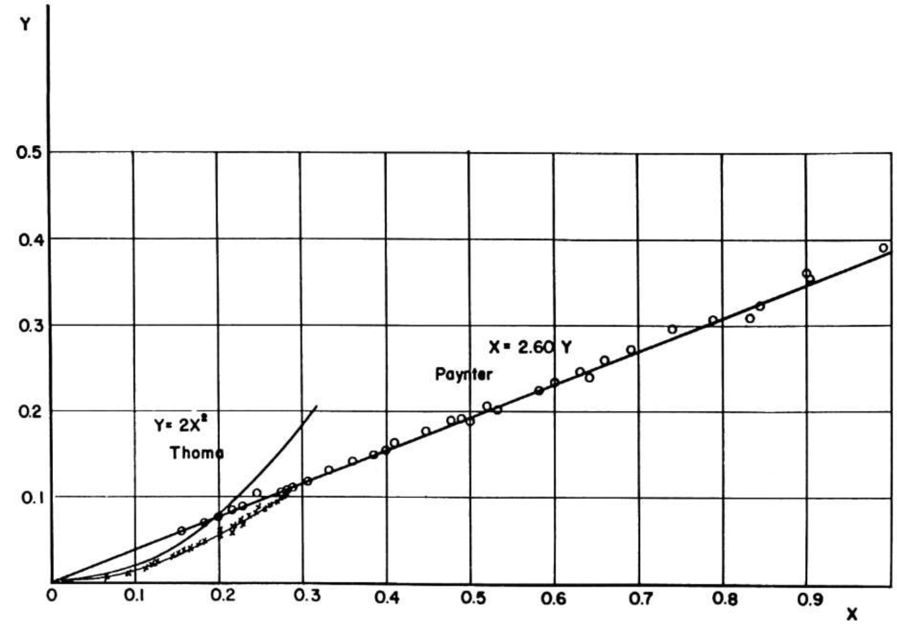

Fig. 7 Experimental data. Points for perpetual oscillations of constant amplitude shown ×. Points for drainage shown ⊙.

vertical tangent to the curve (48) at its point Q of minimum u. The value $u = -{}^4/_{27}X/Y$ is the least velocity in the surge tank for which Equation (49) can hold.

The point Q being a long way from the singularity B $(\alpha, 0)$, the shape of the separatrix in the region PQR is unaffected by B and governed wholly by the singularity A.

The separatrix is a vertical line (of infinite radius of curvature) at Q. It will remain parallel to $w = 0$ for some distance above Q. One may assume that on the separatrix u remains at its value $-{}^4/_{27}X/Y$ at least over the interval $-{}^1/_3 < w < Y$.

The critical condition for inertial drainage is derived by setting the initial point of the motion on the separatrix. The form of the condition must therefore depend on the initial conditions.

For the particular case of instantaneous full load acceptance from an initial zero flow condition in which the first demand of the turbine is met wholly by water from the surge tank, the initial point has co-ordinates ($w = Y$, $u = -Y/X$).

Thus $u = -Y/X = -{}^4/_{27}\, X/Y$ which gives the Paynter condition $X^2/Y^2 = 27/4$ or

$$X = 2.60Y \tag{50}$$

Equation (50) has been derived on the basis of infinitely small X and Y. The derivation depends on the approximation that the section of the separatrix for which $-{}^1/_3 < w < Y$ is the straight line $u = -{}^4/_{27}\, X/Y$.

As discussed in the foregoing the shape of the phase-plane picture in this region is uninfluenced by the position of the double singularity D. The singularity C $(\beta, 0)$ is below $w = -1$. As the friction Y is increased, the only singularity whose movement can effect Equation (50) is the saddle B at $(\alpha, 0)$, and this only after Y becomes very appreciable. Thus at $Y = {}^1/_2$ when B reaches A inertial drainage will be replaced by friction drainage.

Equation (50) can thus be expected to hold for appreciable X and Y. That indeed it does so is shown by the experimental points of Fig. 7 taken from the author's earlier work [5]. Also shown in Fig. 7 are experimental points showing that the Thoma condition $Y = 2X^2$ gives instability.

Conclusion

The phase-plane solutions of the surge-tank equation have been qualitatively mapped by obtaining the solution curves of the equation near each of its singularities and connecting up these curves. The qualitative picture allows quantitative deduction of the types of instability to be expected.

Acknowledgment

The author wishes to acknowledge Mr. Ettore Infante of the Graduate School of the University of Texas. Mr. Infante criticized the author's statement that only the singularities (0, 0) and (α, 0) were of physical significance and suggested that he examine the others. This paper is the result of that examination.

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DISCUSSION

Henry M. Paynter[3]

In this excellent paper the author has demonstrated the analytical basis of the tendency toward drainage instability in certain simple surge tank installations.

Of course, as the author himself points out, the validity of his analysis, as well as his own model experiments and this writer's earlier computer studies of more than a decade ago, is all premised

[3] Massachusetts Institute of Technology, Cambridge, Mass. Mem. ASME.

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upon the assumption of constant hydraulic power into the turbine. In actual hydroelectric installations, in the marginally stable case, the speed governors must ultimately bring the turbine gate momentarily to rest at the maximum gate limit. Beyond this point constant hydraulic power is impossible to maintain, and in practice the unit will drop load if it is tied to a large interconnected system.

Nevertheless, the condition still preserves its practical significance, since a tendency for the tank to drain will seriously jeopardize the value of such installations as sources of system spinning reserve. Actually, the author's Fig. 7 is perhaps more significant when replotted in a slightly different form.

It is fairly easy to demonstrate that the Thoma area A_{Thoma} for any given installation may be simply related to the conduit area A, conduit diameter D, the Darcy-Weisbach friction factor f, and the gross static head H, by the formula:

$$A_{\text{Thoma}} = AD/fH$$

Alternatively, we could determine the *diameter* of the Thoma tank D_{Thoma} by the expression:

$$D_{\text{Thoma}} = D\sqrt{D/fH}$$

Then the actual tank *diameter* D_t can always be expressed relative to the Thoma diameter as:

$$D_t/D_{\text{Thoma}} = D_t/D\sqrt{D/fH}$$

The ratio of the drainage diameter D_{drain} to the Thoma diameter is similarly directly found from the author's results, namely:

$$X_D = 2.6\,Y$$

$$X_T = 0.707\sqrt{Y}$$

$$\therefore \quad \boxed{D_{\text{drain}}/D_{\text{Thoma}} = X_D/X_T = 3.67\sqrt{Y}}$$

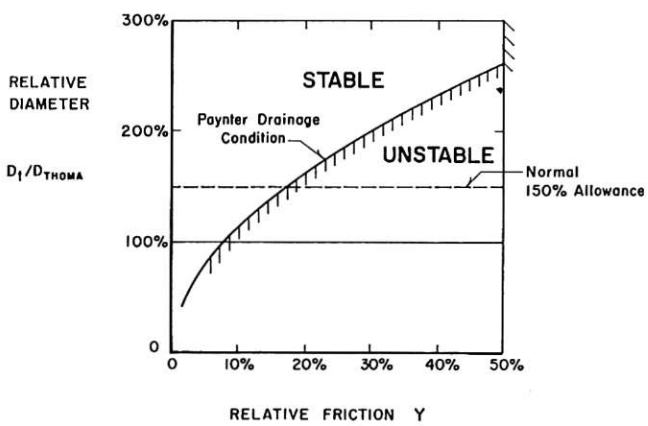

Fig. 8

This plot is indicated as Fig. 8. The drainage condition crosses the 100 per cent line at a value of relative friction Y of 2/27 or 7.4 per cent. This means that for all plants with more than 7 per cent rated friction loss there will be a possibility of incipient drainage if only Thoma areas are used. Moreover, Fig. 8 demonstrates that even the normal 150 per cent allowance will only give protection out to about 15 per cent loss.

As is well appreciated by practitioners in this field, the most acute case arises with low heads and large, long tunnels. If say, D = 15 ft, H = 100 ft, and f = 0.015 then:

$$D_{\text{Thoma}} = 15\sqrt{15/1.5} = 47 \text{ feet}$$

Clearly for such costly installations, it is important to determine the safe minimum size of tank as closely as possible. Moreover, such low head plants are particularly prone to periods of high tailwater during flood season, when they will frequently attempt to generate power under conditions of extreme flow and therefore high percentage friction loss.

For example, the hypothetical plant above, assuming a tunnel conduit 3000 feet long with a velocity head of three feet under worst conditions, would have a relative friction

$$Y = 0.015 \times 3 \times 3000/15 \times 100 = 0.09$$

or 9 per cent loss.

Without question, such a plant might have a drainage problem. *But how practical is an installation with the above specifications?* To summarize the above data, we have assumed:

H = 100 feet
L = 3000 feet
D = 15 feet

This is compatible with a capacity of approximately 25,000 hp. Indeed, such a station is possible but, since it is of an extended fall type, it would have only marginal practical economic benefit.

It is therefore the opinion of the writer that the phenomenon of drainage instability is but little encountered in practice and designers should merely be alerted to this possibility whenever the rated friction loss exceeds 7 per cent.

As for the effects of variable efficiency upon the drainage condition, these are almost certainly less than the gate limit effects mentioned above, since only as the gate position reaches maximum opening will reduced efficiency produce measurable changes in behavior.

Finally, the phenomenon of drainage instability as well as Thoma oscillatory instability can be avoided entirely by proper governing upon load acceptance as we have demonstrated in connection with the current research program in system governing sponsored by the Woodward Governor Company at the Massachusetts Institute of Technology.

Despite all the remarks above, the author should be congratulated for an excellent mathematical analysis of an interesting phenomenon both from a practical and from a topological point of view.

Author's Closure

Dr. Paynter has made a valuable addition to the paper by relating the theoretical instability criteria to current practical situations.

As intimated in the first section, one of the motivations for this paper is to demonstrate the way in which some of the methods of nonlinear mechanics may be applied to real engineering problems. Such methods give much insight into the validity of the classical approximations.

Dr. Paynter's fine discussion leaves little to be added, except the author's thanks.

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