Arbitrariness of Zwanziger’s Parameter
in Stochastic Quantization of Non-Abelian Gauge Fields

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(Received November 25, 1983)

It is shown that there exists another type of Zwanziger’s parameter of the modified Fokker-Planck equation in stochastic quantization of non-abelian gauge fields. The key is the use of the anti-BRS-transformation. It is also pointed out that the parameter has more arbitrariness.

It is known that the stochastic quantization method proposed by Parisi and Wu reproduces the same results as those of the conventional quantizations for non-gauge theories. As far as gauge theories are concerned, however, there are some problems.

By the perturbative calculation Namiki et al. showed the equivalence between the stochastic and the conventional quantization for the non-abelian gauge fields. Zwanziger introduced a modified Fokker-Planck equation and Baulieu and Zwanziger proved that a suitable choice of the parameter contained in this modified Fokker-Planck equation makes the Faddeev-Popov distribution for the covariant gauge condition be an equilibrium solution of the modified Fokker-Planck equation. Moreover, it has been proved in a previous paper that for an arbitrary gauge condition there exists such a parameter, which we call Zwanziger’s parameter in the following.

Therefore, the stochastic quantization method is equivalent to the other conventional ones in the non-abelian gauge theories if the solution of the modified Fokker-Planck equation relaxes on the equilibrium one. We have found in the previous paper that this relaxation is proved if motion of the solution to the Langevin equation is confined to the Gribov region, and have given a criterion for the confinement to the Gribov region. Unfortunately, however, this confinement is not proved yet in the case of the covariant gauge condition with Baulieu and Zwanziger’s choice of the parameter. Thus, the search of other choices of Zwanziger’s parameter should be significant for the proof of the equivalence between the stochastic and the other quantization procedure.

The aim of this paper is to show that there exists another type of Zwanziger’s parameter for the covariant gauge. It is also pointed out that Zwanziger’s parameter has arbitrariness for any gauge condition.

Let us start with the modified Fokker-Planck equation,

\[ \frac{d}{dt} P(A, t) = LP(A, t), \]

\[ L = \int d^4x \left\{ \frac{\delta}{\delta A^a(x)} \left( \frac{\delta}{\delta A^a(x)} \right) \right\}, \]

where \( S_\text{cl} \) is the Euclidean classical action,

\[ S_\text{cl} = \frac{1}{4} \int d^4x (\partial_\mu A^a_\mu - \partial_\mu A^a \partial_\mu A^a), \]

and \( D^a = \delta^a \partial_\mu + g f^{abc} A^b \partial_\mu A^c \). The parameter \( V^a \) is supposed to depend on \( A^a \).

The BRS-invariant total action \( S_\text{tot} \) for an arbitrary gauge condition is given by

\[ S_\text{tot} = S_\text{cl} + \delta K, \]

where \( \delta \) denotes the BRS-transformation which is defined by

\[ \delta = \int d^4x \left\{ D^a A^b c^b + \frac{\delta}{\delta A^a_\mu} (\partial_\mu A^a) \right\}. \]

Here \( B^a, c^a \) and \( c^a \) are an auxiliary and the anti-commuting Faddeev-Popov ghost fields, respectively. \( K \) is an arbitrary function. For the covariant gauge condition we put

\[ K_{(\text{cov})} = \int d^4x \left( \frac{1}{2} B^a + \partial_\mu A^a \right) c^a, \]

and we obtain

\[ S_{\text{tot (cov)}} = S_\text{cl} + \int d^4x \left\{ \frac{1}{2} B^a B^a + \partial_\mu A^a \cdot B^a + \partial_\mu c^a \cdot D^a c^b \right\}. \]
It is proved,\(^4\)\(^5\) that if we put
\[
V^a(x) = -\int dc\, dB \int d^4 y c^a(x) \frac{\delta}{\delta A^a_{\mu}(y)} \left[ \left( \frac{\delta K}{\delta A^a_{\mu}(y)} \right) \exp\left( -S_{\text{tot}} \right) \right] \cdot P_{FP},
\] (8)
the Faddeev-Popov distribution \(P_{FP}\), which is given by
\[
P_{FP} = \int dc\, dB \, \exp\left( -S_{\text{tot}} \right),
\] (9)
satisfies the equilibrium Fokker-Planck equation,
\[
L P_{FP} = 0.
\] (10)

For the covariant gauge condition we have
\[
V_{\text{cov}}^a(x) = \int dc\, dB \int d^4 y c^a(x) \partial_{\mu} c^a(y) \left( \frac{\delta}{\delta A^a_{\mu}(x)} \right) \left[ \left( \frac{\delta K}{\delta A^a_{\mu}(y)} \right) \exp\left( -S_{\text{tot(cov)}} \right) \right] \cdot P_{FP}.
\] (11)
The key of the proof was the BRS-invariance of the total action (4):
\[
\delta S_{\text{tot}} = 0.
\] (12)

The criterion for the confinement to the Gribov region is
\[
\int d^4 x \left\{ \frac{\delta}{\delta A^a_{\mu}(x)} \right\} \left[ \left( \frac{\delta K}{\delta A^a_{\mu}(x)} \right) \exp\left( -S_{\text{tot}} \right) \right] = 0.
\]
As mentioned before, we failed to prove this equation for the covariant gauge with the choice of Eq. (11). Thus, we should search other possibilities.

It is known that there exists another type of the BRS-transformation (anti-BRS-transformation)\(^8\) which is defined by
\[
\delta = \int d^4 x \left\{ D_{\mu}^{ab} c^b \cdot \frac{\delta}{\delta A^a_{\mu}} \right\} - \left( B^a + g f^{abc} c^b c^c \right) \frac{\delta}{\delta c^a} - \frac{g}{2} f^{abc} c^b c^c \frac{\delta}{\delta B^c} - \frac{\delta}{\delta B^a} \right\}.
\] (13)

This expression can be rewritten into a more symmetric form to the BRS-transformation (5) by the shift of \(B^a\):
\[
B^a = B^a + gf^{abc} c^b c^c.
\] (14)

We obtain instead of (13),
\[
\delta = \left\{ \int d^4 x \left\{ D_{\mu}^{ab} c^b \cdot \frac{\delta}{\delta A^a_{\mu}} \right\} - \left( B^a + g f^{abc} c^b c^c \right) \frac{\delta}{\delta c^a} - \frac{g}{2} f^{abc} c^b c^c \frac{\delta}{\delta B^c} - \frac{\delta}{\delta B^a} \right\}.
\] (15)

This coincides with \(\delta \) of (5) where \(c^a, \bar{c}^a\) and \(B^a\) are substituted by \(\bar{c}^a, -c^a\) and \(B^a\), respectively. Unitarity of physical \(S\)-matrix is assured also on the basis of the anti-BRS-invariance of the total action \(S_{\text{tot}}\). The anti-BRS-invariant total action is obtained by
\[
S_{\text{tot}} = S_{\text{cov}} + \delta \tilde{K}
\] (16)
with an arbitrary function \(\tilde{K}\). For this anti-BRS-invariant action the parameter \(V^a\) which corresponds to \(V^a\) of (8) is given by
\[
\tilde{V}^a(x) = -\int dc\, dB \int d^4 y c^a(x) \cdot \frac{\delta}{\delta A^a_{\mu}(y)} \left[ \left( \frac{\delta K}{\delta A^a_{\mu}(y)} \right) \exp\left( -S_{\text{tot(cov)}} \right) \right] \cdot P_{FP}.
\] (17)

Equation (10) holds with the above parameter \(\tilde{V}^a\). The proof is given in the same manner as in the previous paper\(^8\) by making use of the anti-BRS-transformation instead of the BRS-one.

One should note that the BRS-invariant action (7) for the covariant gauge condition is also anti-BRS-invariant.\(^8\) This is easily seen by the substitution of
\[
\tilde{K} = \int d^4 x \left\{ -\frac{g}{2} c^a B^a - \partial_{\mu} A^a_{\mu} \cdot c^a \right\}
\] (18)
into (16). Then we obtain the anti-BRS-invariant total action which is equal to (7). The parameter \(\tilde{V}^a\) in this case is given by
\[
\tilde{V}_{\text{cov}}^a(x) = -\int dc\, dB \int d^4 y c^a(x) \cdot \frac{\delta}{\delta A^a_{\mu}(y)} \left[ \left( \frac{\delta K}{\delta A^a_{\mu}(y)} \right) \exp\left( -S_{\text{tot(cov)}} \right) \right] \cdot P_{FP}.
\] (19)

The above \(\tilde{V}_{\text{cov}}^a\) is unequal to \(V_{\text{cov}}^a\) of (11) because \(S_{\text{tot(cov)}}\) of (7) is asymmetric to the exchange of \(c^a\) and \(\bar{c}^a\) for \(\bar{c}^a\) and \(-c^a\), respectively.

In contrast to this, if we put
\[
K = \delta \left\{ -\frac{1}{2} \int d^4 x (a c^a c^a \cdot A^a_\mu A^a_\mu) \right\}
\] (20)
we obtain instead of (13),
\[
K = \frac{1}{2} \int d^4 x \left\{ a B^a c^a + \frac{g}{2} f^{abc} c^b c^c \right\}
\] (21)
\[ S_{\text{tot}} = S_{\text{cl}} + \frac{1}{2} \int d^4 x \]
\[ \times \left\{ a B^a B^a - \frac{1}{a} (\partial_\mu A_\mu)^2 \right\} \]
\[ + a \left( \frac{g}{2} f^{abc} c^b c^c \right)^2 + D_\mu c^a \partial_\mu c^a \]
\[ + \partial_\mu c^a \cdot D_\mu c^a , \]
\[ (21) \]

where we put
\[ B^a = B^a + \frac{g}{2} f^{abc} c^b c^c - \frac{1}{a} \partial_\mu A_\mu c^a . \]
\[ (22) \]

In this case we find \( V^a = \tilde{V}^a \). With this total action (21), however, we cannot carry out the functional integration with respect to \( c^a \) and \( \tilde{c}^a \) because of the quartic ghost coupling in the action (21).

Since the modified Fokker-Planck equation (9) contains \( V^a \) linearly, a linear combination of \( V_{(\text{cov})}^a \) and \( \tilde{V}_{(\text{cov})}^a \) such as
\[ V_{(\text{cov})}^a = \beta V_{(\text{cov})}^a + (1 - \beta) \tilde{V}_{(\text{cov})}^a , \]
\[ (23) \]
where \( \beta \) is a constant, is also a solution of the inverse problem of the Fokker-Planck equation (10).

Before ending, we point out that the parameter \( V^a \) of (8) has more arbitrariness; we can add a BRS-invariant term to \( K \) in (4) without changing the total action \( S_{\text{tot}} \). On the other hand, \( V^a \) of (8) is varied if this additional term contains \( c^a \) and \( A_\mu c^a \).

The two types of arbitrariness of the parameter \( V^a \) mentioned above and their combination may be useful for proving the confinement of the solution of the Langevin equation to the Gribov region.