A Dynamical Macroscopic Description of Isovector Giant Multipole States

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(Received January 23, 1984)

An irrotational fluidodynamical method based on the generalized scaling approximation is applied to the description of isovector giant multipole states. For the case of a simple square density model for the ground state, analytic solutions of the equation of motion are derived and contrasted with those of the familiar hydrodynamical model. It is shown from model calculations that the present description provides a fair approximation to microscopic RPA results on gross features of isovector giant vibrations.

§ 1. Introduction

Macroscopic fluidodynamical (FD) models of collective motion provide interesting possibility of understanding basic features of collective states using a few dynamical variables without entering into detailed microscopic calculations. During the last decade, stimulated by an attempt of Bertsch to approximately reformulate the RPA equation in terms of a macroscopic displacement (generalized scaling) field, FD descriptions have found renewed interest especially in the application to giant multipole states. It has been shown that the scaling approach can take account of the dynamical quadrupole distortion in momentum space and thus serves to avoid the assumption of local equilibrium underlying the standard hydrodynamical model. In our previous work we have demonstrated that the irrotational FD approach to nuclear dynamics in terms of a single velocity (or displacement) potential is successfully applicable to the description of isoscalar giant vibrations with low multipolarity. Recently this approach has been also used to calculate collective excitations in the continuum with a good success. In the present paper we shall study basic features of isovector giant multipole states resulting from this irrotational description. The present work thus is concerned with a dynamical extension of the variational study of Ref. 9) where a specially parametrized form is assumed for the velocity potential.

In §2 we briefly summarize the formalism leading to the macroscopic equation of motion for the dynamical variable, using Skyrme-type effective interactions. On the assumption of a square density distribution for the ground state, analytic solutions of the equation of motion are derived in §3. These solutions are then contrasted with those of the familiar hydrodynamical model based on the first sound dynamics. In §4 properties of eigenmodes resulting from the present description are discussed and compared with microscopic RPA results.

§ 2. Equation of motion

We start with constructing a time-dependent many-body state $|\Psi(t)\rangle$ as

\[ |\Psi(t)\rangle = \sum_n C_n(t) |\phi_n\rangle \]

where $|\phi_n\rangle$ are the microscopic eigenstates of the Hamiltonian and $C_n(t)$ are the coefficients that evolve according to the equation of motion.

\[ \frac{dC_n}{dt} = \int d^3x \left[ \frac{1}{2m} \nabla^2 C_n(x,t) - V(x) C_n(x,t) \right] \]

where $V(x)$ is the effective potential.

\[ \left[ \frac{1}{2m} \nabla^2 - V(x) \right] C_n(x,t) = \frac{1}{\hbar} \sum_{n'} \langle \phi_n| \left[ \frac{1}{2m} \nabla^2 - V(x) \right] |\phi_{n'}\rangle C_{n'}(t) \]

The solution of this equation can be found using the method of Green's functions.
in terms of the velocity potential \( \xi(\xi^* = \dot{\xi}) \) and the displacement (generalized scaling) field \( s \). Here \( |\Psi_0\rangle \) is a time-reversal invariant and spherically symmetric HF ground state, \( m \) the nucleon mass and \( \tau_z \) the \( z \)-component of the isospin Pauli matrix. On the assumption of small amplitude oscillations we then subject the wave function of Eq. (2.1) to the time-dependent variational principle:

\[
\delta \int_{t_1}^{t_2} \langle \Psi(t)| \left( i \frac{\partial}{\partial t} - H \right) |\Psi(t)\rangle dt = 0
\]  

(2.2)

with \( \delta \Psi(t_1) = \delta \Psi(t_2) = 0 \). As the effective interaction in the total Hamiltonian \( H \), we employ the Skyrme-type interaction:

\[
v_{12} = t_0(1 + x_0 P_0) \delta(r_{12}) + \frac{1}{2} t_1(1 + x_1 P_0)(k'^2 \delta(r_{12}) + \delta(r_{12}) k'^2)
\]

\[+
 t_3(1 + x_3 P_0) k \cdot \delta(r_{12}) k + \frac{6}{1} t_3(1 + x_3 P_0) \rho^s(R_{12}) \delta(r_{12}),
\]  

(2.3)

where \( k = (1/2i)(\overrightarrow{r}_1 - \overrightarrow{r}_2) \), \( k' = -(1/2i)(\overrightarrow{r}_1 - \overrightarrow{r}_2) \), \( r_{12} = r_1 - r_2 \) and \( R_{12} = (r_1 + r_2)/2 \). The Coulomb and spin-orbit interactions are omitted and \( N = Z \) systems are considered.

The variation with respect to \( \xi(r, t) \) leads to the equation of continuity:

\[
m \dot{\rho}_0(r) \cdot (\rho_0(r) s(r)) + m \rho \cdot ((1 + x(r)) \rho_0(r) \rho \xi(r, t)) = 0,
\]  

(2.4)

where \( \rho_0(r) \) denotes the ground state density and \( x(r) \) the local ‘enhancement factor’ defined by

\[
1 + x(r) = (m/m^*(r))(1 - 4 b_r m^*(r) \rho_0(r))
\]  

(2.5)

with the effective mass \( m^*(r) = (1/m + 4 b_0 \rho_0(r))^{-1} \). Here \( b_0 \) and \( b_r \) are the coefficients of the current-current interaction in the isoscalar and isovector spin-independent channels (0- and \( \tau \)-channels), respectively:

\[
\begin{bmatrix}
 b_0 = [3 t_1 + 5 t_2 (1 + 4 x_2/5)] / 32, \\
 b_r = [- t_1 (1 + 2 x_1) + t_3 (1 + 2 x_2)] / 32.
\end{bmatrix}
\]

Equation (2.4) is obviously satisfied by choosing

\[
\dot{a}(t) s(r) + (1 + x(r)) \rho \xi(r, t) = 0.
\]  

(2.6)*

By assuming a harmonic time-dependence of \( a(t) \) and \( \xi(r, t) \) and taking account of Eq. (2.6), the variation with respect to \( a(t) \) yields

\[
\omega^2 = C[s]/B[s].
\]  

(2.7)

Here the restoring force coefficient \( C[s] \) and the inertia parameter \( B[s] \) are given by

\[
C[s] = \frac{1}{2} \left[ \frac{d^2}{da^2} \langle \Psi_0 | \exp(-a D[s]) H \exp(a D[s]) |\Psi_0\rangle \right]_{a=0}
\]  

(2.8)

* Generally it is possible to add a term \((1/\rho_0(r)) \rho \times A(r, t)\) to this equation. In the present paper, however, we leave out this additional possibility.
with \( D[s] = \sum_j(s(r_j) \cdot \nabla_j + \nabla_j \cdot s(r_j)) \tau_2(j)/2 \), and
\[
B[s] = \frac{m}{2} \int d^3 r \rho_0(r) s(r) \cdot s(r)/(1 + x(r)).
\]
(2.9)

It follows from Eq. (2.6) that
\[
s(r) = (1 + x(r)) \nabla F(r)
\]
(2.10)

with \( \xi(r, t) = -\dot{a}(t) F(r) \). The displacement field is no longer irrotational except for the case \( x(r) = \text{constant} \), although the convection part \( v(r, t) \) of the velocity field is, by construction, irrotational \( v(r, t) = \nabla \xi(r, t) \). The variation with respect to \( F(r) \) reads
\[
\frac{\delta C[s]}{\delta F(r)} = \omega^2 \frac{\delta B[s]}{\delta F(r)} = 0.
\]
(2.11)

This is the basic equation of our dynamical description.

Before proceeding to the next section, we show the expression of the restoring force coefficient \( C[s] \): Let

\[
a_0(\rho) = \frac{3}{4} \left[ t_0 + \frac{(a+1)(a+2)}{2} \frac{1}{6} t_0 \rho^a \right],
\]

\[
a_\tau(\rho) = \frac{1}{4} \left[ t_0(1+2x_0) + \frac{1}{6} t_3(1+2x_3) \rho^a \right],
\]

\[
c_0 = \frac{1}{64} \left[ 9t_1 - 5t_2 \left( 1 + \frac{4}{5} x_2 \right) \right],
\]

\[
c_\tau = \frac{1}{64} \left[ 3t_1(1+2x_1) + t_2(1+2x_2) \right]
\]

and

\[
\tau_{as}(r) \equiv \{ \nabla a(1) \nabla a(2) \rho_0(r_1, r_2) \}_{r_1=r_2=r},
\]

\( \rho_0(r_1, r_2) \) being the ground state density matrix; then
\[
C[s] = \int d^3 r \left\{ \frac{1}{2m^2(r)} \tau_2(r)
\right.
\]

\[
- \frac{1}{2} a_0(\rho_0(r)) (s(r) \cdot \nabla \rho_0(r)) \nabla \cdot (\rho_0(r) s(r)) + \frac{1}{2} a_\tau(\rho_0(r)) \left[ \nabla \cdot (\rho_0(r) s(r)) \right]^2
\]

\[
- b_0 s(r) \cdot (\nabla \tau_{as}(r)) \nabla \cdot (\rho_0(r) s(r)) + 2b_\tau \tau_1(r) \nabla \cdot (\rho_0(r) s(r))
\]

\[
- c_0(\nabla^2 \rho_0(r)) \nabla \cdot [s(r) \nabla \cdot (\rho_0(r) s(r))] + c_\tau [\nabla \cdot (\rho_0(r) s(r))]^2 \}
\]
(2.12)

where

\[
\tau_1(r) = 2(\nabla \rho_0(r)) \tau_{as}(r) + \nabla \cdot (\tau_{as}(r) s(r)) + \frac{1}{2} (\nabla \rho_0(r) \cdot \nabla s(r)) (\nabla \rho_0(r)),
\]

\[
\tau_2(r) = [(\nabla \rho_0(r))(\nabla \rho_0(r)) + (\nabla a \nabla \rho_0(r))(\nabla a \nabla \rho_0(r)) - s(r) \cdot \nabla (\nabla a \nabla \rho_0(r))] \tau_{as}(r)
\]

\[
+ \frac{1}{2} \nabla \cdot [s(r) \nabla \cdot (\tau_{as}(r) s(r))] + 2 \nabla \cdot [s(r)(\nabla a \nabla \rho_0(r)) \tau_{as}(r)]
\]
with the summation convention over repeated vector indices. We note that \( \alpha_0(\mathbf{r}) \), \( \alpha_1(\mathbf{r}) \) and \( \alpha_2(\mathbf{r}) \) are related to the isoscalar \((\theta = 0)\) and isovector \((\theta = 1)\) kinetic energy densities of the time-even state \( \exp\{aD[s]\}\psi\). by

\[
\langle \psi|e^{-aD[s]}\sum_{i}^{n}i\mathbf{P}_{i}\cdot\delta(\mathbf{r}-\mathbf{r}_{i})|\psi\rangle
= (1-\theta)(\alpha_0(\mathbf{r})+a^2\alpha_2(\mathbf{r})+\cdots)+\theta(\alpha_1(\mathbf{r})+\cdots).
\]

The equation of motion (2.11) in our dynamical description can be solved numerically under the appropriate boundary conditions using Eqs. (2.9) and (2.12) and the HF densities, \( \rho_0(\mathbf{r}) \) and \( \alpha_0(\mathbf{r}) \).

§ 3. Square density model

Since the exact solutions of the equation of motion (2.11) are given only numerically, analytic solutions based on a simplified geometry are very useful to investigate characteristic features of the eigenmodes. In this section, we assume the square density distribution:

\[
\rho_0(\mathbf{r})=\frac{2k_F^3}{3\pi^2}\theta(R-r)=\rho_0\theta(R-r),
\]

\[
\alpha_a(\mathbf{r})=\alpha_a(\mathbf{r})-\frac{1}{4}\mathbf{P}_{a}\cdot\rho_0(\mathbf{r})=\frac{\delta_{ab}}{3}\frac{3k_F^2}{5}\rho_0\theta(R-r)=\frac{\delta_{ab}}{3}\alpha_b(\mathbf{r})
\]

and employ a semiclassical estimate for the restoring force parameter \( C[s] \) of Eq. (2.8). This estimate is given by

\[
C_{sc}[s]=\frac{1}{2}\int\rho_0^W(\mathbf{r},\mathbf{p})\left\{ D^W(\mathbf{r},\mathbf{p}),\frac{1}{i}\delta U_{sc}^W(D)+\{H_0^W(\mathbf{r},\mathbf{p}),D^W(\mathbf{r},\mathbf{p})\}\right\}d^3rd^3p
\]

with \( D^W(\mathbf{r},\mathbf{p})=i\mathbf{P}(\mathbf{r})\cdot\mathbf{p} \), where the superscript \( W \) indicates the Wigner transform, \( \{ , \} \) the Poisson bracket and \( H_0 \) the unperturbed HF Hamiltonian; \( \delta U_{sc}^W(D) \) stands for the semiclassical ‘transition potential’ associated with the generalized scaling transformation

\[
\delta U_{sc}^W(D)=2b_{\tau}\mathbf{P}\cdot(\rho_0(\mathbf{r})\mathbf{s}(\mathbf{r}))p^2+a_{\tau}(\rho_0(\mathbf{r}))\mathbf{P}\cdot(\rho_0(\mathbf{r})\mathbf{s}(\mathbf{r}))
\]

\[
+\left(b_\tau-\frac{1}{2}c_{\tau}\right)p^2\mathbf{P}\cdot(\rho_0(\mathbf{r})\mathbf{s}(\mathbf{r}))
\]

\[
+2b_{\tau}[2(\mathbf{P}\cdot\alpha_0(\mathbf{r}))\alpha_0(\mathbf{r})+\mathbf{P}\cdot(\alpha_0(\mathbf{r})\mathbf{s}(\mathbf{r}))].
\]

The resulting semiclassical restoring force parameter reads

\[
C_{sc}[s]=\int\frac{\alpha_0(\mathbf{r})}{2m^*(\mathbf{r})}[(\mathbf{P}\cdot\alpha_0(\mathbf{r}))(\mathbf{P}\cdot\alpha_0(\mathbf{r}))+2(\mathbf{P}\cdot\alpha_0(\mathbf{r}))(\mathbf{P}\cdot\alpha_0(\mathbf{r})-\mathbf{s}(\mathbf{r})(\mathbf{P}\cdot\alpha_0(\mathbf{r})))]
\]
For consistency with the assumption of the square density distribution, we neglect the last two terms in \(C_{sc}[s]\). Equation (3·1) applies also to the isoscalar case if we replace \(a_\tau(\rho_0), b_\tau\), and \(c_\tau\) by \(a_0(\rho_0), b_0\), and \(c_0\), respectively. In this case the most diverging terms involving \((\partial\rho_0(r)/\partial r)^2\) or \((\partial \bar{\sigma}_{aa}(r)/\partial r)(\partial\rho_0(r)/\partial r)\) cancel out and the remaining surface terms which contain \((\partial\rho_0(r)/\partial r)\) or \((\partial \bar{\sigma}_{aa}(r)/\partial r)\) can be eliminated by using the semiclassical equilibrium condition:

\[
\frac{\delta}{\delta s(r)}\left[\int \rho_0^w(r, p) i[H_0^w(r, p), D^w(r, p)]d^3r \frac{d^3p}{(2\pi)^3}\right] = 0.
\]

On the contrary, for isovector modes we have by no means such cancellations among the most diverging terms and we are thus led to require

\[
\bar{\sigma} \cdot s(r)|_{r=R} = 0
\]

in order to avoid infinite \(C_{sc}[s]\). Equation (3·2) is nothing but the boundary condition of the Steinwedel-Jensen model.¹⁰

Let us now introduce the nuclear Lamé constants \(\lambda\) and \(\mu\) by

\[
\begin{align*}
\lambda &= \tau_0/3m^* + a_\tau(\rho_0)\rho_0^2 + 4b_\tau(5/3)\tau_0\rho_0 = (\tau_0/3m)(m/m^*)(1 + 5F_0'/3), \\
\mu &= \tau_0/3m^* = (\tau_0/3m)(m/m^*),
\end{align*}
\]

where \(F_0'\) is the Landau parameter in \(r\)-channel. Since we are interested in the equation of motion in the interior and the boundary conditions at the surface for the inside solutions we can put \(s(r) = (1 + x(r))\bar{\sigma} F(r) = (1 + x_\tau)\bar{\sigma} F(r)\) in Eq. (3·1).

The restoring force coefficient \(C[s]\) in the square density model can then be reduced to the form:

\[
C[s] = \frac{1}{2} (1 + x_\tau)^2 \int_\theta (R - r)[(\vec{\sigma})^2 F(r)]^2 + 2\mu(\vec{\sigma} \cdot \vec{V}_F F(r))(\vec{\sigma} \cdot \vec{V}_F F(r))d^3r
\]

under condition (3·2). The inertia parameter \(B[s]\) of Eq. (2·9) becomes
Here the enhancement factor \( x_r \) is related to the Landau parameter \( F_1' \) and the effective mass \( m^*(m^*/m=1+F_1'/3) \) by

\[
1+x_r=(m/m^*)(1+F'_1/3)=(1+F'_1/3)/(1+F_1/3).
\]

By using Eqs. (3.4) and (3.5) and taking account of condition (3.2), we evaluate the variation \( \delta \{ C[s]-\omega^2 B[s]-\int \epsilon (\vec{r}) (\partial F(r)/\partial r) \delta (R-r) d^3 r \} \) with \( \epsilon \) being a Lagrange multiplier. We find

\[
\begin{align*}
\delta F \{ C[s]-\omega^2 B[s]-\int \epsilon (\vec{r}) (\partial F(r)/\partial r) \delta (R-r) d^3 r \} \\
= m \rho_0 (1+x_r)^2 \int \delta (R-r) \left[ \left( \frac{\lambda+2\mu}{m \rho_0} + \frac{\omega^2}{1+x_r} \right) \vec{F}(r) \right] \delta \vec{F}(r) d^3 r \\
- m \rho_0 (1+x_r)^2 \int \delta (R-r) G_1(\vec{r}) \delta \vec{F}(r) d^3 r \\
+ \int \delta (R-r) ((1+x_r)^2 \left[ \lambda \vec{F}^2 + 2\mu (\partial^2/\partial r^2) \right] F(r) - \epsilon (\vec{r}) ) \delta (\partial F(r)/\partial r) d^3 r \\
= 0,
\end{align*}
\]

(3.6)

where \( G_1(\vec{r}) \) is a linear function of derivatives of \( F(r) \) (see Eq. (3.11b)). From Eq. (3.6) we immediately obtain the equation of motion in the nuclear interior \((r<R)\):

\[
\left( \frac{\lambda+2\mu}{m \rho_0} + \frac{\omega^2}{1+x_r} \right) \vec{F}(r) = 0.
\]

(3.7)

For a given multipolarity \( L \), this equation has regular solutions of the form:

\[
F(r) = [A_L(r/R)^L + j_L(qr)] Y_{L0}(\vec{r})
\]

(3.8)

with the dispersion relation

\[
\omega^2 = \frac{\lambda+2\mu}{m \rho_0} (1+x_r)q^2 = \left( \frac{k_F}{m^*} \right)^2 \left( \frac{3}{5} + \frac{1}{3} F_1' \right) \left( 1 + \frac{1}{3} F_1'/3 \right) q^2.
\]

(3.9)

The components \( v^{(\pm)}(r) \) of the velocity field \( \vec{v}(r)=\sum_{\pm} v^{(\pm)}(r) Y_{L0}(\vec{r}) \) are then proportional to

\[
\begin{align*}
v^{-}(r) \propto \sqrt{\frac{L}{2L+1}} [(2L+1)A_L(r/R)^{L-1} + z j_{L-1}(qr)], \\
v^{+}(r) \propto \sqrt{\frac{L+1}{2L+1}} z j_{L+1}(qr),
\end{align*}
\]

(3.10)

where \( z = qR \). Two independent parameters \( A_L \) and \( q \) in \( F(r) \) are determined by boundary conditions at the surface \((r=R)\). In the present case \( \partial F(r)/\partial r \) is fixed by condition (3.2) but \( F(r) \) is free, so that from Eq. (3.6) we have to require

\[
\partial F(r)/\partial r |_{r=R} = 0,
\]

(3.11a)
By substituting Eqs. (3·8) and (3·9) into these equations, we obtain

\[ G_1(\mathbf{r})_{r=R} = \left[ \left( \frac{\lambda + 4\mu}{m\rho_0} \frac{\partial}{\partial r} \mathbf{r}^2 - \frac{2\mu}{m\rho_0} \left( \frac{\partial^2}{\partial r^2} - \frac{1}{r} \frac{\partial}{\partial r} \mathbf{r}^2 + \frac{3}{r^2} \frac{\partial^2}{\partial r^2} \right) + \frac{\omega^2}{1 + \chi} \frac{\partial}{\partial r} \right) F(\mathbf{r}) \right]_{r=R} = 0. \]  

(3·11b)

By substituting Eqs. (3·8) and (3·9) into these equations, we obtain

\[ L^2 + \frac{L^2}{(L-1)(L+1)} A_{L_2} - \frac{L+1}{(L-1)} A_{L_1} + \frac{L+1}{(L-1)} A_{L_1} = 0, \quad (L^2 + \frac{L^2}{(L-1)(L+1)} A_{L_2}) (L+1) j_{L}(z) + (L+1) z j_{L-1}(z) = 0 \]

with \(a^2 = (\lambda + 2\mu)/2\mu = (5/2)(3/5 + F_0'/3)\). The dispersion relation is reexpressed as

\[ \omega^2 = \left( \frac{m}{m^*} \right)^2 \left( \frac{3}{5} + \frac{1}{3} F_0' \right) \left( 1 + \frac{1}{3} F_i' \right) z^2 \mathcal{Q}^2, \]

(3·13)

where a sort of harmonic oscillator frequency \(\mathcal{Q}\) has been defined by

\[ \mathcal{Q}^2 = 2\langle \frac{T}{m} \rangle = 5\tau_0 / 3 m^2 \rho_0 R^2 \]

with \(\langle T \rangle = \langle \Psi_0 | \sum_i \rho_i \frac{1}{2m} | \Psi_0 \rangle / |\Psi_0\rangle \) and \(\langle r^2 \rangle = \langle \Psi_0 | \sum_i r_i^2 | \Psi_0 \rangle / |\Psi_0\rangle\). Obviously the eigenfrequency \(\omega\) of Eq. (3·13) has \(A^{-1/3}\)-dependence for a given effective interaction.

We close this section with brief comparison of our results (3·8), (3·9) and (3·11) (3·12)) with the standard hydrodynamical model results:

\[ \xi(\mathbf{r}) \propto j_{L}(q r) Y_{L0}(\hat{r}), \quad \omega^2 = \left( \frac{k_r}{m^*} \right)^2 \left( \frac{1}{3} + \frac{1}{3} F_0' \right) \left( 1 + \frac{1}{3} F_i' \right) \mathcal{Q}^2, \quad \frac{d j_{L}(z)}{d z} = 0. \]

(3·14a) (3·14b) (3·14c)

We first note that the present description based on the generalized scaling approximation admits the incompressible term \(r^2 Y_L\) in the velocity or displacement potential. Obviously, the divergence free term in the displacement potential causes no density change in the interior but the quadrupole distortion of the local Fermi surface for \(L>1\). This Fermi surface distortion is not taken into account in the hydrodynamical description, and its effect is also reflected in the difference of the sound speeds (compare Eq. (3·9) with Eq. (3·12b)). Concerning the wave number \(q\) which characterizes the eigenfrequencies, we can show that \(z = q R\) determined from Eqs. (3·12a) and (3·12b) for \(L \neq 0\) is smaller than the corresponding hydrodynamical value \(z_{\text{HD}}\). For the monopole vibration \(L = 0\) our boundary condition (3·12a) agrees with the hydrodynamical one (3·14c), so that \(z = z_{\text{HD}}\). Finally we notice that in the limit \(F_0' \to \infty (a^2 \to \infty)\) Eq. (3·12) yields

\[ A_L = 0, \quad \frac{d j_{L}(z)}{d z} = 0. \]

recovering the hydrodynamical solutions. This is in marked contrast with the case of the isoscalar excitations, where the lowest mode with \(L \geq 1\) finds the Tassie solution in the incompressible limit \(F_0 \to \infty\). 5)

\section*{§ 4. Results and discussion}

In this section we employ pure zero-range interactions \((t_1 = t_2 = 0)\) for the sake of simplicity. The power \(a\) of the density dependence is chosen to be 1/6; \(t_0\) and \(t_2\) are
adjusted to give nuclear matter properties as \( k_F = 1.26 \text{ fm}^{-1} \) (\( \rho_{\text{NM}} = 0.135 \text{ fm}^{-1} \)) and \( E/A = -13.8 \text{ MeV} \). Concerning the parameters \( x_0 \) and \( x_3 \) which prescribe the isovector properties (volume and surface symmetry energies, \( \varepsilon_{s^v} \) and \( \varepsilon_{s^s} \)) we use two different sets:

(i) \((x_0, x_3) = (0.503, 0.6)\) leading to \( \varepsilon_{s^v} = 25.5 \text{ MeV} \) \((F_0' = 1.32)\) and \( \varepsilon_{s^s} = -20 \text{ MeV} \) according to the semiclassical description of Ref. 12),

(ii) \((x_0, x_3) = (-0.289, -0.5)\) leading to \( \varepsilon_{s^v} = 32.9 \text{ MeV} \) \((F_0' = 2.0)\) and \( \varepsilon_{s^s} = -51 \text{ MeV} \).

We first discuss results from the square density model described in §3. Table I shows properties of the lowest two \( L=1 \) and \( 2 \) states determined from Eqs. (3-9) and (3-12) for several values of \( F_0' \) in comparison with the hydrodynamical (HD) results; \( c/v_F \) stands for the reduced sound speed \( c/v_F = \sqrt{3/5 + F_0'}/3 \) for zero range forces, so that \( \omega/Q = (c/v_F)z \), and similarly for the HD case \( c_{\text{HD}}/v_F = \sqrt{1/3 + F_0'}/3 \). The rows of \( EWS/m_1 \) denote the ratio of the energy weighted strength (EWS) to the linear energy-weighted sum rule value \( (m_1) \) for the multipole operator \( r^L Y_L \tau_z \). It is seen in Table I that the value of \( z = qR \) for finite \( F_0' \), which is determined from the boundary conditions (3-12), is smaller than the corresponding one of the hydrodynamical model as discussed in §3. The increase of the reduced sound speed \( c/v_F \) from the hydrodynamical one \( c_{\text{HD}}/v_F \) is thus partially cancelled by the reduction of the wave number in the excitation energy, yielding \( \omega^{(1)}/\omega^{(1)}_{\text{HD}} = 1.07 \) \((1.11)\) for the first \( L=1(2) \) state in the case \( F_0' = 1.32 \), as compared with \( c/c_{\text{HD}} = 1.16 \). For the second mode the reduction of the wave number is negligibly small and therefore the increase of the sound speed is directly reflected in the increase of the excitation energy from the hydrodynamical one. In this connection we recall that for the monopole vibration we always obtain the relation \( \omega/\omega_{\text{HD}} = c/c_{\text{HD}} = \sqrt{9/5 + F_0'}/(1 + F_0') \). Finally we note that in Table I the ratio \( EWS/m_1 \) in our model is almost independent of \( F_0' \) and is

<table>
<thead>
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<th>( F_0' )</th>
<th>0.0</th>
<th>0.5</th>
<th>1.32</th>
<th>2.0</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>( c/v_F )</td>
<td>0.77</td>
<td>0.88</td>
<td>1.02</td>
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<tr>
<td>( c_{\text{HD}}/v_F )</td>
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<td>0.71</td>
<td>0.88</td>
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<td>( L=1 )</td>
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<td>( z^{(1)} )</td>
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<td>1.92</td>
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<tr>
<td>( \omega^{(1)}/Q )</td>
<td>1.38</td>
<td>1.62</td>
<td>1.96</td>
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<tr>
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<td>( \omega^{(1)}_{\text{HD}}/Q )</td>
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<tr>
<td>( z^{(2)} )</td>
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<td>5.94</td>
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<td>5.94</td>
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<tr>
<td>( \omega^{(2)}/Q )</td>
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<tr>
<td>( EWS/m_1 )</td>
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<tr>
<td>( \omega^{(2)}_{\text{HD}}/Q )</td>
<td>3.43</td>
<td>4.20</td>
<td>5.23</td>
<td>5.94</td>
<td>5.94</td>
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<tr>
<td>( L=2 )</td>
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<tr>
<td>( z^{(1)} )</td>
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<td>3.15</td>
<td>3.21</td>
<td>3.23</td>
<td>3.34</td>
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<tr>
<td>( \omega^{(1)}/Q )</td>
<td>2.40</td>
<td>2.76</td>
<td>3.27</td>
<td>3.64</td>
<td>3.64</td>
</tr>
<tr>
<td>( EWS/m_1 )</td>
<td>78.2</td>
<td>78.0</td>
<td>77.8</td>
<td>77.8</td>
<td>77.4</td>
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<tr>
<td>( \omega^{(1)}_{\text{HD}}/Q )</td>
<td>1.93</td>
<td>2.36</td>
<td>2.94</td>
<td>3.34</td>
<td>3.34</td>
</tr>
<tr>
<td>( z^{(2)} )</td>
<td>7.28</td>
<td>7.28</td>
<td>7.28</td>
<td>7.28</td>
<td>7.28</td>
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<tr>
<td>( \omega^{(2)}/Q )</td>
<td>5.64</td>
<td>6.38</td>
<td>7.43</td>
<td>8.20</td>
<td>8.20</td>
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<tr>
<td>( EWS/m_1 )</td>
<td>7.9</td>
<td>8.0</td>
<td>8.2</td>
<td>8.2</td>
<td>8.5</td>
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<tr>
<td>( \omega^{(2)}_{\text{HD}}/Q )</td>
<td>4.21</td>
<td>5.15</td>
<td>6.41</td>
<td>7.29</td>
<td>7.29</td>
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</table>
close to the hydrodynamical value: EWS/m1 = 85.7(77.4)% for the first L=1(2) state and 6.0(8.5)% for the second L=1(2) state.

Let us now compare the results from the present macroscopic description with those from microscopic HF-RPA. We consider two model systems: A=184 and 4096. Although the latter is very fictitious, it is expected to exhibit the bulk feature of giant vibrations in a large A limit. As in Ref. 5) we employ the response function method\(^{13}\) for the RPA calculation. In most cases the RPA transition strength for isovector excitations is fragmented to several states. Since we are not interested in detailed features of individual nuclei, we smooth out fine structure by taking a proper average over these states and regard the position of the resulting peak as the RPA energy \(\omega_{\text{RPA}}\). The RPA transition density and velocity field are also averaged around the peak. In Table II, we show the energy \(\omega_N\) of the first excited state resulting from numerical solutions\(^*\) of Eq. (2·11) in comparison with the RPA energy for \(L=1\) and 2 states in each model system. It is seen that the eigen-frequency \(\omega_N\) in our dynamical description using HF densities gives a satisfactory agreement with the microscopic RPA result \(\omega_{\text{RPA}}\). By comparing \(\omega_N/\Omega\) with the corresponding square density result \(\omega_{\text{sq}}/\Omega\):

(i) \(\omega_{\text{sq}}/\Omega = 1.96 (L=1)\) and 3.27 \((L=2)\),

(ii) \(\omega_{\text{sq}}/\Omega = 2.19 (L=1)\) and 3.64 \((L=2)\),

we notice that the effect of the smooth surface leads to significant reduction in the eigenfrequency in the smaller system \(A=184\). As expected, this effect is particularly large for force (ii) leading to a more negative surface symmetry energy and thus to a less stiff surface for isovector deformations compared with case (i). In Table II we have also included the results \(\omega_{\text{sc}}\) from the simple scaling ansatz \(F(\mathbf{r}) = r^L Y_{\ell \alpha}(\hat{\mathbf{r}}) L \neq 0\), which corresponds to the familiar Goldhaber-Teller model\(^{14}\) in the case of the isovector giant dipole resonance. For pure zero-range interactions \(\omega_{\text{sc}}\) is given by

\[
\omega_{\text{sc}}^2 = \begin{cases} 
\frac{1}{3mA} \int d^3r [a_1(\rho_0) - a_0(\rho_0)](d\rho_0(r)/dr)^2, & (L=1) \\
\frac{1}{mA<r>^2} \left\{ \frac{2}{m} \int d^3r \bar{\tau}_{\text{eq}}(r) + \frac{2}{5} \int d^3r [a_1(\rho_0) - a_0(\rho_0)] r^2(d\rho_0(r)/dr)^2 \right\}, & (L=2)
\end{cases}
\]

(4·1)

**Table II.** Excitation energies \(\omega\) of isovector giant dipole and quadrupole states in the model systems \(A=184\) and 4096 for two different forces (i) and (ii); \(\omega_{\text{RPA}}, \omega_N\) and \(\omega_{\text{sc}}\) are defined in the text and given in MeV.

<table>
<thead>
<tr>
<th>A</th>
<th>L</th>
<th>(\omega_{\text{RPA}})</th>
<th>(\omega_N/\Omega)</th>
<th>(\omega_{\text{sc}})</th>
</tr>
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<tr>
<td>184</td>
<td>1</td>
<td>(i) 12.1</td>
<td>13.2(1.80)</td>
<td>16.8</td>
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<tr>
<td></td>
<td></td>
<td>(ii) 12.7</td>
<td>13.6(1.85)</td>
<td>16.4</td>
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<tr>
<td></td>
<td>2</td>
<td>(i) 21.1</td>
<td>21.8(2.97)</td>
<td>25.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(ii) 21.6</td>
<td>21.9(2.97)</td>
<td>24.8</td>
</tr>
<tr>
<td>4096</td>
<td>1</td>
<td>(i) 4.6</td>
<td>5.1(1.90)</td>
<td>9.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(ii) 5.2</td>
<td>5.6(2.06)</td>
<td>9.3</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>(i) 8.3</td>
<td>8.6(3.18)</td>
<td>14.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(ii) 8.9</td>
<td>9.2(3.41)</td>
<td>13.6</td>
</tr>
</tbody>
</table>

*\(^*\) Throughout the present calculation we have used the boundary condition appropriate for bound states.*
which approximately yields
\[(\omega_{sc}^L)^2 \approx (\alpha L A^{-1/3})^2 + (\beta L A^{-1/6})^2,\]
where
\[\alpha L^2 = \frac{4\langle T \rangle}{3mr_0^2}(L-1)(2L+1),\]
\[\beta L^2 = L \frac{4\pi r_0^2}{3m} \int dr [a_\rho(\rho_0) - a_0(\rho_0)] (d\rho_0(r)/dr)^2\]
with \(R = r_0 A^{1/3}\). Equation (4·1) shows that in this scaling ansatz a significant contribution to the restoring force comes from the surface term containing \((d\rho_0(r)/dr)^2\); in particular for \(L=1\) only the surface contribution is present, as often noted previously.\(^{15}\) Naturally the deviation of \(\omega_{sc}\) from \(\omega_N\) grows larger with the increase of the mass number.

Fig. 1. Transition densities of the isovector giant dipole (a) and quadrupole (b) states in the \(A=184\) model system: \(\delta\rho_{\text{RPA}}\), the full RPA result; \(\delta\rho_n\), the result from the numerical solution of Eq. (2·11); \(\delta\rho_T\), the Tassie result. The vertical scale has no significance.
A.

We now proceed to the comparison of the transition densities and velocity fields. Figure 1 displays the transition densities of the isovector \( L = 1 \) and 2 states in \( A = 184 \) for force (ii). The behavior of the RPA transition densities \( \delta \rho_{\text{RPA}} \) is fairly well reproduced by the numerical results \( \delta \rho_{\text{N}} \) of our dynamical description based on Eq. (2.11). It is also noted that both \( \delta \rho_{\text{RPA}} \) and \( \delta \rho_{\text{N}} \) show rather strong volume characters. This is a contrast to the surface-peaked Tassie transition density \( \delta \rho_{\text{T}}(r) \propto r^{L-1}d\rho_{0}(r)/dr \) obtained from the ansatz \( F(r) = r^{L}Y_{L} \), although the difference between \( \delta \rho_{\text{T}} \) and \( \delta \rho_{\text{RPA}}(\delta \rho_{\text{N}}) \) may be exaggerated in the present calculation because pure zero-range forces yield HF density with surface thickness 30% smaller than the standard value. Turning to the comparison of the velocity fields \( (L = 1 \) and 2; \( A = 184 \) ) displayed in Fig. 2, we again find that our dynamical description provides a fair approximation to the microscopic RPA results.

Fig. 2. Velocity fields \( v = \sum_{\lambda} \nu^{\lambda}(r) Y_{LL+\lambda}(\hat{r}) \) of the isovector dipole (a) and quadrupole (b) states in the \( A = 184 \) model system: \( V_{\text{RPA}} \), the full RPA result; \( v_{\nu}^{(\pm)} \), the result from the numerical solution of Eq. (2.11). Crosses (x) indicate the prediction Eq. (3.10) of the square density model. Again, the vertical scale has no significance.
is seen in Fig. 2 that the velocity fields $v_{n}^{(\pm)}$ obtained from the numerical solution of Eq. (2'11) deviate significantly from the square density model results ($\times$). This indicates importance of the effect of the smooth nuclear surface.

In conclusion, we have applied the irrotational fluidynamical method to the description of isovector giant multipole states. For the square density distribution analytic solutions of the equation of motion have been derived and compared with those of the hydrodynamical model. The dynamical quadrupole distortion in momentum space is taken into account in our description and leads to the sound speed higher than the hydrodynamical one. The wave number determined by the boundary conditions is somewhat smaller than the hydrodynamical value, so that the difference in the eigenfrequency is not so large as expected from the increase in the sound speed. Numerical solutions of the equation of motion using HF densities have been shown to reproduce the basic features of microscopic RPA results for the considered model systems. The smooth nuclear surface has significant effects on the isovector giant vibrations and these effects have been analysed within the framework of the droplet model.\textsuperscript{15}~\textsuperscript{17} It is interesting to investigate the role of surface effects in a transparent way on the basis of the present model.

One of the authors (S.N.) thanks Soryushi Shyogakukai for financial support. Numerical computations were done on FACOM M382 computer at the Data Processing center of Kyoto University. This work was financially supported in part by Research Center for Nuclear Physics, Osaka University.

References