A Confining Potential Model for a Quark Core and Its Surrounding Pion Clouds

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We discuss a confining potential model for a baryon, preserving spontaneously broken chiral symmetry. Every quark in the baryon is described independently of each other by the Dirac equation with a linear confining scalar potential, and pions are introduced as Nambu-Goldstone bosons so as to realize spontaneously broken chiral symmetry. Within the one loop approximation for the pion interaction and the first order perturbation in the color coupling constant $a_s$, we obtain good fits for static quantities of a nucleon such as the $G_A/G_V$ ratio, $g_A$, charge radii, anomalous magnetic moments and a mass, and $a_s$ turns out to be near one, which is compared with the value $a_s=2.2$ of the MIT bag model.

§ 1. Introduction and summary

It is a very hard task to calculate hadron properties from the original QCD Lagrangian. The Monte Carlo simulation of lattice QCD appears to be confronted with the problem of large size effects of the lattice, though numerical values of hadron masses obtained so far are not unreasonable. It seems not so easy also to see the conventional structure of hadrons only from such numerical experiments. There have been many attempts, therefore, to construct effective Lagrangians or model Hamiltonians, which incorporate the confinement and other important ingredients of QCD: The MIT bag model is a typical example. In the bag model quarks are confined in a (spherically) finite cavity, which is pressured by the outside super-vacuum. Inside the cavity quarks obey the free Dirac equation independently of each other. This motion could be regarded as the one by the Dirac equation with a scalar infinite square-well potential, whose center is nailed down to a fixed space point. The square-well potential do not seem to be natural, however, though it reflects well the confinement phenomena and the sharp surface may be only a mathematical fiction. For example, the charge distribution has an unnatural discontinuity at the surface. Introduction of a finite thickness of the surface to smear the sharp surface would induce, however, additional parameters besides the universal bag constant and the complicated situation at the boundary conditions.

Recently, Carlson, Kogut and Pandharipande have discussed the mass spectra of light hadrons within a semi-relativistic quark model by use of a linear potential. The linear potential is introduced as an energy of the chromoelectric flux tube with the string tension $\sigma=1.0$ GeV/fm, which is also predicted from the Monte Carlo simulation of lattice QCD. They observed as one of their conclusions that the root mean square radii of light baryons and mesons are much smaller than the observed charge radii; their results are $\langle r^2 \rangle^{1/2} \sim 0.3$ fm. They suggested that their model might be concerned with a quark core inside a hadron. If this is the case, the following questions would arise: What cloud-like objects surround the quark core? What types of interactions are there between the core and clouds? Why do such interactions not disturb the original fit of the calculated mass spectra to the hadronic ones?
The realization of spontaneously broken chiral symmetry (in the $u$ and $d$ sector at least) is thought to be one of the characteristic features of the strong interaction derived from QCD as well as the asymptotic freedom and the confinement. Since MIT bag model breaks chiral symmetry,$^6$ two types of the chiral bag model, the little$^7$ and cloudy$^8$ bag model, have been proposed, where an elementary pion field is introduced as Nambu-Goldstone boson in order to recover explicit breaking of chiral symmetry. Extensively discussed are the mass values and electromagnetic properties of ground state baryons, though the discontinuity at the surface is not removed at all.$^9$

We propose in this paper a potential model for a baryon, which preserves spontaneously broken chiral symmetry: Each quark is the source of the chromoelectric flux tube, and the three tubes merge at a point, the junction, to neutralize the color. We assume that the role of each flux tube can be substituted by that of a linearly rising potential $M(r)$ acting on the source quark,$^{10,11}$ where $r$ is a distance from the junction to the quark, and that the quark in the core is described independently of each other by the Dirac equation with the scalar potential $M(r)$. The linearly rising term should be fixed by the universal string tension, which is taken to be 0.8 GeV/fm, corresponding to $\alpha' = 1.0 \text{ GeV}^{-2}$ of the Regge slope. Although our model is inspired by the flux tube or string model, it may be also interpreted as the Hartree field.$^{10,11}$

Fig. 1. The potential model for baryons inspired by the flux tube model. (a) The flux tube model. (b) The independent particle model with the potential $M(r_J)$, where $r_J = |r_i - r_j|$ and $r_j$ is the position of the junction and is taken as the origin of the potential.

We notice here that the existence of the scalar potential breaks the conservation of the axial vector current of the quarks. The scalar potential may be regarded as an effective variable mass of the quark. We then assume that explicit breaking of chiral symmetry is recovered by introducing pions as Nambu-Goldstone bosons. The quark-pion interaction is thus fixed.$^{7,8,11,12}$ Consequently the model does not need any additional free parameters besides the ones used for the potential.

It is further assumed as in the bag model$^{7,8}$ and potential model$^{9,11}$ that the ground state baryons are described well only by the lowest $1s_{1/2}$ quark state: We ignore higher excited quark states as well as sea $q\bar{q}$ and gluon excitations. This may be problematic as discussed in Refs. 14$^\sim 16$).

We calculate the static quantities of a nucleon within the perturbation theory up to the one-loop approximation for the pion interaction and the first order in the color coupling constant $\alpha_s$. We summarize our main results on the ground state non-strange baryons, the nucleon $N$ and the delta $\Delta$.

(i) The bare pion-nucleon coupling constant satisfies the Goldberger-Treiman relation,$^{17}$

\begin{align*}
\text{Fig. 1. The potential model for baryons inspired by the flux tube model. (a) The flux tube model. (b) The independent particle model with the potential } M(r_J), \text{ where } r_J = |r_i - r_j| \text{ and } r_j \text{ is the position of the junction and is taken as the origin of the potential.}
\end{align*}
A Confining Potential Model for a Quark Core

\[ f_{\pi}^{(0)} = \frac{m_{\pi}}{2f_{\pi}} g_{A}^{(0)}, \]  

where \( f_{\pi} \) is the pion decay constant, which is taken to be 92 MeV, and \( g_{A} \) is the \( G_{A}/G_{V} \) ratio. We can calculate the bare \( g_{A}^{(0)} \) and get

\[ g_{A}^{(0)} = 1.27 \sim 1.37, \]  

which is much improved from the MIT value, 1.09. Using the ratio of the renormalization constants for \( N, Z_{1} \sim 0.9 \) and (1·2) we have

\[ g_{A} = 1.16 \sim 1.22 \]  

for the renormalized \( G_{A}/G_{V} \) ratio and

\[ f_{\pi}^{2}_{\pi N}/4\pi = 0.062 \sim 0.069 \]  

for the renormalized \( \pi N \) coupling constant. These values are very encouraging, since they are just a little small compared with the experimental values, \( g_{A} = 1.25 \) and \( f_{\pi}^{2}_{\pi N}/4\pi = 0.08 \).

(ii) The quark distribution has no longer any discontinuity and the root mean square (r.m.s.) radius of the core is about 0.6 fm, while the pion distribution extends from the inside of the core to the outside and the r.m.s. radius is nearly 1.1 fm. The charge carried by the pion cloud is less than 20 %, and the resultant proton charge radius turns out to be about 0.83 fm and the neutron radius about \(-0.47 \) fm, where the minus sign means a negative \( \langle r^{2} \rangle \). These values are satisfactory, though the observed neutron radius is about \(-0.34 \) to \(-0.37 \) fm.

(iii) As for the magnetic moment, our calculation is concerned with the anomalous part, because the static approximation should be taken for the form factors. The isovector part is given as

\[ \mu^{V} = 1.75 \sim 1.85 \text{ n.m.} \]  

in a unit of nuclear magneton, which agrees well with the experimental value, 1.85 n.m. The pion current contribution is \( 0.5 \sim 0.9 \) n.m. On the other hand, the isoscalar part is

\[ \mu^{S} = 0.17 \sim 0.19 \text{ n.m.}, \]  

which has the wrong sign.

(iv) We can deduce the strong coupling constant \( \alpha_{s} \) of the quark-gluon interaction from the mass difference between \( N \) and \( \Delta \). Our model gives \( \alpha_{s} \sim 1 \), that may be more favorable than the value \( \alpha_{s} = 2.2 \) of the original MIT bag model.

This paper is organized as follows: In the next section the model Hamiltonians are introduced. The bare masses, coupling constants and the renormalization effects on them are calculated in §3. We discuss the electromagnetic properties of the nucleon in §4 and some discussions are given in §5.

§ 2. The model Hamiltonian

2.1. Chiral invariant effective Lagrangian

Since the quark has a local mass, the axial current composed of the quark is not
conserved; the axial quark current

\[ A_\mu^a(x) = \bar{\phi}(x) (\tau_\mu / 2) \gamma^a \gamma_5 \phi(x) \]  

(2.1)

has a non-vanishing four divergence

\[ \partial_\mu A_\mu^a(x) = i \bar{\phi}(x) \tau_\mu \gamma_5 M(r) \phi(x), \]  

(2.2)

where \( M(r) \) is the linear confining potential and \( r \) is the radial variable. In order to recover chiral symmetry, we introduce the elementary pion as Nambu-Goldstone field and then we have

\[ L_{\text{eff}}(x) = \bar{\phi}(i \gamma_5 - M(r)) \phi - (M(r)/f_\pi) \bar{\phi} \tau_\mu \gamma_5 \phi_\lambda + 1/2 (\partial_\mu \phi_\lambda)(\partial^\mu \phi_\lambda) \]  

(2.3)

as the linearized effective Lagrangian, which is constructed by the standard ways.\(^7,8,11)-13\) The axial current is, then, written as

\[ A_\mu^a = \bar{\phi}(\tau_\mu / 2) \gamma^a \gamma_5 \phi + f_\pi \partial^\mu \phi_\lambda \]  

(2.4)

within the same linearization approximation.

2.2. The model Hamiltonian

Due to the assumption stated in §1 the model space for the quark is restricted only to the ground \( 1s_{1/2} \) state with the energy \( E_0 \). Consequently, the quark in the Hamiltonian looks like a static particle such as a nucleon in the Chew-Low theory,\(^19\) in spite of the massless quark.

The model Hamiltonians are then written as

\[ H_\pi = E_0 b_0^\dagger b_0, \]  

(2.5)

\[ H_\pi = \sum_{\lambda=1}^3 \int d^3 k \omega a_{\lambda k \lambda} a_{\lambda k \lambda}^\dagger, \]  

(2.6)

\[ H_{\pi \pi} = \sum_{\lambda=1}^3 \int \frac{d^3 k}{(2\pi)^{3/2}} \sqrt{2\omega} (V_{k \lambda} a_{\lambda k \lambda} + \text{h.c.}) b_0^\dagger b_0, \]  

(2.7)

where \( b_0(a_{\lambda k \lambda}) \) is the destruction operator for the \( 1s_{1/2} \) quark (pion with momentum \( k \) and isospin index \( \lambda \)), and

\[ V_{k \lambda} = (1/f_\pi) \int d^3 x M(r) \bar{\psi}_0(x) i \tau_\mu \gamma_5 \psi_0(x) \exp(i k \cdot x), \]  

(2.8)

denotes the \( q\pi \pi \) vertex. We use \( \omega = \sqrt{k^2 + m_\pi^2} \) by adding the mass term, \( m_\pi^2 \phi_\lambda \phi_\lambda^* \), to (2.3). In the above \( \psi_0 \) is the solution of the Dirac equation,

\[ \psi_0(x) = \frac{1}{\sqrt{4\pi r}} \left( i f(r) \sigma \cdot \hat{r} g(r) \right) \chi_m, \]  

(2.9)

where \( \hat{r} \) is the unit vector of \( x \), \( \chi_m \) is the two component Pauli spinor with \( m = \pm 1/2 \). The normalization of \( \psi_0 \) is defined as

\[ \int_0^\infty dr \{ f^2(r) + g^2(r) \} = 1. \]  

(2.10)
2.3. The πN and πΔ interactions

The pion-quark coupling (2.8) can be rewritten as

\[ V_{\pi q} = i(f_\pi^{(0)}/m_\pi)v(k)(\sigma \cdot k)\tau_1, \]

where the bare pion-quark coupling constant is given as

\[ f_\pi^{(0)} = -(2m_\pi/3f_\pi)\int_0^\infty drrM(r)f(r)g(r), \]

and the form factor as

\[ v(k) = \int_0^\infty drrM(r)f(r)g(r)(3j_1(kr)/kr)/\int_0^\infty drrM(r)f(r)g(r). \]

Using the spin-flavor wave functions for N and Δ, we can easily write down the πNN, πNΔ and πΔΔ vertices as follows:

\[ V_{\pi NN} = i(f_\pi^{(0)}/m_\pi)v(k)(\gamma_\pi \cdot k)\tau_1, \]

\[ V_{\pi N\Delta} = i(f_\pi^{(0)}/m_\pi)v(k)(S_N \cdot k)T_{N\Delta}^1, \]

\[ V_{\pi \Delta\Delta} = i(f_\pi^{(0)}/m_\pi)v(k)(S_\Delta \cdot k)T_{\Delta\Delta}^1, \]

where the SU(6) ratio among the coupling constants holds as follows:

\[ f_\pi^{(0)}(=5/3f_\pi^{(0)}) : f_\pi^{(0)} = 1 : 12\sqrt{2}/5 : 1/5 \]

under the spin and isospin matrices defined as

\[ (S_N)^{a\beta} = C_{3/2,1}(\beta', a; 1/2, \beta) = (T_{N\Delta}^a)_{\beta'\beta}, \]

\[ (S_\Delta)^{a\beta} = \sqrt{15}C_{3/2,1}(\beta', a; 3/2, \beta) = (T_{\Delta\Delta}^a)_{\beta'\beta}. \]

In this case the core is treated as if it were a simple bare particle and the model Hamiltonians are given by

\[ H_{core} = \sum_{a=N,\Delta} m_a^{(0)}a^\dagger a, \]

\[ H_{int} = \sum_{a, a'} \int \frac{d^3k}{(2\pi)^3} \{ V_{ka,\beta}^{a_s}a^\dagger a a_{\beta a} + h.c. \} \]

with the same \( H_\pi \) of (2.6), where \( m_a^{(0)} \) is the bare core mass.

2.4. Color magnetic and Coulomb interactions

The static color magnetic interaction between \( i \) and \( j \) quarks is described as

\[ H_{Mij} = -\sum_{a=1}^3 \int d^3x d^3y \frac{G^2}{4\pi|x-y|} j^i_a(x) \cdot j^j_a(y), \]

where

\[ j^i_a(x) = \bar{\psi}_0(x)(\lambda_i^a/2)\gamma_\mu \psi_0(x). \]

We also give here the Coulomb (electric) interaction term between \( i \) and \( j \) quarks,
The color magnetic and Coulomb interactions between different two quarks are treated as a perturbation, because they are the residual interactions after incorporating the non-perturbative gluon interactions into the color flux tube or the linearly rising potentials. Furthermore, they are two-body forces which break the independent particle approximation. The self-energy term by the gluon exchange for a single quark is, however, ignored, because it is related to the mass of the quark and is supposed to be absorbed into the linear scalar potential, an effective local mass of the quark.

§ 3. Masses and coupling constants

3.1. Potential

The potentials examined in this paper have phenomenological forms,

$$M(r) = \sigma (r - r_0) \quad \text{for all } r$$  \hspace{1cm} (3.1)

where the universal string tension $\sigma$ is taken to be 0.8 GeV/fm and 1.0 GeV/fm, and $r_0$ is changed as Table I. The latter form (3.2) is interesting, because it could be interpreted as if chiral symmetry were restored in the region $r < r_0$. For small $r_0$ there is no essential difference between (3.1) and (3.2). Five potentials are shown in Fig. 2 and parameters are listed in Table I.

The wave functions $f(r)$ and $g(r)$ behave like Gaussian functions asymptotically owing to the linearity of $M(r)$, and $f(r) \sim O(r)$ and $g(r) \sim O(r^2)$ at the origin for the $1s_{1/2}$ state. We

<table>
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<th>Table I. Parameters for potentials in GeV units.</th>
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solv the Dirac equation numerically from the origin to 2.0 fm by adjusting the eigenvalue $E_0$ so as to satisfy the boundary condition. Obtained values for $E_0$ are listed in Table II.

### 3.2. Bare masses and r.m.s. radii of the core

Unfortunately we are not sure how to evaluate the spurious motion of the center of mass, which is inherent to the independent particle model. Ignoring the CM correction, we have $3E_0$ for the bare core mass, while if we employ a simple method to evaluate the CM correction, which is advocated in Refs. 21) and 22), we have

$$M_{\text{core}}^2 = (\Sigma E_{oi})^2 - \langle (\Sigma p_i)^2 \rangle$$

$$= (3E_0)^2 - 3\langle p^2 \rangle$$

(3·3)

for the bare core mass, where $\langle p^2 \rangle$ is the mean square of the momentum of a single quark. Even $M_{\text{core}}$ is much larger than $(M_N + M_d)/2$, but the excess will be consumed by the self-energy owing to the pion cloud.

The size of the core, which is measured by r.m.s. radius, $\langle r^2 \rangle^{1/2}$, is about 0.6 fm, which is much larger than the size of the little bag and rather near the r.m.s. radius of the cloudy bag model with $R=0.82$ fm.\(^8\)

### Table II. Bare quantities of the core. The pion decay constant $f_\pi$ is taken to be 92 MeV.

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<tbody>
<tr>
<td>$E_0$ (GeV)</td>
<td>0.648</td>
<td>0.590</td>
<td>0.487</td>
<td>0.724</td>
<td>0.585</td>
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<tr>
<td>$3E_0$ (GeV)</td>
<td>1.94</td>
<td>1.77</td>
<td>1.46</td>
<td>2.17</td>
<td>1.75</td>
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<tr>
<td>$M_{\text{core}}$ (GeV)</td>
<td>1.73</td>
<td>1.55</td>
<td>1.23</td>
<td>1.93</td>
<td>1.49</td>
</tr>
<tr>
<td>$\sqrt{\langle p^2 \rangle}$ (GeV)</td>
<td>0.51</td>
<td>0.50</td>
<td>0.46</td>
<td>0.57</td>
<td>0.53</td>
</tr>
<tr>
<td>$\sqrt{\langle r^2 \rangle}$ (fm)</td>
<td>0.63</td>
<td>0.66</td>
<td>0.73</td>
<td>0.56</td>
<td>0.62</td>
</tr>
<tr>
<td>$m_{\pi i}^0$ (GeV)</td>
<td>0.47</td>
<td>0.41</td>
<td>0.31</td>
<td>0.53</td>
<td>0.39</td>
</tr>
<tr>
<td>$m_{\rho i}^0$ (GeV)</td>
<td>0.40</td>
<td>0.32</td>
<td>0.17</td>
<td>0.44</td>
<td>0.24</td>
</tr>
<tr>
<td>$g_{\rho i}^a$</td>
<td>1.37</td>
<td>1.34</td>
<td>1.27</td>
<td>1.37</td>
<td>1.29</td>
</tr>
<tr>
<td>$f_{\pi i}^{\rho i}/4\pi$</td>
<td>0.086</td>
<td>0.082</td>
<td>0.074</td>
<td>0.086</td>
<td>0.077</td>
</tr>
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</table>
If an appropriate CM correction is employed, our r.m.s. radius may be reduced somewhat. Nevertheless, our r.m.s. radii may be larger than those of Ref. 4. The difference would be attributed to the following factors besides the broader size of our potential wells. In the latter the potential is defined by minimizing the energy $\sum_{i=1}^{3}M(|r_i - r_j|)$ with respect to $r_j$, the position of the junction, under keeping all the $r_i$'s fixed and is rewritten by a sum of three two-body potentials. Furthermore, the color Coulomb term is incorporated into the potential term, while in our treatment it is used as a perturbation. All of the above factors play a role reducing the r.m.s. radii.

We are now interested in how to estimate an effective mass of the constituent quark. One is to take an average value of $\langle M(r) \rangle$,

$$m_{\text{eff}}^{(1)} = \langle f^2(r) + g^2(r) \rangle,$$

(3.4)

The other is to define the mass as

$$m_{\text{eff}}^{(2)} = \sqrt{E^2 - \langle p^2 \rangle}.$$  

(3.5)

In the latter case the constituent quark is seen like a free particle with the same momentum as that of the massless quark in the potential. We observe that these values are within the range 0.2~0.5 GeV, which are usually taken as the constituent quark mass. Consequently the constituent quark mass may be interpreted as the effect of the energetic color flux tube accompanying the massless quark.

### 3.3. The $G_A/G_V$ ratio and the $\pi NN$ coupling constant

The axial vector coupling constant or $G_A/G_V$ ratio, $g_A$, is given as

$$g_A^{(0)} = \frac{5}{3} \int_0^{\infty} dr \left\{ f^2(r) - \frac{1}{3} g^2(r) \right\},$$  

(3.6)

for the bare quark current. The pion current in (2.4) does not contribute to $g_A$, because in our model the pion field is allowed to invade the whole space, but vanishes at the origin and infinity. The calculated $g_A^{(0)}$ is 1.27~1.37, which is much improved compared with the MIT bag model prediction, 1.09, and the non-relativistic quark model prediction. 20 It has already been pointed out\(^{11}\) that for $M(r) = \alpha r^n$ with $n=2$ and 3 we can have $g_A^{(0)} = 1.25$ and 1.21, respectively.

The bare $\pi NN$ coupling constant is calculable from the original definition, which is reduced to the Goldberger-Treiman relation\(^{17}\)

$$f_{\pi NN}^{(0)} = \frac{2f_\pi g_A^{(0)}}{M_{\pi}},$$

(3.7)

because both are proportional to the matrix element of $\sum \sigma_i \cdot k_{\ell4}$ between the spin-flavor wave function of the nucleon. Using (3.7) we have

$$f_{\pi NN}^{(0)} / 4\pi = 0.074 \sim 0.087,$$

(3.8)

where $f_{\pi}$, the pion decay constant, is taken as 92 MeV. The values of $g_A^{(0)}$ and $f_{\pi NN}^{(0)} / 4\pi$ are very encouraging, though the renormalization will reduce these values somewhat.
3.4. Renormalization owing to the pion cloud

The renormalization of the $\pi NN$ coupling constant and the nucleon and delta masses can be calculated by using $2\cdot10 \sim 2\cdot20$ in parallel with the Chew-Low theory or Ref. 8.

The renormalized quantities are listed in Table III. The wave function renormalization constant $Z_2$ for $N$ is given as

$$Z_2^{-1}=1+\frac{3}{\pi} \frac{f^{(0)}_{\pi NN}}{4\pi} \frac{1}{m_\pi^2} \int \frac{dk^4 v^2(k)}{\omega} \left\{ \frac{1}{\omega^2+32/25+(\omega+\Delta m)^2} \right\},$$

where $f^{(0)}_{\pi NN}/f^{(0)}_{\pi NN}=12\sqrt{2}/5$ is substituted and $\Delta m$ is the mass splitting between the physical $N$ and $\Delta$, which is taken to be 0.3 GeV throughout this paper. Similarly the vertex renormalization constant $Z_1$ is written as

$$Z_1^{-1}=1+\frac{1}{3\pi} \frac{f^{(0)}_{\pi NN}}{4\pi} \frac{1}{m_\pi^2} \int \frac{dk^4 v^2(k)}{\omega} \left\{ \frac{1}{\omega^2+256/25+(\omega+\Delta m)^2} \right\}.$$  

As stressed in Ref. 8) the renormalization of the coupling constant is not so significant; $Z_1^{-1}Z_2$ is about 0.9 in our case, too. Since the same renormalization holds for $g_A$ as for $f_{\pi NN}$, the renormalized $g_A$ is written as $Z_1^{-1}Z_2g_A^{(0)}$; $g_A$ is in the range 1.16~1.22, just a little smaller than the experimental value 1.25. These values are, however, much improved compared with the chiral bag models.

In order to obtain the mass correction and electromagnetic effects by the pion cloud, we substitute the renormalized coupling constants obtained simply by multiplying the same $Z_1^{-1}Z_2$, though strictly speaking the renormalization constants for $\Delta$ differ slightly from those for $N$. That is to say, we keep the $SU(6)$ ratio (2·17) after the renormalization.

The self-energies by the pion cloud for $N$ and $\Delta$ are written as

$$\delta m_N^{\pi}=-\frac{3}{\pi} \frac{f^{(0)}_{\pi NN}}{4\pi} \frac{1}{m_\pi^2} \int \frac{dk^4 v^2(k)}{\omega} \left\{ \frac{1}{\omega^2+32/25} \right\},$$

$$\delta m_\Delta^{\pi}=-\frac{3}{\pi} \frac{f^{(0)}_{\pi NN}}{4\pi} \frac{1}{m_\pi^2} \int \frac{dk^4 v^2(k)}{\omega} \left\{ \frac{1}{\omega^2+8/25+\mathfrak{D}} \right\},$$

where $\mathfrak{D}$ denotes the principal value of the integration.

The absolute values of the self-energies would depend on the model space for the quarks and the pion structure, but the mass splitting between the two ground states, $N$ and $\Delta$, is expected to be less affected by the model space. Especially, the contribution from the one pion exchange diagrams between different quarks is given irrespectively of the model space, provided that the configuration mixing is ignored. These diagrams contribute to the mass splitting. Our calculation shows, however, that the mass difference owing to the pion self-energies is rather small; only about 10% of $\Delta m$ can be attributed to the pion self-energy diagrams. This is because $\Delta m$ itself, appearing in the denominators in (3·11) and (3·12), makes the contribution to $\Delta m$ small. The rest of the mass difference should be supplied by the color magnetic interactions.

The color magnetic energy between $i$ and $j$ quarks is written as

$$\Delta E_M^{ij}=-\alpha_s(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \sum_{a=1}^4 \langle \lambda_i^a \lambda_j^a \rangle \int_0^\infty dr \frac{\mu'(r)\mu(r)}{r^3},$$

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Table III. Renormalization effects owing to the pion cloud and gluon interactions. $a_s$ is determined so as to satisfy $\Delta m=0.3$ GeV, so that $m_\pi$ is always larger than $m_\pi$ by $\Delta m=0.3$ GeV.

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<tbody>
<tr>
<td>$Z_i^{-1} Z_i$</td>
<td>0.89</td>
<td>0.90</td>
<td>0.93</td>
<td>0.88</td>
<td>0.90</td>
</tr>
<tr>
<td>$a_s$</td>
<td>1.22</td>
<td>1.20</td>
<td>1.18</td>
<td>1.21</td>
<td>1.16</td>
</tr>
<tr>
<td>$f_{\text{bare}}/4\pi$</td>
<td>0.069</td>
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</tr>
<tr>
<td>$\delta m_{\pi}^{\text{prob}}$(GeV)</td>
<td>-0.35</td>
<td>-0.27</td>
<td>-0.181</td>
<td>-0.49</td>
<td>-0.30</td>
</tr>
<tr>
<td>$\delta m_{\pi}^{\text{dom}}$(GeV)</td>
<td>-0.33</td>
<td>-0.25</td>
<td>-0.178</td>
<td>-0.44</td>
<td>-0.28</td>
</tr>
<tr>
<td>$\delta m_{\pi}^{\text{OGE}}$(GeV)</td>
<td>-0.14</td>
<td>-0.14</td>
<td>-0.15</td>
<td>-0.12</td>
<td>-0.14</td>
</tr>
<tr>
<td>$\delta m_{\pi}^{\text{OGE}}$(GeV)</td>
<td>-0.26</td>
<td>-0.23</td>
<td>-0.17</td>
<td>-0.24</td>
<td>-0.19</td>
</tr>
<tr>
<td>$a_s$</td>
<td>0.93</td>
<td>0.91</td>
<td>0.88</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>$m_N=3E_0+\delta m$</td>
<td>1.19</td>
<td>1.13</td>
<td>0.96</td>
<td>1.32</td>
<td>1.12</td>
</tr>
<tr>
<td>$m_N=3E_0+\delta m$</td>
<td>0.98</td>
<td>0.91</td>
<td>0.73</td>
<td>1.08</td>
<td>0.86</td>
</tr>
</tbody>
</table>

where $\mu'(r)=d\mu(r)/dr$ is the magnetic moment density of a single quark current,

$$j=-\frac{3}{4\pi}r\times\sigma\frac{\mu'(r)}{r^3} \quad (3.14)$$

with

$$\mu'(r)=-2/3r f(r) g(r). \quad (3.15)$$

Since the boundary condition for $f(r)$ and $g(r)$ gives $\mu'(r)\sim O(r^4)$ at the origin and a Gaussian behavior at infinity, the partial integration gives

$$\delta m_{\pi}^{\text{OGE}}(\Delta)=-\delta m_{\pi}^{\text{OGE}}(N)=12a_s \int_0^\infty dr \frac{\sigma^2(r)}{r^4} \quad (3.16)$$

for a baryon, which has the same form as the MIT bag model except that there is not the term $-3\mu^2(R)/R^3$ and the upper bound of the integration is not $R$, but an infinity. Both differences come from the sharp discontinuity at the bag surface: If any smooth distribution is assigned to the quark, the one gluon exchange energy should be reduced to (3.16).

The calculated values for $a_s$ are less than 1.0 in our model, which should be compared with 2.2 in the MIT bag model. The extrapolation from the heavy quarkonia indicates that $a_s$ stays near one, so that our result is encouraging.

The perturbative energy by the color Coulomb interactions among different quarks is written, by use of (2.23), as

$$\delta m_{\pi}^{\text{OGE}}=-2a_s \int_0^\infty dr \rho^2(r) \quad (3.17)$$

where

$$\rho(r)=\int_0^r dr' (f^2(r')+g^2(r')). \quad (3.18)$$

This term is irrelevant to the mass splitting. The calculated values are listed in Table III.
§ 4. Electromagnetic properties of a nucleon

4.1. Static limit

A nucleon in this model cannot move about, since it is nailed down to a fixed point in the world. Therefore, we have to define the static limit of the electromagnetic vertices in order to study the electromagnetic properties of the static nucleon. The photon-nucleon vertex is written as

\[
\Gamma^{\mu}(q) = \bar{u}(p')[\gamma^{\mu}F_1(q^2) + i\sigma^{\mu\nu}q_{\nu}F_2(q^2)]u(p),
\]

(4.1)

where \( q_{\nu} = p'_{\nu} - p_{\nu} \) with \( q_{0} = 0 \). The two form factors are composed of the isovector and isoscalar parts:

\[
F_i(q^2) = F_i^S(q^2) + \tau_3 F_i^V(q^2) \quad \text{for } i = 1, 2.
\]

(4.2)

Each of the form factors is normalized as follows:

\[
F_i^S(0) = F_i^V(0) = e/2,
\]

\[
F_i^S(0) = -0.06\mu_N,
\]

\[
F_i^V(0) = 1.85\mu_N,
\]

where \( \mu_N = e/(2m_N) \) is the nuclear magneton.

Now, the static limit to forbid the recoil of the nucleon is defined by taking the zeroth order term of \( u(p) \) and \( \bar{u}(p') \) in the expansion by \( 1/m_N \). In this expansion \( \mu_N \) cannot be counted as the \( m_N^{-1} \) term, because \( \mu_N \) is merely a unit. The charge current \( j^\mu(q) \), composed of the pionic and the core ones, is identified to the static limit of \( F_i^\mu(q) \):

\[
\lim_{m_N \to \infty} \Gamma^{\mu}(q) = \langle N | j^\mu(q) | N \rangle.
\]

(4.5)

We then have

\[
\langle N | \rho(q) | N \rangle = F_1^S(q^2) + F_1^V(q^2)\tau_3,
\]

(4.6)

where \( \rho(q) = j^0(q) \), and

\[
\langle N | j(q) | N \rangle = i\sigma \times q[F_2^S(q^2) + F_2^V(q^2)\tau_3],
\]

(4.7)

and \( | N \rangle \) denotes the dressed nucleon state. Using the Fourier transform of \( j^\mu(q) \), we can write

\[
F_1(q^2) = \langle N | \int d^3r e^{iqr}\rho(r) | N \rangle,
\]

(4.8)

\[
F_2(q^2) = \langle N | -i\frac{\sigma \times q}{2q^2} \int d^3r e^{iqr}j(r) | N \rangle.
\]

(4.9)

4.2. Charge distribution

The pionic charge distribution is given by the contribution from the second order diagrams, provided that the renormalized coupling constants are substituted at the vertices and intermediate states are dressed.
The charge distribution is, then, written as
\[
\rho_n(r) = \frac{2e}{\pi^3} \left( \frac{f_{\pi NN}^2}{4\pi} \right) \frac{1}{m^3} \int dk \int dk' \frac{F(k, r) F(k', r)}{\omega + \omega'} \times \left\{ \frac{1}{\omega \omega'} - \frac{16}{25} \frac{1}{(\omega + \Delta m)(\omega' + \Delta m)} \right\},
\] (4.10)
where \( F(k, r) = k^3 v(k) j_i(kr) \). The total charge carried by the pion cloud is given by
\[
Q_x = \frac{2}{\pi} \left( \frac{f_{\pi NN}^2}{4\pi} \right) \frac{1}{m^3} \int dk \frac{k^4 v^2(k)}{\omega} \left\{ \frac{1}{\omega^2} - \frac{16}{25} \frac{1}{(\omega + \Delta m)^2} \right\},
\] (4.11)
in units of \( e (>0) \).

The core charge distribution is not altered by the renormalization due to the pion cloud, because the correction, equal to \( Z_2^{-1} \), is cancelled by the wave function renormalization constant \( (Z_2^{1/2})^2 \). Thus we have
\[
4\pi r^2 \rho_c(r) = eQ_c(f^2(r) + g^2(r)),
\] (4.12)
where \( Q_c \) is the core charge probed by a photon. The core charge is defined by the relation,
\[
Q_e^p + Q_x = 1
\] (4.13)
for the proton, and
\[
Q_e^n - Q_x = 0
\] (4.14)
for the neutron. The charge distributions, \( 4\pi r^2 \rho(r) \), are drawn in Fig. 4.

The mean square radii for the proton and neutron are expressed as
\[
\langle r^2 \rangle_p = Q_x \langle r^2 \rangle + Q_e^p \langle r^2 \rangle_c
\]
\[
= Q_x \langle r^2 \rangle - \langle r^2 \rangle_c + \langle r^2 \rangle_c
\] (4.15)
for the proton, and

| Table IV. Electromagnetic quantities of the static nucleon. |
|-----------------|---|---|---|---|---|
|                | I  | II | III | IV | V  |
| \( Q_x/e \)     | 0.18 | 0.15 | 0.11 | 0.22 | 0.16 |
| \( \langle r^2 \rangle_p \text{ fm} \) | 0.73 | 0.75 | 0.80 | 0.68 | 0.72 |
| \( \langle r^2 \rangle_n \text{ fm} \) | 0.38 | 0.45 | 0.53 | 0.37 | 0.34 |
| \( \mu_s \text{ n.m.} \) | 0.76 | 0.67 | 0.53 | 0.88 | 0.67 |
| \( \mu^p \text{ n.m.} \) | 1.75 | 1.76 | 1.85 | 1.77 | 1.76 |
| \( \mu^n \text{ n.m.} \) | 0.17 | 0.18 | 0.19 | 0.17 | 0.18 |
| \( \mu_p \text{ n.m.} \) | 1.92 | 1.94 | 2.04 | 1.94 | 1.94 |
| \( \mu_e \text{ n.m.} \) | −1.58 | −1.57 | −1.66 | −1.60 | −1.58 |
Confining Potential Model for a Quark Core

\[
\langle r^2 \rangle_n = -Q_\pi \langle r^2 \rangle_\pi + Q_c \langle r^2 \rangle_c \\
= -Q_\pi \langle \langle r^2 \rangle_\pi - \langle r^2 \rangle_c \rangle
\]  
(4.16)

for the neutron, where \( \langle r^2 \rangle_\pi (\langle r^2 \rangle_c) \) is the charge radius squared of the pion (core) distribution and defined as

\[
\langle r^2 \rangle_\pi = \int d^3 r r^2 \rho_\pi(r)/Q_\pi 
\]  
(4.17)

and

\[
\langle r^2 \rangle_c = \int_0^\infty drr^2 (f^2(r) + g^2(r)).
\]  
(4.18)

The latter, \( \langle r^2 \rangle_c \), is the same as the core size discussed in the preceding section. Since \( \langle r^2 \rangle_\pi^{1/2} \sim 1.1 \) fm, the resultant proton charge distribution is broaden and has \( \langle r^2 \rangle_\pi^{1/2} \sim 0.75 \) fm and the neutron radius, \( \langle r^2 \rangle_n \), is negative. Numerical values are listed in Table IV. Here we note that experimental charge radii squared are usually given as \( \langle r^2 \rangle_{ch}^2 = \langle r^2 \rangle_1 + 6F_2(0)/2m_N \), where \( \langle r^2 \rangle_1 \) is given by (4.15) or (4.16), and then we have \( \{\langle r^2 \rangle_{ch}^2\}^{1/2} = 0.81 \sim 0.88 \) fm and \( \{-\langle r^2 \rangle_{ch}^2\}^{1/2} = 0.48 \sim 0.46 \) fm from Table IV.

4.3. Magnetic moments

First, we point out that the magnetic moment discussed in our model is concerned only with the anomalous magnetic moment, but not the total one. The reason is that the current distribution \( j \) can contribute only to \( F_2(0) \) in the static limit as discussed in §4.1. (See (4.9).) The normal part comes from the Zitterbewegung of the total charge of a spin \( 1/2 \) Dirac particle, irrespectively of its compositeness. Thus it is sufficient to add the unit of \( \mu_N \) in order to get the total magnetic moment of the proton for example.

The pion magnetic moment \( \mu_\pi \) is written as

\[
\mu_\pi = \frac{4e}{3\pi} \left( f_\pi^2/4\pi \right) \frac{1}{m_\pi^3} \int dk \frac{k^2 b^2(k)}{\omega^3} \left\{ \frac{1}{\omega} + \frac{8}{25} \frac{(\omega + \Delta m/2)}{(\omega + \Delta m)^2} \right\}.
\]  
(4.19)

The magnitude of \( \mu_\pi \) is less than 1.0 \( \mu_N \).

The bare magnetic moment of the core is given by

\[
\mu_c^p(0) = -\frac{3}{2} \mu_c^\pi(0) = -\frac{2}{3} e \int_0^\infty drrf(r)g(r)
\]

\[
= -\frac{e}{4E_0} \left( 1 + \frac{3}{5} g_{\pi(0)} \right).
\]  
(4.20)

In our model \( E_0 \) is larger than or close to 0.5 GeV, so that the bare magnetic moment of the core is less than 1.70 \( \mu_N \) for the proton. Due to the spin structure of \( j_c \), there remains the pion correction to the core magnetic moment.

The resultant anomalous magnetic moments are tabulated in Table IV. The isovector part \( \mu^V \) is in the range 1.76 ~ 1.85 \( \mu_N \) and should be compared with the observed value, 1.85 \( \mu_N \). On the other hand, the isoscalar part is positive and larger by a factor of 3. It should be noted that the lowest order diagrams of pion current contributing to the isoscalar part are three-pion diagrams. When we consider that our model has almost no adjustable parameters, our results are encouraging.
§ 5. Discussion

We have discussed the structure of the ground states of the non-strange baryons in this paper. In order to extend the model to strange and charm baryons, we have to introduce bare masses for the strange and charm quarks, the effects of which change only the constant term of the potential. If we are concerned with $SU(2) \times SU(2)$ chiral symmetry breaking, there are only the pionic corrections to the quark core. Out of the strange baryons the $\Omega^-$ state is of special interest, because there is no pion cloud, and we could deduce the strange quark mass from $\Omega^-$. For example, if we adopt the potential I for the strange quark and II for the nonstrange quarks, we can estimate the $\Omega^-$ mass: Using $M_{\text{core}} = 1.73$ GeV, $\delta m_{\text{OGE}} = -0.26$ GeV $\times (0.91/0.93)$ and $\delta m_{\text{OGE}} = +0.14$ GeV $\times (0.91/0.93)$ from Tables II and III, where the ratio $(0.91/0.93)$ comes from the substitution of the value $a_s$ of I by the one of II, we have about 1.61 GeV for the resultant $\Omega^-$ mass and 0.91 GeV for $N$. In this case the constituent quark mass difference between the strange and non-strange quarks obtained from $m_{\Omega^0}$ seems to be much less than the usually assigned value 150 MeV.

The extension to $SU(3) \times SU(3)$ chiral symmetry induces the kaon and $\eta$ contributions, but these are not so appreciable owing to much larger masses than the pion mass. Further analysis will be made elsewhere.

We have restricted the model space of the Hamiltonian to the ground $1s_{1/2}$ state for the quark and the ground $N$ and $\Delta$ for the baryon, while the pion has been described as an elementary particle. This is the same as the model space employed by the cloudy bag model.  

The extension of the model space implies the divergent contributions to the self-energies of $N$ and $\Delta$ from higher excited nucleons and delta states, $N^*$ and $\Delta^*$, and also from $q\bar{q}$ pairs.  

Introduction of a $q\bar{q}$ creation and destruction would destroy the elementarity of the pion. We are, then, confronted with the old but yet unsolved problem; what is a pion? The divergence of the self-energy coming from the higher excited states would be strongly dependent on the pion structure. It has been pointed out that incorporating the finite size both in space and time directions removes the divergence within the bag model with a fixed $R$. On the other hand, the fuzzy bag model is introduced to smear out the sharp surface and to make the self-energy finite only by the space-like extension of the pion.

In our model a single quark excitation induces the increase of the mean square radius. Asymptotically the Dirac equation with a linearly rising scalar potential resembles the Schrödinger equation for the harmonic oscillator. This gives the Regge behavior like $E,^2 \propto l$ and $\langle r^2 \rangle \propto l,^{28}$ where $l$ is the orbital angular momentum. Increasing $\langle r^2 \rangle$ is expected to reduce strongly the high $l$ contributions more than a model with a fixed $\langle r^2 \rangle$.

We thank our colleague, I. Fukui for helpful discussions. The numerical calculations were done at the Computer Center, Kyushu University, and SALS developed by T. Nakagawa and Y. Oyanagi has been used.

References

A Confining Potential Model for a Quark Core


