Meson Exchange Current Contribution to the Deuteron Magnetic Moment

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Meson exchange current (MEC) contribution to the deuteron magnetic moment is reexamined on the basis of nucleon-nucleon (N-N) interaction model dependent effects and $\rho \pi \gamma$ process. MEC contribution is found to be an order of magnitude smaller than the value obtained previously by Adler. It also strongly depends on the choice of the $N-N$ interaction model, in particular on the percentage of $D$-state and the radius of the repulsive core.

The search for relationship between the deuteron magnetic moment $\mu_d$; the deuteron $D$-state probability $P_0$ and the nucleon-nucleon tensor force has considerable antiquity. It has been established beyond doubt MEC plays the important role in determining the electromagnetic structure of deuteron. However, quantitative estimates of correction to $\mu_d$ arising due to various meson exchange processes and hence $P_0$ needed to fit observed value of $\mu_d$ are sometime inconclusive.

The purpose of this brief report is to refine the calculation reported by Adler on $\rho \pi$ MEC contribution to $\mu_d$. This refinement is mainly based on two distinct areas of concern.

(i) The two-body wave function or, equivalently more recent nucleon-nucleon interaction model with varied “core” radius and $D$-state admixture.

(ii) The current operator in particular the $\rho \pi \gamma$ coupling constant.

Previous workers also used large value of $\rho \pi \gamma$ coupling constant ($g_{\rho \pi \gamma}$) in this problem.

The well-known connection between deuteron $D$-state and deuteron magnetic moment $\mu_d$ given in nuclear magneton is

$$\mu_d = \mu_d' + \Delta \mu_d, \quad (1)$$

where

$$\mu_d' = \mu_n + \mu_p - \frac{3}{2} (\mu_n + \mu_p) P_0 \quad (2)$$

is obtained in terms of nucleonic components (impulse approximation) and the percentage of $D$-state is

$$P_0 = \int w^2(r) dr / \int (u^2 + w^2) dr \quad (3)$$

with $u$ and $w$ being $S$- and $D$-state deuteron wave functions. $\Delta \mu_d$ is the contribution to $\mu_d$ arising from relativistic and meson exchange effects and the presence of velocity dependent terms in nucleon-nucleon interaction. The experimental value of $\mu_d = 0.8577406$ (1) $\mu_N$, produces $P_0 = 3.9\%$ if $\Delta \mu_d = 0$.

Earlier, Jaus showed that two boson exchange current is as important as the relativistic effect for the magnetic moment of deuteron. But the validity of Jaus's calculation has recently been questioned by Sato et al., who showed that $D$ component of the deuteron wave function, which was neglected by Jaus, reduces the exchange current effects on the magnetic moment by about 30%. These results indicate the importance of a consistent treatment of exchange current and nuclear force.

In the present investigation we are only concerned with $\rho \pi$ MEC contribution to $\mu_d$, which was overestimated by Adler in a much earlier study. In addition to this we have explored the interaction model dependence of the said contribution. Further, in contrast to other meson exchange contribution $\pi \pi$ process does not contribute to $N-N$ potential and hence double counting could be avoided while estimating total $(\Delta \mu_d)$ from velocity dependent term in $N-N$ potential as well. Therefore, quantitative evaluation of $\rho \pi$
MEC contribution to $\Delta \mu_d$ and its relative magnitude from other one pion exchange processes will serve a very useful purpose.

In order to estimate $\rho\pi\gamma$ MEC contribution to $\Delta \mu_d$, one first considers the correction of the said process to the magnetic form factor for electron-deuteron $(e-d)$ scattering. For zero momentum transfer, this correction is then associated with a mesonic contribution to the deuteron static magnetic moment. The $\rho\pi\gamma$ process in $e-d$ scattering shown in Fig. 1 and contribution to $\Delta \mu_d$ arising due to this process was obtained by Adler in terms of a one dimensional weighted integral over $S$- and $D$-state wave functions of the deuteron as follows:

$$
(\Delta \mu_d)_{MEC} = \frac{8\Gamma}{3\pi(m_{\rho}^2 - m_{\pi}^2)} \left[ \int_0^\infty \left( m_{\rho}^2 \frac{\exp(-m_{\rho}r)}{r} - m_{\pi}^2 \frac{\exp(-m_{\pi}r)}{r} \right) \right] 
$$

$$
\times \left( u - \frac{1}{2} w^2 \right) dr + \int_0^\infty \left( m_{\rho}^2 \left( 1 + \frac{3}{m_{\rho}r} + \frac{3}{m_{\pi}r^2} \right) \exp(-m_{\rho}r) \right) 
$$

$$
- m_{\pi}^2 \left( 1 + \frac{3}{m_{\rho}r} + \frac{3}{m_{\pi}r^2} \right) \times \exp(-m_{\pi}r) \right) \Gamma^2 w \left( u + \frac{w}{\sqrt{2}} \right) dr,
$$

where $m_{\rho}$ and $m_{\pi}$ are the masses of the rho and pi mesons respectively and $\Gamma$ is given by

$$
\Gamma = \frac{G g_{\rho\pi\gamma}(3)}{16 m_{\rho}^2}.
$$

However, there is an uncertainty regarding the "sign" of this contribution. Here $G$ is the pion-nucleon coupling constant ($G^2/4\pi = 14$) and $g_{\rho\pi\gamma}$ is the $\rho\pi\gamma$ coupling constant whose value is obtained from the experimental value of the electromagnetic decay width for $\rho \rightarrow \pi + \gamma$ decay; employing the relation

$$
\Gamma_{\rho\pi\gamma} = \frac{1}{24} \left( \frac{g_{\rho\pi\gamma}^2}{4\pi} \right) m_{\rho} \left( 1 - \frac{m_{\pi}^2}{m_{\rho}^2} \right)^3.
$$

Further $\alpha$ is the isovector nucleon form factor $\alpha$.

Table I. Values of $(\Delta \mu_d)_{MEC}$ for various $N-N$ interaction model. The subscript 'M' denotes the modified version of the potential.

<table>
<thead>
<tr>
<th>Potential</th>
<th>Adler</th>
<th>Present work</th>
<th>$P_0$ in $N-N$ interaction</th>
<th>$Q_0$ quadrupole moment (mb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yale</td>
<td>0.0188</td>
<td>0.00479</td>
<td>6.96</td>
<td>2.770</td>
</tr>
<tr>
<td>(Yale)$_M$</td>
<td>0.00469</td>
<td>0.00429</td>
<td>7.00</td>
<td>2.820</td>
</tr>
<tr>
<td>HJ</td>
<td>0.0180</td>
<td>0.00424</td>
<td>6.93</td>
<td>2.860</td>
</tr>
<tr>
<td>(HJ)$_M$</td>
<td>0.00361</td>
<td>6.47</td>
<td>2.791</td>
<td></td>
</tr>
<tr>
<td>RSC</td>
<td>0.00333</td>
<td>5.84</td>
<td>2.824</td>
<td></td>
</tr>
<tr>
<td>(RSC)$_M$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Due to the uncertainties in the theory concerning the effects of MEC, this $P_0$ is expected to be somewhere $3.5\sim 10\%$. Since the relative strengths of the central and non-central components in the $N-N$ interaction are not well determined by the $N-N$ scattering data, it should be possible to read just the parameters of any one of the above-mentioned potential in such a way as to affect the $D$-state probability while leaving unchanged the values of the other quantities calculable from the potential. This may be accomplished by decreasing the relative contribution of the tensor component to the two nucleon potential to the total potential. This was done by Breit and collaborators\(^{6,9}\) for Yale, HJ and RSC potentials retaining the quality of agreement of other calculable quantities almost the same. The deuteron wave function obtained with modified Yale, HJ, RSC potentials have expected normal behaviour.

We summarize our results for different $N-N$ potential models in Table I. In the present calculation we have used a new value of $\rho\gamma$ coupling constant $(g^\rho_{\rho e}/4\pi)=1.24\times 10^{-3}$ and $a=1$, which corresponds to two pole fit to isovector nucleon form factor. It has been pointed out by Brodsky and Chertok\(^{10}\) that with more than one pole contribution to the amplitude for nucleon form factors one can get better fit to the deuteron form factor data.

We see from Table I that $(\Delta\mu_d)_{\text{MEC}}$ in $\rho\gamma$ process obtained by us for various $N-N$ interaction model are smaller by more than a factor of 4 than those obtained by Adler, who used a very large value of $(g^\rho_{\rho e}/4\pi)=0.018$. Sato et al.\(^{5}\) found that the magnitude of the two boson exchange contribution to deuteron magnetic moment is $0.032$ corresponding to Reid soft core potential and $(g^\rho_{\rho e}/4\pi)=1.80\times 10^{-2}$. This value is close to our value for modified version of Reid soft core potential though the value of $(g^\rho_{\rho e}/4\pi)$ used by Sato et al. is larger than the value used by us.

It was found by Friar\(^{11}\) and Hadjimichael\(^{12}\) that total meson exchange current contribution arising from all possible meson exchange processes ranges between 0.01 and 0.03 as $P_0$ changes from $4.5\%$ to $7.5\%$. It appears from our calculation that $\rho\gamma$ MEC contribution is nearly $10\%$ of the entire MEC contribution. It shows a sensitivity towards the percentage of $D$-state present in the $N-N$ interaction model. However, this sensitivity is less pronounced than the total contribution of the meson exchange currents. It is also interesting to note that $\rho\gamma$ MEC contribution estimated with soft core potential is nearly $25\%$ smaller than that obtained with hard core potential. $(\Delta\mu_d)_{\text{MEC}}$ obtained for $\rho\gamma$ process is of the same order of magnitude as obtained for velocity dependent term in $N-N$ interaction, namely, $(\Delta\mu_d)_{\text{MEC}}$. Here 'MS' stands for minimal substitution $p=(p-eA/e)$. This is in agreement with the results obtained by Horikawa et al.\(^{13}\) for Ueda-Green\(^{14}\) and Bryan-Scott\(^{15}\) potentials. It appears that Breit’s modified version of Yale, HJ and Reid soft core potentials which have proper deuteron components give consistently slightly smaller values of $(\Delta\mu_d)_{\text{MEC}}$. Whether the presence of the proper deuteron component in $N-N$ interaction model and more accurate value of $g^\rho_{\rho e}$ improve the meson exchange correction can be tested only when we have accurate estimations of all the corrections needed to explain the observed deuteron magnetic moment. It is desirable to redo the calculation with Paris potential,\(^{17}\) which explicitly considers the effects of two pion exchange, omega and rho meson exchanges. However, one must avoid double counting.

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