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Simulation of Groundwater Response by Conceptual Models

– Three Case Studies

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A conceptual runoff model is modified and applied to groundwater observations in an unconfined till aquifer, a confined aquifer under a clay deposit and a large unconfined esker aquifer. The results show that these types of aquifers can be modelled by one general model structure with only a few options. For the confined case the model is feasable for response simulation only while it gives a fair estimate of recharge of the unconfined aquifer.

Introduction

Modelling of groundwater responses from climatological data has much in common with conceptual runoff modelling. Both problems require reliable routines for snow accumulation and melt, soil moisture accounting and a response function. It was therefore felt natural to modify an existing conceptual runoff model and apply it to groundwater observations in different types of aquifers. The model may very well be classified as a conceptual groundwater response model.

The General Model

The starting point for this work is the HBV runoff model (Bergström and Forsman 1973 and Bergström 1976) which has been widely used in Sweden and Norway since the mid 1970's. The snow and soil moisture routines of the model have been unchanged but the configuration of the response function is modified to suit

S_{sm} – computed soil moisture storage
ΔP – contribution from rainfall or snowmelt
ΔQ – contribution to the response function/ runoff
Fc – maximum soil moisture storage
β – empirical coefficient
E_p – potential evapotranspiration
E_a – computed actual evapotranspiration
L_p – limit for potential evapotranspiration

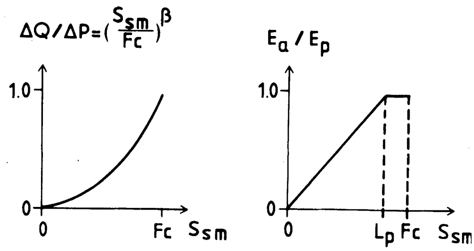


Fig. 1. Schematic presentation of the soil moisture accounting subroutine in the general model.

different types of aquifers. As will be shown later the various response functions have many components in common and can be expressed by a rather simple algorithm with few options.

The snow routine of the model is essentially a degree-day approach with a meltfactor C_o and with a liquid water holding capacity of dry snow delaying meltwater. This capacity is generally fixed at 10 %. Snow is accumulating and added to the modelled snow pack if precipitation is reported at temperatures below a critical value, T_o. Solid precipitation is further subject to a snowfall correction factor C_{sf} which is accounting for both aerodynamic losses at the gauge and winter evaporation.

The soil moisture accounting procedure is based on three parameters, β, L_p and Fc, as shown in Fig. 1. β is controlling the contribution to the response function ($\Delta Q/\Delta P$) or increase in soil moisture storage ($1-\Delta Q/\Delta P$) from each millimetre of rainfall or snowmelt, L_p is the threshold value above which evapotranspiration reaches its potential value, and Fc is the maximum soil moisture storage in the model. In order to avoid problems with non-linearity the soil moisture routine is fed by snowmelt and rainfall mm by mm.

In addition to the parameters of the snow and soil moisture routines a general precipitation correction factor, P_{corr}, is used in some cases.

Data Base

The model is run on a daily basis with daily totals of precipitation and daily mean air temperature as input together with monthly standard values (30 years means) of potential evapotranspiration (Wallén 1966). Output are daily groundwater levels or storage variations. These are compared to observations in piezometer tubes or wells, normally with 14 days resolution in time. The model also delivers snow storage, runoff, actual evapotranspiration and soil moisture index as by-products.

Calibration Technique

In all applications of the model a combination of subjective trial and error technique and sensitivity analysis has been used to estimate optimum parameter values. If possible an independent test period has been saved for final test of the model performance.

When simulating groundwater response there will always be a problem of estimating the effective porosity (storage coefficient) of the aquifer. This problem can be isolated from the estimation of other parameters if a dimensionless criterion of fit, such as the coefficient of correlation, is used. The coefficient of correlation will yield the best co-variation between the model and the record regardless of errors in amplitude or reference levels.

In principle the calibration of the groundwater response model has thus been performed in two steps.

1) An optimum set of parameter values is sought on the basis of best coefficient of correlation between the model simulation and the observations.

2) Values of the effective porosity and refence levels are found by linear regression (scatter diagrams) between modelled and observed values of groundwater storage.

It has generally been necessary to go through steps 1 and 2 several times both with sensitivity analyses of pairs of parameters and subjective trial and error until an acceptable agreement between the model simulations and the observations was found. The problem was particularily complex in cases where variable values of the effective porosity with depth below ground were used.

Application of the Models

For this presentation results from three test sites of quite different characters have been chosen. Their locations are shown on the map in Fig. 2. All test sites are part of the Swedish national groundwater network described in detail by Nordberg and Persson (1974).

Special attention will be given to the description of the configuration of the response functions. The snow- and soil moisture accounting routines are normally not participating in the calibration procedure as their coefficients are chosen by experience from previous work on conceptual runoff modelling. In Table 4 (page 83) in the last section a summary of the model parameters in the snow-, soil moisture- and response calculations can be found.

Malgomaj - Unconfined Aquifer

The Malgomaj test site consists of 6 piezometer tubes in till. They are spread over a quite large area with a maximum distance between two tubes of almost 40 km as

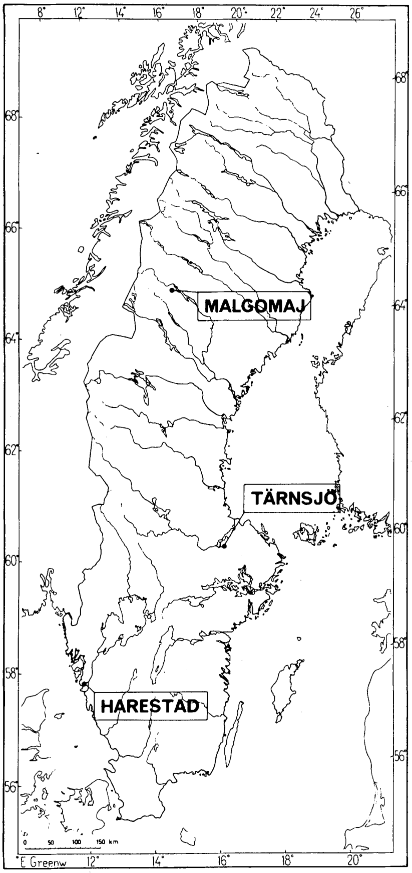


Fig. 2. Location of the test sites.

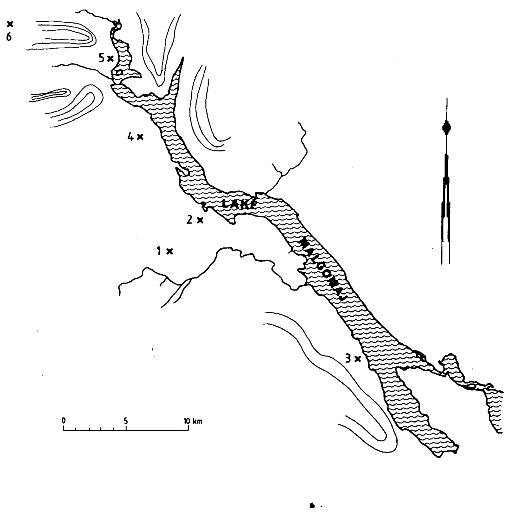


Fig. 3. Location of the stations in the Malgomaj area.

shown in Fig. 3. Two precipitation and temperature stations and six groundwater stations were used for the simulation. Both groundwater storage and level variations were modelled in Malgomaj.

The model structure is shown in Fig. 4. It has combined the snow- and soil moisture routines from the general model with a simple response function where runoff is increasing in steps as the groundwater level raises. This, contrary to the original HBV-model, means that runoff is generated from one single saturated zone representing groundwater, either close to surface or deeper depending on the hydrological situation. This structure is supported by recent hydrological research represented by, for example, Rodhe (1981) who showed the importance of the saturated zone for runoff generation in this type of basins. The drainage components are calculated as follows (see Fig. 4)

$$Q_0 = \begin{cases} K_0\,(UZ-L_0-L_1) & \text{if} \quad UZ > (L_0+L_1) \\ 0 & \text{if} \quad UZ \leq (L_0+L_1) \end{cases} \tag{1}$$

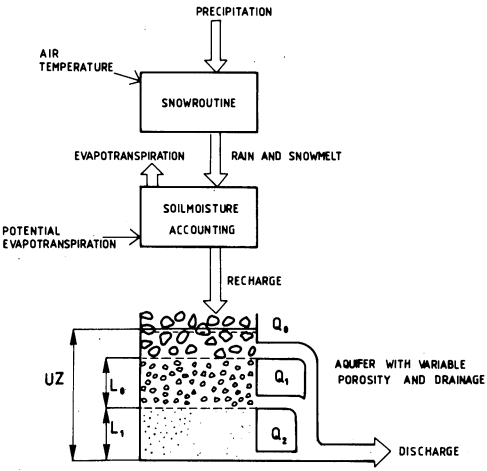


Fig. 4. Schematic presentation of the model structure for an unconfined aquifer in till.

$$Q_1 = \begin{cases} K_1 L_0 & \text{if} \quad UZ > (L_0+L_1) \\ K_1 (UZ-L_1) & \text{if} \quad (L_0+L_1) \geq UZ > L_1 \\ 0 & \text{if} \quad UZ \leq L_1 \end{cases} \tag{2}$$

$$Q_2 = \begin{cases} K_2 \; L_1 & \text{if} \quad UZ > L_1 \\ K_2 \; UZ & \text{if} \quad UZ \leq L_1 \end{cases} \tag{3}$$

where K_o, K_I *and* K_2 are recession coefficients telling the proportion of respective storage which will empty in 24 hours.

When the model is used for simulation of changes in groundwater *storage* the recorded groundwater levels are multiplied by an estimate of the effective porosity to arrive at comparable units. In the model there are options for variable effective porosity with depth below ground. The reference levels for stepwise increase in runoff may coincide with the limits for effective porosity but they may also be determined individually. If *levels* are to be modelled the modelled groundwater storage is turned to levels by the opposite procedure, i.e. division by estimates of the effective porosity.

When simulating the groundwater storage the model was calibrated to the average value of the 6 stations, with the effective porosity estimated individually for each station. This means that individual values of the effective porosity at different depths were sought for each station on the basis of a best linear relationship between the station and the model but the final comparison was made between the model and the average of the 6 stations. The resulting values of the model parameters are summarized in Table 4. It must be stressed that the values of the effective porosity are model parameters and may very well be effected by shortcomings in the model structure. One example is the lack of a routine for

Table 1 – Results of the simulation of groundwater storage and levels expressed as correlation coefficient in the Malgomaj area.

	calibration period	independent period	calibration period if snow routine is calibrated independently for each station
groundwater storage			
station No. 1	0.8124	0.7739	0.8629
station No. 2	0.9439	0.9243	0.9449
station No. 3	0.9182	0.8844	0.9269
station No. 4	0.8765	0.9222	0.8845
station No. 5	0.8536	0.9091	0.9312
station No. 6	0.9055	0.9030	0.9055
average groundwater storage	0.9496	0.9304	
average groundwater levels	0.8883	0.9369	

capillary flux to the surface and subsequent evapotranspiration.

When simulating the groundwater levels average values of the effective porosity from the storage simulation were applied to the response function of the model and turned the model storage to level fluctuations.

The results of the simulation of the independent test period of the average variation of groundwater storage and levels of the 6 stations is shown as a continuous plotting in Fig. 5. In this connection it has to be noted that a visual comparison of the two series of different resolution may give the false impression that the one with better resolution has a quicker response. Therefore only days with observations should be compared.

The values of the correlation coefficient of calibration and test periods for each station and their average is shown in Table 1. In the same table is shown the results if parameters of the snowroutine are calibrated separately for each station. As the snowmelt conditions are very susceptible to variations in the locations it is not surprising that this correlation is somewhat better.

It may be argued that the use of a variable effective porosity by which the recorded values are multiplied is an illegitimate manipulation of data unworthy hydrological modelling. We are fully aware of this argument but it is difficult to find another way of transforming levels to volumes in these variable soils with limited background data.

Harestad - Confined Aquifer

Records from 3 stations were chosen for the simulation in Harestad. They are all penetrating the clay deposit and showing the piezometric head in the underlying friction material. The thickness of the clay deposit is varying from a few metres to

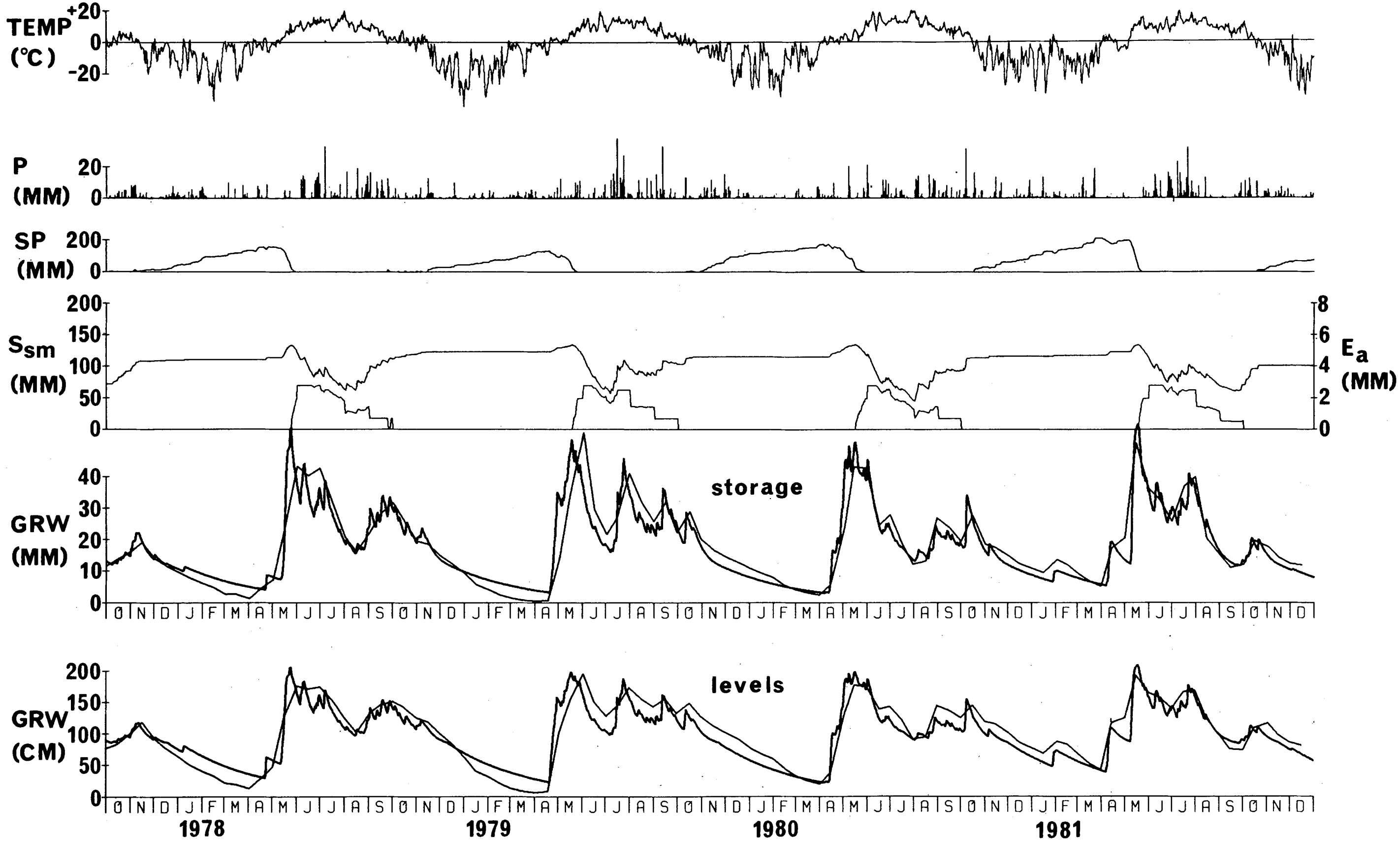


Fig. 5. Results of the simulation of average groundwater storage and levels in the Malgomaj area. Independent test period. The upper simulation represents groundwater storage fluctuations and the lower groundwater levels.

TEMP – mean daily air temperature
P – daily total precipitation
SP – computed snowpack (mm water eqv.)
S_{sm} – computed soil moisture storage
E_a – computed actual evapotranspiration
GRW – groundwater storage or level
Thick line – simulation
Thin line – observations

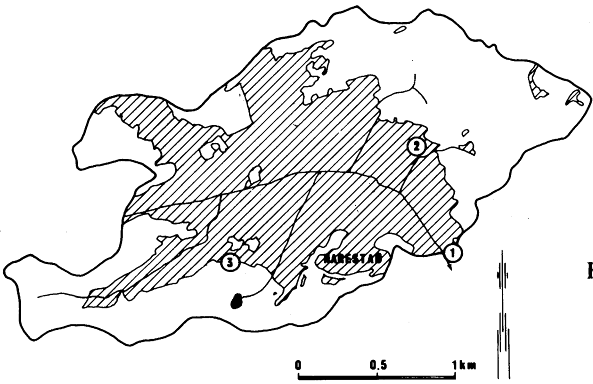


Fig. 6. Location of the stations in the Harestad area. The shaded area represents the clay deposit.

20 with its bottom approximately 18 m below sea level. The location of the stations is shown schematically in Fig. 6. Station No. 1 is located in the bottom of the valley with artesian conditions while the others are in more peripherical locations. Station No. 3 is situated only some tens of metres from the edge of the clay deposit. Precipitation and temperature data are from Säve a few km away.

As is shown schematically in Fig. 7, the basic model for simulation of unconfined conditions is complemented with a near-linear reservoir representing the confined aquifer. According to the notations in Fig. 7 the drainage components are calculated as follows

$$Q_1 = \begin{cases} K_1\,(UZ-L_1) & \text{if} \quad UZ > L_1 \\ 0 & \text{if} \quad UZ \leq L_1 \end{cases} \tag{4}$$

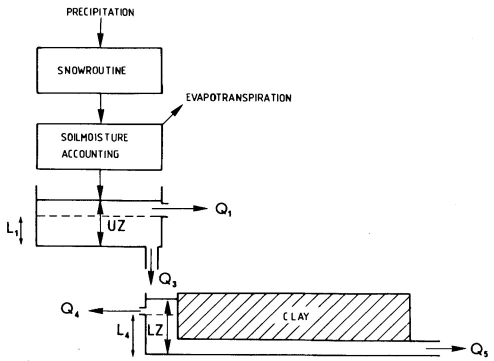
$$Q_3 = \begin{cases} K_3\,(L_1+0.01\,(UZ-L_1)) & \text{if} \quad UZ > L_1 \\ K_3\;UZ & \text{if} \quad UZ \leq L_1 \end{cases} \tag{5}$$

$$Q_4 = \begin{cases} K_4\,(LZ-L_4) & \text{if} \quad LZ > L_4 \\ 0 & \text{if} \quad LZ \leq L_4 \end{cases} \tag{6}$$

$$Q_5 = K_5\;LZ \tag{7}$$

Note that the effect of an increase of UZ on the recharge of the aquifer is reduced if UZ is greater than L_1.

Due to the fact that little is known about recharge areas and the storage and drainage characteristics of the confined aquifer this model requires a more empirical approach than the ones for unconfined conditions. The linear regression analysis previously used for the estimation of effective porosity is now used for the estimation of a truly empirical coefficient, p, representing the bulk relation between inflow in mm from an undefined recharge area and storage conditions in the



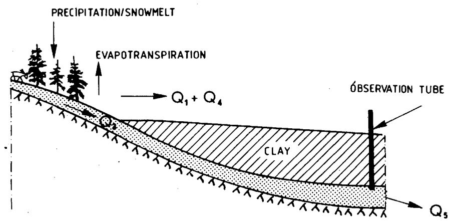


Fig. 7. Schematic presentation of the model structure for a confined aquifer.

aquifer. Thanks to the character of the correlation coefficient, p can be determined independently and after the assessment of all other model parameters. In practice the best correlation between LZ and the observations is first sought regardless of amplitude and refence levels. Thereafter p and the correct reference levels are estimated by regression between LZ and the observations in each piezometer tube separately before the results are plotted.

The observations in Harestad showed some inhomogeneities which forced us to exclude the first years of record. In order to get a sufficient amount of hydrological events we also had to use all remaining years for calibration. Therefore we have no proper independent period but only a general feeling that the risk for over-fit is small.

The calibration of the model to the observations in Harested proved to be a complicated procedure due to all possible interactions between model parameters. First after a lengthy combination of sensitivity analyses and visual inspection of graphs we reached an acceptable goodness of fit. There is still, however, no guarantee that we have not reached local optima only.

The results of the simulations are presented separately for each station in Fig. 8, results expressed as correlation coefficients in Table 2.

The use of an empirical coefficient, p, in the Harestad model when transforming model storage to aquifer response means that this type of models should be restricted to response analysis only. It will be very difficult to use the model for estimations of recharge of the aquifer.

Table 2 – Results expressed as correlation coefficient in the Harestad area.

	calibration period
station No. 1	0.8398
station No. 2	0.8756
station No. 3	0.8854

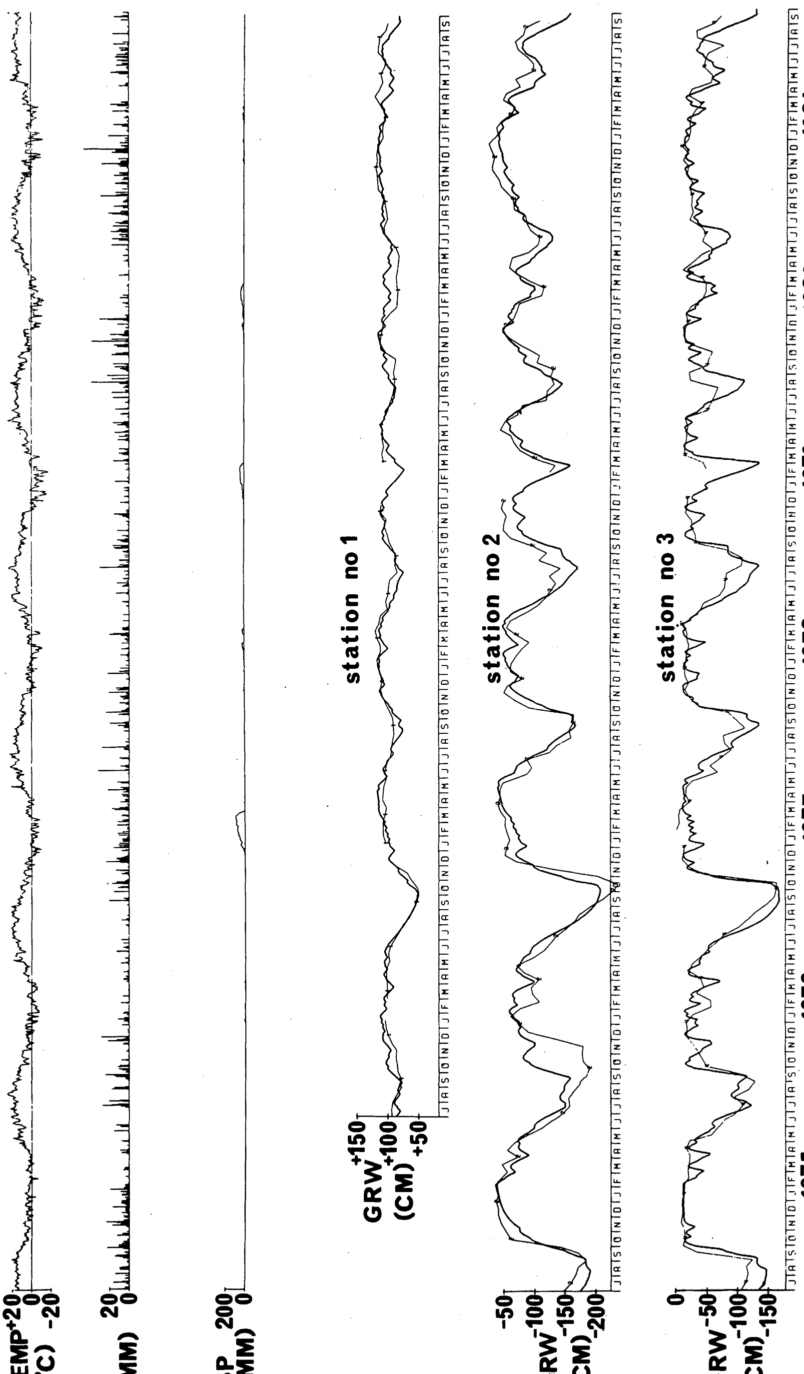


Fig. 8. Results of the simulation of groundwater level response in the Harestad area. Calibration periods.

TEMP – mean daily air temperature
P – daily total precipitation
SP – computed snowpack (mm water eqv.)
GRW – groundwater levels above (+) or below (–) ground level
Thick line – simulation
Thin line – observations

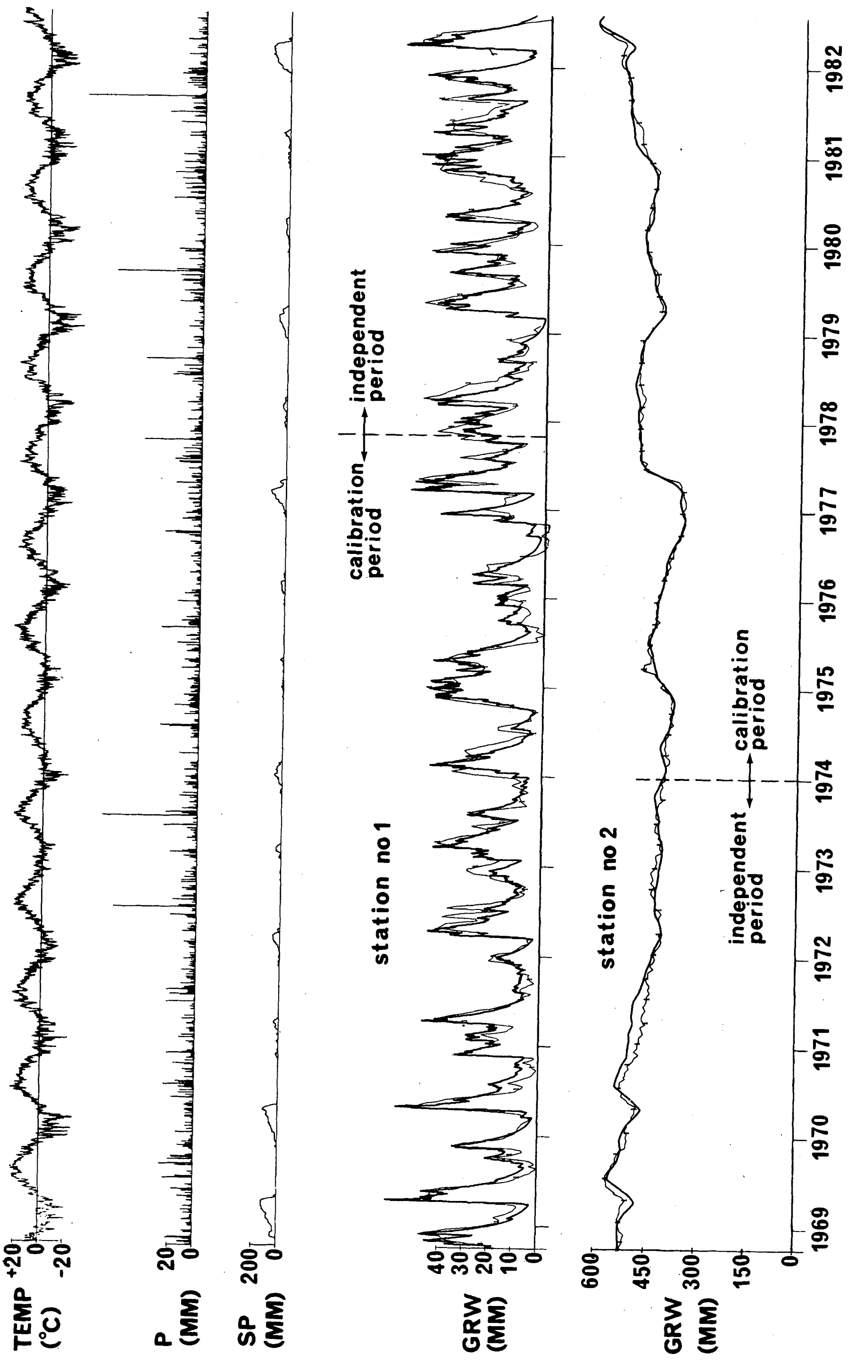


Fig. 10. Results of the groundwater simulations in the Tärnsjö area.

TEMP – mean daily air temperature
P – daily total precipitation
SP – computed snowpack (mm water eqv.)
GRW – groundwater storage
Thick line – simulation
Thin line – observations

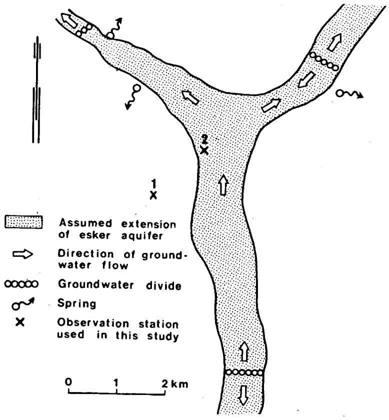


Fig. 9. Location of the stations in the Tärnsjö area (after Aneblom and Persson 1979).

Tärnsjö-Unconfined Aquifers

The Tärnsjö area is dominated by a large esker rising up to 30-50 m above the surrounding terrain. Two observation stations are used in this study, one (No. 1) representing a small aquifer in till a short distance from the esker and the other (No. 2) representing the large aquifer in the esker itself as shown in Fig. 9. Precipitation data were collected from Ljusbäck a few km away and temperature data from Folkärna some 30 km from the site.

The simulation of station No. 1 was made by a model of similar type as the one used in Malgomaj (Fig. 4) but without variations in the effective porosity with depth below surface. When simulating the esker aquifer the delayed recharge caused by transport in the unsaturated zone had to be taken into account. A study of the vertical moisture flux around tube No. 2 carried out by Aneblom and Persson (1979) indicated that the velocity of the moving wet zones may be in the order of 1.5-3.0 m/month for the upper 7-8 m of the esker and that the velocity is governed not only by the properties of the geological material but also by the moisture content in the material.

Two attempts were made to model the delayed recharge caused by the deep unsaturated zone in the esker. First a single linear reservoir was introduced and secondly a time-lag and smoothing of the rainfall/snowmelt-pulse was used. The latter technique proved to give a slightly better representation particularily of the timing of recharge.

The model was calibrated over a 9 years period for station 1 and an 8 years period for station 2 leaving the remaining 5 as independent test periods.

Due to the extremely slow response of station 2 and the fact that the different sets of parameters require different initial conditions it was necessary to exclude the first years of simulation when performing the sensitivity analysis. Therefore it was necessary to use the second part of the record for calibration and the first one as independent test period when calibrating the model to station 2.

Table 3 – Results expressed as correlation coefficient in the Tärnsjö area.

	calibration period	independent period
station No. 1	0.8762	0.7807
station No. 2	0.9858	0.9318

Table 4 – Summary of optimum sets of parameters for the aquifers. Note that the recession coefficients, K_o-K_5, are indicating the proportion of respective storage emptied in 24 hours.

Area	Malgomaj	Harestad			Tärnsjö	
Station No	1-6	1	2	3	1	2
P_{corr}	1.06	1.00	1.00	1.00	0.97	0.97
C_{sf}	1.02	0.80	0.80	0.80	0.86	0.86
T_o (°C)	0.1	0.0	0.0	0.0	0.0	0.0
C_o (mm/°C day)	2.0	2.5	2.5	2.5	2.5	2.5
Fc (mm)	150	200	200	200	200	200
L_p (mm)	100	150	150	150	150	150
β	2.5	2.5	2.5	2.5	3.0	1.5
K_o (day^{-1})	0.50	–	–	–	–	–
K_1 (day^{-1})	0.06	0.60	0.60	0.60	0.45	–
K_2 (day^{-1})	0.01	–	–	–	0.025	0.0015
K_3 (day^{-1})	–	0.22	0.04	0.06	–	–
K_4 (day^{-1})	–	0.04	–	0.15	–	–
K_5 (day^{-1})	–	0.01	0.03	0.04	–	–
L_o (mm)	30	–	–	–	–	–
L_1 (mm)	12	1.7	15	15	43	–
L_4 (mm)	–	28	–	10	–	–
eff.porosity (%)	0.5-12.6*	–	–	–	2.1	37
p (empirical transformation from LZ to groundwater response)	–	30	86	156	–	–

* variable with depth and between stations

When delaying the rainfall/snowmelt for station 2 the pulse was evenly spread over the 94 days following the date of the event. This means a very smooth recharge of 94 days duration from one day with rainfall or snowmelt.

The results from the simulations in the Tärnsjö area are summarized in Fig. 10 with correlation coefficients in Table 3. Optimum parameter sets are given in tabular form in Table 4.

As can be seen in Fig. 10 the response of these aquifers of quite different types can be modelled with model structures of great similarity. Again the interpretation of the effective porosity has to be made with great care as station No. 2 is only representing one point in a very large aquifer and is therefore more an index of the storage than an absolute value. As can be seen in Table 4 the modelling of station No. 2 is made with only one recession coefficient. This means that the response function is a true single linear reservoir.

Summary of the Results and Discussion

A summary of optimum parameter settings for all stations is given in Table 4. It is noticeable that the various types of aquifers can be modelled by one general model structure with a few options. Much of the variability in response-pattern can be accounted for by a few recession coefficients.

As long as the complexity of the model is kept low, as in the case of unconfined aquifers, parameter estimation is no great problem but for a confined aquifer the increased complexity and parameter interaction can easily lead to confusing results and modelling may turn to curve-fitting. As a consequence the model for confined conditions is feasable for response simulation only while in the unconfined case the models give a fair estimate of aquifer recharge as well.

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