Journal of Applied Mechanics

Discussion

Discussion: "On the Relationship Between the L-Integral and the Bueckner Work-Conjugate Integral" (Shi, J. P., Liu, X. H., and Li, J., 2000 ASME J. Appl. Mech., 67, pp. 828–829)

Y. Z. Chen
Division of Engineering Mechanics, Jiangsu University, Zhenjiang, Jiangsu 212013, P.R. China

K. Y. Lee
Department of Mechanical Engineering, Yonsei University, Seoul 120-749, South Korea

Three wrong expressions in the paper ([1]) have been found. Equations (4) and (5) in the paper are written in the forms

$$\varphi^{(\mathrm{II})}(z)=-i\varphi'(z),\quad \psi^{(\mathrm{II})}(z)=-iz\psi'(z)+2i\bar{z}\varphi'(z),\tag{1}$$

$$u_i^{(\mathrm{II})}=yu_{i,x}-xu_{i,y}\tag{2}$$

$$\sigma_{ij}^{(\mathrm{II})}=y\sigma_{ij,x}-x\sigma_{ij,y}+\frac{1}{2}\int\sigma_{ij,x}dy-\frac{1}{2}\int\sigma_{ij,y}dx\quad(i,j=1,2).\tag{3}$$

1 Complex potentials suggested by Muskhelishvili should be an analytic function ([2]). However, since the argument $\bar{z}$ is involved in the second term of $\psi^{(\mathrm{II})}(z)$ in Eq. (1), $\psi^{(\mathrm{II})}(z)$ cannot be an analytic function. Therefore, $\psi^{(\mathrm{II})}(z)$ in Eq. (1) is a wrong expression.

2 In the complex variable function method, the displacement components can be expressed as ([2])

$$\begin{aligned}2G(u+iv)&=\kappa\varphi(z)-z\overline{\varphi'(z)}-\overline{\psi(z)}\\&=\kappa\varphi(z)+z\{-\overline{\varphi'(z)}\}-\overline{\psi(z)}\end{aligned}\tag{4}$$

where G is the shear modulus of elasticity, $\kappa=(3-\nu)/(1+\nu)$ is for the plane stress problem, $\kappa=3-4\nu$ is for the plane strain problem, and ν is the Poisson's ratio, and $\varphi(z)$ and $\psi(z)$ are two analytic functions.

Equation (4) reveals a rule that in a real displacement expression of plane elasticity, if the function after the elastic constant κ is $\varphi(z)$, the term after z in Eq. (4) should be $-\overline{\varphi'(z)}$.

On the other hand, from Eq. (4) we have

$$2G\left(\frac{\partial u}{\partial x}+i\frac{\partial v}{\partial x}\right)=(\kappa\varphi'(z)-\overline{\varphi'(z)})-(z\overline{\varphi''(z)}+\overline{\psi'(z)})$$

$$2G\left(\frac{\partial u}{\partial y}+i\frac{\partial v}{\partial y}\right)=i\{(\kappa\varphi'(z)-\overline{\varphi'(z)})+(z\overline{\varphi''(z)}+\overline{\psi'(z)})\}.\tag{5}$$

Therefore, from Eqs. (2) and (5), the displacement components in Eq. (2) can be expressed as

$$\begin{aligned}2G(u^{(\mathrm{II})}+iv^{(\mathrm{II})})&=2G\left(y\left(\frac{\partial u}{\partial x}+i\frac{\partial v}{\partial x}\right)-x\left(\frac{\partial u}{\partial y}+i\frac{\partial v}{\partial y}\right)\right)\\&=\kappa\{-iz\varphi'(z)\}+z\{i(\overline{\varphi'(z)}-\bar{z}\overline{\varphi''(z)})\}\\&\quad-i\bar{z}\overline{\psi'(z)}.\end{aligned}\tag{6}$$

From the fact that

$$-\overline{\frac{d}{dz}\{-iz\varphi'(z)\}}=-i(\overline{\varphi'(z)}+\bar{z}\overline{\varphi''(z)})\neq i(\overline{\varphi'(z)}-\bar{z}\overline{\varphi''(z)})\tag{7}$$

and the rule mentioned above, the displacements $u^{(\mathrm{II})}$ and $v^{(\mathrm{II})}$ shown in Eq. (2) are not an elasticity solution. Therefore, the displacement shown in Eq. (2) is also a wrong expression.

3 In Eq. (3) an indefinite integral is used to express the stress components. In the continuum medium of elastic body, the integral should be path-independent. Also, it is well known that if a function $F(x,y)$

$$F(x,y)=\int_{(x_o,y_o)}^{(x,y)}p(x,y)dx+q(x,y)dy\tag{8}$$

is a path independent integral, the following condition must be satisfied:

$$\frac{\partial p(x,y)}{\partial y}=\frac{\partial q(x,y)}{\partial x}\quad\text{or}\quad\frac{\partial q(x,y)}{\partial x}-\frac{\partial p(x,y)}{\partial y}=0.\tag{9}$$

If Eq. (3) were true, substituting $p(x,y)=-\sigma_{ij,y}/2$ and $q(x,y)=\sigma_{ij,x}/2$ into Eq. (9) yields the following:

$$\frac{\partial^2\sigma_{ij}}{\partial x^2}+\frac{\partial^2\sigma_{ij}}{\partial y^2}=0.\tag{10}$$

However, the stress components σ_{ij} are not a harmonic function in general. Thus, the $\sigma_{ij}^{(\mathrm{II})}$ shown by Eq. (3) is also a wrong expression.

References

[1] Shi, J. P., Liu, X. H., and Li, J. M., 2000, "On the Relation Between the L-integral and the Bueckner Work-Conjugate Integral," ASME J. Appl. Mech., **67**, pp. 828–829.
[2] Muskhelishvili, N. I., 1953, *Some Basic Problems of Mathematical Theory of Elasticity*, Noordhoof, Dordrecht, The Netherlands.

Discussion: "A Critical Reexamination of Classical Metal Plasticity" (Wilson, C. D., 2002, ASME J. Appl. Mech., 69, pp. 63–68)

C. J. Lissenden
Department of Engineering Science and Mechanics,
The Pennsylvania State University, 212 Earth-Engineering Sciences Building, University Park, PA 16802

The author correctly identifies the backbone of metal plasticity as the Mises yield criterion, the Prandtl-Reuss flow law, and isotropic/kinematic hardening. However, there has always been the qualification that these simplifications of plasticity work well for "most metals" or "some metals." It is noteworthy that while the author has devoted a section of his paper to Richmond's work refuting the widespread use of the assumption of pressure-independent flow in metals, he did not reference the keystone work of Spitzig and Richmond [1], where they provide additional results for 1100 aluminum. This would have further reinforced his point. Spitzig and Richmond found 1100 aluminum to exhibit pressure-dependence but not a strength-differential. Here the term strength-differential means a tension-compression asymmetry (e.g., compressive yield strength larger than tensile yield strength), which is different from a Bauschinger effect. The yield function that Spitzig and Richmond used can be written in the forms

$$f = aI_1 + \sqrt{3J_2} - c$$

$$f = \alpha I_1 + \frac{\sqrt{3J_2}}{c} - 1$$

where I_1 and J_2 are the usual stress invariants and $\alpha = a/c$, a is the pressure coefficient, and c is the strength coefficient. The strength-differential depends only on the parameter a, but pressure-dependence is affected by both a and c. While a and c were shown to be strain-dependent, α was not ([1]). In fact, $\alpha = a/c$ for aluminum was approximately three times that of iron-based materials.

Based on the tensile and compressive yield strengths reported by Wilson for 2024-T351 aluminum, presumably using the 0.2% offset strain definition; the yield function parameters can be calculated and compared with results from Spitzig and Richmond in Table 1.

Table 1 Yield function parameters

Material	a	c (MPa)	$\alpha = a/c$ (TPa)
2024-T351 aluminum	0.0296	791	37
1100 aluminum ([1])	0.0014	25	56
Aged maraging steel ([1])	0.037	1833	20

The pressure-dependence of 1100 and 2024-T351 is similar, but 2024-T351 exhibits a strength-differential ($2a$) of 5.9%, while 1100 does not exhibit an appreciable strength-differential. While Wilson did not measure volume change, Spitzig and Richmond did, and found there to be no significant dilation; indicating that an associated flow rule will not correctly predict plastic strain. This is also the case for frictional materials, where it is common to employ a nonassociated flow rule.

We have observed strength-differential in laboratory experiments using aged Inconel 718 (a precipitation strengthened nickel-base alloy) ([2,3]), 6061-T6 aluminum and 6092/SiC/17.5-T6 (a particulate reinforced aluminum alloy) ([4]). The Mises yield criterion does not apply well to these materials either. Our work on Inconel 718 ([3]) indicates that a J_2-J_3 yield function, which we called a threshold function because we were working in the realm of viscoplasticity, along the lines of that proposed by Drucker [5] for an aluminum alloy was most suitable.

Finally, while it is fairly obvious, it is worth pointing out that the Drucker-Prager yield criterion predicts more flow for the same tensile stress than the Mises yield criterion simply due to the presence of the positive I_1 term. Thus, the finite element results of Wilson for Mises and Drucker-Prager yield criteria are self-consistent. It would be interesting to know the range of I_1 for a particular notch geometry.

References

[1] Spitzig, W. A., and Richmond, O., 1984, "The Effect of Pressure on the Flow Stress of Metals," Acta Metall., **32**, pp. 457–463.
[2] Gil, C. M., Lissenden, C. J., and Lerch, B. A., 1999, "Yield of Inconel 718 by Axial-Torsional Loading at Temperatures Up to 649C," J. Test. Eval., **27**, pp. 327–336.
[3] Iyer, S. K., and Lissenden, C. J., 2002, "Viscoplastic Model Accounting for the Strength-Differential in Inconel 718," submitted for publication.
[4] Lissenden, C. J. and Lei, X., 2002, "A More Comprehensive Method for Yield Locus Construction for Metallic Alloys," submitted for publication.
[5] Drucker, D. C., 1949, "Relation of Experiments to Mathematical Theories of Plasticity," ASME J. Appl. Mech., **16**, pp. 349–357.