curve. Four approximations to the deflection of a member are presented so that a satisfactory approximation of the loads for a given depth of yielding can be computed. The results presented in this paper are believed to be sufficient to justify the following conclusions:

1 The interaction curve is constructed for a given member subjected to a combination of axial load and bending moment by locating two or more points on the curve. Each point is located by computing the values of P and M for the stress distribution that is assumed to occur for the given depth of inelastic strain. Because of the ease of construction and use, the interaction curve was derived to give the range of corresponding values of moment and load which would produce a given depth of inelastic strain. The tension and compression stress-strain curves for the material are approximated by two straight lines, Figs. 8 and 9.

2 The results obtained from tests of eccentrically loaded rectangular tension members of annealed rail steel and aluminum alloys 24S-T4 and 75S-T6 gave good correlation between theoretical and experimental interaction curves.

3 In designing an eccentrically loaded tension member for a given depth of yielding, the lateral deflection in Equation [2] for the member must be known so that the relation between axial load and bending moment can be determined. Four approximations to the moment-load curve are presented. All approximations give values of P and M that are not greatly different from the values which correspond to the actual conditions assumed in the member.

4 The test results show that for beams of rectangular cross section subjected to combined axial tension and bending loads, more than 50 per cent increase in load at the threshold of the occurrence of inelastic strain in the most highly stressed fibers of the beam is required to cause inelastic strains in the most stressed cross section to occur to a depth of $^1/_2$ the cross section.

5 The results of this investigation are not generally applicable to such members as described in the foregoing conclusions when the axial load is a compressive force. This problem is one of unstable equilibrium because the axial load reaches a maximum and at this load the member will collapse due to inelastic buckling. It is concluded that further research, in which moment-load curves such as OBD and OFG in Fig. 13 must be used, will be required to solve this problem. The method of analysis developed in this report will be of great value in analyzing the behavior of a member subjected to combined bending and axial compressive load.

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Discussion

J. M. English.[6] The authors have presented an interesting paper on the interaction curve for bending and direct stress excluding stability. A valuable contribution lies in the approach they took. Ultimately it may lead to a better understanding of the column-interaction curve which does involve stability.

When stability is not a factor, the criterion for failure must be defined by stress. The authors do this by limiting arbitrarily the extent of the inelastic behavior of the cross section of the beam to one half the depth. With this criterion it is then possible to obtain a relationship between P and M which can be plotted as an interaction curve. The interaction curve may be plotted nondimensionally by dividing by chosen reference values of P and M. Because the development of the curve was started from the well-known elastic relation

$$\sigma = P/A + Mc/I$$

it was quite natural to choose P_e and M_e as the reference values. However, a more useful representation of the interaction curve may be obtained by choosing the ultimate moment M_u. The ultimate value of moment, defined by an arbitrary proportion of the section exceeding the elastic limits, will be in excess of the elastic limit value M_e. Consequently, as is shown in the plotted curves, the intercept on the moment axis exceeds unity when M_e is the reference.

If the ultimate value M_u is chosen as the reference moment value, the usefulness of the interaction curve is enhanced. Some interesting relationships become immediately apparent. The ultimate value of the moment for pure bending may be defined either, as before, in terms of a percentage of the section depth behaving inelastically or alternatively as an experimental ultimate. Essentially it makes little difference to the resulting interaction curve.

Consider the case of idealized plasticity, $\alpha = 0.0$, and a fully plastic section. The M_u will be 1.5 M_e. The curve, labeled "fully plastic" in Fig. 2 of the paper, will then be scaled down by this factor to fit the framework of a one-one diagram. Its equation can be shown to be

$$\frac{M}{M_u} + \left(\frac{P}{P_e}\right)^2 = 1 \quad [9]$$

If the curves of "partial depth yielded" are plotted on the same diagram as Equation [9], the upper portion of the interaction curve in each case is also a parabola and thus is coincident with that of the fully plastic curve.

This fact may readily be seen with reference to Fig. 14 of this discussion, which represents the stress distribution in the cross section for some value of P and M. The only way for P to increase and still satisfy the original postulate, is for the sloping line

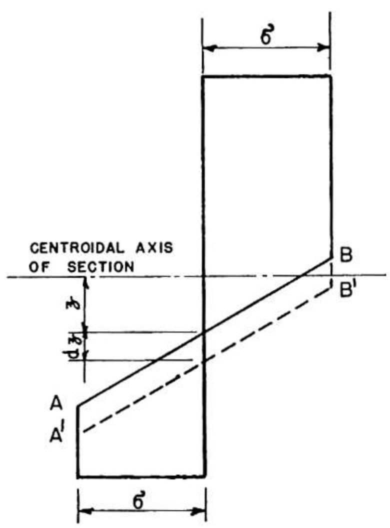

Fig. 14 Stress Distribution Across Section

[6] Department of Engineering, University of California, Los Angeles, Calif.

A-B to shift to A'-B'. Hence the increase in P is

$$dP = 2\sigma dz$$

Likewise, the change in moment will be

$$dM = -2\sigma z dz$$

Integrating each of these expressions and eliminating z, a parabola

$$\tfrac{1}{4}P^2 + M = c$$

is obtained.

The only effect of the arbitrary limitation of the depth of plasticity is to change the slope of the straight portion of the interaction curve. This also establishes the intersection of the straight line and the parabola. The resulting interaction curve is shown

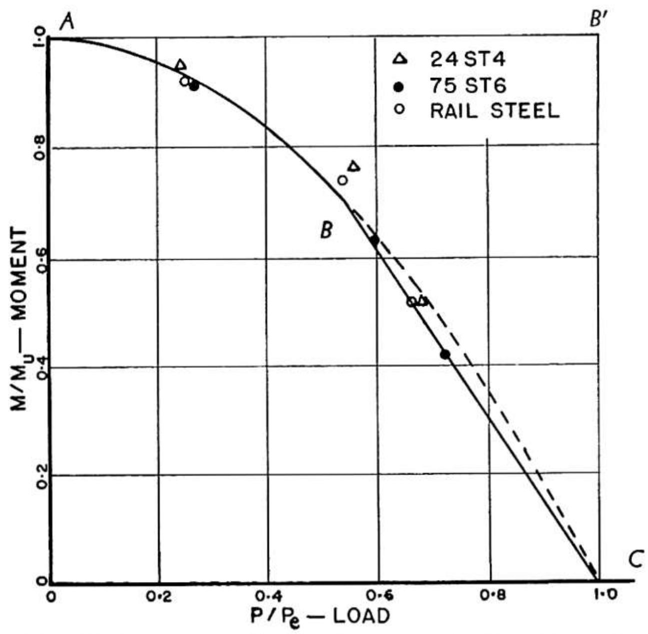

Fig. 15 Interaction Curve—Rectangular Cross Section

in Fig. 15, herewith, for the case of $^1/_2$ the depth yielded. The difference between this and the one having a fully plastic stress in the section is seen by comparison with the completed parabola shown dotted.

The authors kept the problem general by not basing their development on the idealized flat-top stress-strain relation. However, it is interesting to note the results of the tests as plotted [7] on the one-one interaction diagram in Fig. 15. Even for the case of annealed rail steel which did not exhibit a horizontal region in the stress-strain curve, the results lie remarkably close to the interaction curve with the parabolic upper portion and the straight-line lower portion. It may be that this simplified interaction relation will provide a suitable general expression for engineering applications.

[7] The data for replotting the authors' test results were taken directly from the preprint of the paper. There may have been a loss of accuracy in the transcribing.

Authors' Closure

The authors are grateful to Mr. J. M. English for his interesting discussion. His method of representing an interaction curve for a member made of a material with $\alpha = 0.0$ is a simpler relation than that given by the authors, and the one-to-one plot, Fig. 15 of his discussion, is perhaps more appealing to the engineer. Interaction curves for rectangular members made of materials with $\alpha \neq 0.0$ also may be plotted on the same diagram. Curve A-B-C is the interaction curve for $^1/_2$ depth of yielding of a member made of a material with $\alpha = 0.0$. If $\alpha = 1.0$, the corresponding interaction curve is A-B'-C. Interaction curves for rectangular members made of materials having other values of α would lie in the area bounded by A-B'-C-B-A. As indicated in Fig. 3 of the paper, the co-ordinates of point B, the end of the straight line B-C, is very important. In Fig. 3 the co-ordinates of B for a given α were obtained by linear interpolation between points B and B'. Although the abscissa of point B'' for $\alpha \neq 0$ may be obtained by linear interpolation between the abscissa of point B and the abscissa of point B', the ordinate must be obtained from the relation

$$B'' = \frac{BM_u + (B'M'_u - BM_u)\,\alpha}{M_u + (M'_u - M_u)\,\alpha}$$

where B'' is the ordinate of the point for the required α and M_u and M'_u are the ultimate values of the moments for $\alpha = 0.0$ and $\alpha = 1.0$, respectively. The ultimate value of the moment is the magnitude of the pure moment which will produce the given depth of yielding.

The representation of the interaction curves as given by Mr. English has two disadvantages. In the first place the values of M_u would have to be given for each set of curves. These values could be given in equation, tabular, or graphical form. Also, the curve as represented in Fig. 15 of the discussion, conceals the relative increase in load which may be required to produce a given depth of yielding.

There is one point on which the authors wish to differ with the discusser. The criterion for failure was not defined by stress. If α is equal to zero, there is no way in which the magnitude of stress can be associated with depth of yielding, since each depth of yielding results in the same value for the maximum stress.