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9 "Correlation of Plastic Deformation During Metal Cutting With Tensile Properties of the Work Material," by J. T. Lapsley, R. C. Grassi, and E. G. Thomsen, Trans. ASME, vol. 72, 1950, pp. 979–986.

10 "Discussion to Paper 'Yielding and Fracture of Medium-Carbon Steel Under Combined Stress,' " by C. W. MacGregor and L. F. Coffin, *Journal of Applied Mechanics*, Trans. ASME, vol. 68, 1946, pp. 70–71.

Discussion

L. P. Tarasov.[8] The results of this paper will be of great interest not only to those concerned with the details of the grinding process but also to the solid-state physicists, who long have maintained that the intrinsic strengths of metals, calculated from a knowledge of interatomic forces, are many times greater than found in ordinary mechanical testing. Perhaps suitably conducted grinding experiments, combined with refinements of the theory, will make it possible to obtain the theoretically predicted strengths for a variety of metals.

The use of extremely low table speeds by the authors, which were necessary if undesirable inertia effects were to be avoided, made it possible to observe the region in which the grinding energy remains constant with changing depth of cut, provided this is kept below 0.001 in. At the considerably higher table speeds normally used, this effect is absent because the critical depth of cut below which the energy is constant is too small to permit successful experimentation. For example, we have found in surface-grinding hardened high-speed steel at a table speed of 60 fpm with a 32A46-H8VBE wheel that the specific grinding energy (calculated from the input power to the motor) decreased with depth of cut between 0.00025 and 0.003 in., at first rapidly and then progressively less rapidly.

In ordinary grinding practice the chip thickness usually will exceed the critical size because in those operations where a low work speed is employed, the wheel depth of cut is made very high, and in finishing operations, where the depth of cut is low, a high work speed is employed.

With regard to Fig. 12 of the paper, it might be inferred from the text that the steel was SAE 1112, but the microstructure some distance below the grinding scratches indicates a considerably higher percentage of carbon than corresponds to this steel, as evidenced by the large amount of what appears to be pearlite. An explanation of this would be desirable.

B. T. Chao[9] and K. J. Trigger.[10] The authors' treatment of the grinding and micromilling data is a logical approach to the analysis of chip formation under complicated cutting conditions. Their discussion of the so-called "size effect" not only has a direct bearing on many metal-cutting phenomena but also serves to clarify one of the fundamental topics in the strength of materials and the physics of solids.

The authors' method of evaluating the chip-length ratio r_L from the micromilling data can be simplified. A more direct and better method is to plot the chip length l_c against the undeformed chip length l under the cutting conditions prescribed by the authors. According to Equation [16] of the paper, the slope of the resulting curve at any assigned value of l will represent the corresponding value of r_L. Appropriate values for the tooth depth of cut t, also can be obtained easily. It is not necessary to employ Equations [17] to [19] in the evaluation of the chip-length ratio. As an illustration, Table 3, herewith, has been prepared for the case when the rake angle of the cutter is 0 deg.

TABLE 3 EVALUATION OF CHIP-LENGTH RATIO FROM MICROMILLING DATA

Wheel depth of cut, d, 1/1000 in.	Chip length,[a] l_c, in.	Undeformed chip length, $l = \sqrt{Dd}$, in.	Chip length ratio, r_L	Tooth depth of cut,[b] t, microinches
0.4	0.0175	0.0490	0.357	231
0.8	0.0248	0.0693	0.380	326
1.2	0.0309	0.0849	0.397	399
1.6	0.0362	0.0980	0.416	462
2.0	0.0411	0.1095	0.444	516

[a] Values taken from Fig. 5(*b*) of paper.
[b] Calculated from Equation [15] of paper.

Values of r_L in Table 3 are determined from the slopes of the curve shown in Fig. 18, herewith. It is seen that the relationship between l_c and l is not quite linear. Consequently, the chip-length ratio is not independent of the changes in t as the authors believe. This is illustrated by the last two columns in Table 3.

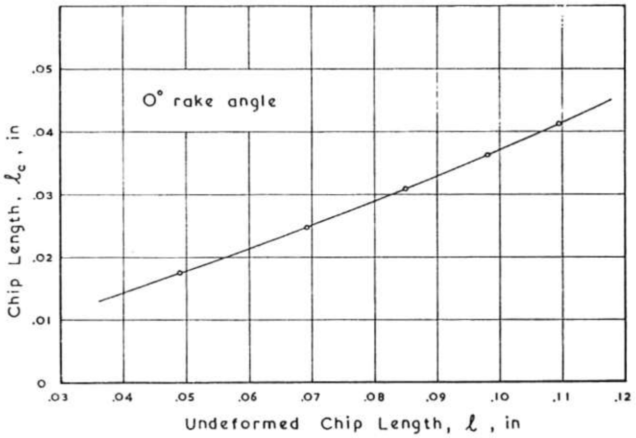

Fig. 18 Variation of Chip Length With Undeformed Chip Length

Merchant's plasticity equation, which follows:

$$2\,(\text{shear angle}) + (\text{friction angle}) - (\text{rake angle}) = C$$

has been found by many investigators to agree with experimental results to a first degree of approximation. That this is also true in micromilling operations can be seen from Table 4 of this discussion. Tool forces given by the authors, Table 1, are used to obtain the pertinent data for Table 4.

It is evident that, in spite of the pronounced size effect which exists in the formation of the extremely thin chips during the micromilling operation, the agreement between experimental results and Merchant's plasticity equation is good. However, it

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TABLE 4 MACHINING CONSTANT IN MICROMILLING

(For 0-deg-rake-angle cutter)

Tooth depth of cut, t, microinches	Tool forces[a]		Friction angle, $\tan^{-1} \mu$, deg	Chip-length ratio,[b] r_L	Shear angle, ϕ, deg	Machining constant C, deg	
	F_H, lb	F_V, lb				When r_L varies with t (writer's)	When r_L is taken as a constant (authors')
200	90.5	93.6	45.8	0.349	19.2	84.2	85.6
250	105.7	99.7	43.3	0.362	19.9	83.1	83.1
300	119.2	105.0	41.4	0.374	20.5	82.4	81.2
350	130.8	109.6	39.9	0.387	21.2	82.3	79.7
400	140.5	113.3	38.9	0.400	21.8	82.5	78.7
450	148.7	116.6	38.1	0.413	22.5	83.1	77.9
500	155.7	118.8	37.3	0.426	23.1	83.5	76.1

[a] Taken from Table 1 of the paper.

[b] Values taken from a plot of r_L against t (using data given in Table 3 of this discussion).

is recognized that there is doubt with regard to the validity of some assumptions used in the formulation of the plasticity equation. A detailed discussion on this latter subject will be given in a forthcoming paper.[11]

The fact that r_L is not a constant with respect to changes in t indicates the need of some modifications of the authors' conclusions.

NOTE: The combined Authors' Closure to the discussions on the following papers appears on pages 83–86 of this issue of the Transactions: (*A*) "Forces in Dry Surfaces Grinding"; (*B*) "The Size Effect in Metal Cutting"; (*C*) "Surfaces Temperatures in Grinding." In the closure the papers will be referred to as (*A*), (*B*), (*C*).

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[11] "Thermo-Physical Aspects of Metal Cutting," by B. T. Chao, K. J. Trigger, and L. B. Zylstra, presented at the Annual Meeting, Atlantic City, N. J., November 25–30, 1951, of The American Society of Mechanical Engineers, Paper No. 51—A-41.