Thermal and Dynamical Evolution of Intergalactic Clouds

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Assuming that the intergalactic clouds with the mass $10^7 \sim 10^{13} M_\odot$ were born at the epoch $z=5 \sim 10$, their thermal and dynamical evolution is numerically explored. The clouds with the mass greater than $10^9 M_\odot$ had collapsed by the present epoch, and their evolution weakly depends upon the mass. We may assign these collapsed clouds to quasars and galaxies. The clouds with the mass smaller than $10^9 M_\odot$ can keep expansion, and their evolution is characterized as pressure confined clouds. Physical quantities of these expanding clouds well correspond to the Lyman alpha absorbers of QSO lights. Some relevant problems to the present calculations such as the formation of a massive halo and the mass spectrum of intergalactic clouds are discussed briefly.

§ 1. Introduction

The high-resolution spectral observations of QSOs have clarified that there are numerous absorption lines on the shorter wavelength side of the $L\alpha$ emission line. Sargent et al. investigated such QSO absorption lines and concluded that they consist of the single $L\alpha$ absorption lines and the metallic line systems, both of which are produced by cosmologically distributed intervening material not associated with QSOs. Especially the $L\alpha$ absorption clouds must be an intergalactic population which is not also associated with galaxies. Ostriker and Ikeuchi (henceforth, referred to as OI) examined the observational and theoretical constraints for the intergalactic clouds and determined the allowed ranges of the temperature and density of the intergalactic medium (hereafter, abbreviated as IGM). Further, Ikeuchi and Ostriker (henceforth, IO) investigated its thermal history. They also examined the dynamical evolution of the intergalactic clouds by the use of the virial theorem.

The scenario of the evolution of intergalactic clouds is as follows. At first, clouds are formed by the fragmentation of the dense shell which is formed in the vicinity of the shock front generated by an explosion or a negative density perturbation in the expanding medium. More strictly speaking, once the IGM was reheated up to a higher temperature than $10^6K$ and cooled down lower than $10^6K$, the thermal instability occurs and divides the IGM into two phases having different temperatures. The region with the lower temperature is compressed, so that it cannot follow the Hubble expansion and cools more rapidly than the ambient gas, leading to the formation of clouds. The following evolution depends on the cloud mass and initial expanding velocity. Roughly speaking, however, it is substantially determined by the mass. The cloud with the mass $\geq 10^{9-10} M_\odot$ is to collapse at the epoch $z=0 \sim 4$ and that with the mass $\leq 10^9 M_\odot$ continues to expand keeping the pressure balance with the ambient IGM. Physical properties of such expanding clouds are closely similar to those of the $L\alpha$ absorption clouds. The collapsed clouds may be thought to evolve to QSOs, galaxies and blue compact galaxies, according to the epoch of collapse.

When the virial theorem is used (IO), the structure of clouds is inevitably simplified. Therefore, such clouds as on the border of expansion or collapse may not be examined.
correctly. In the present paper, we explore in detail the thermal and dynamical evolution of the intergalactic clouds, taking their structure into account, with the spherically symmetric model. Our principal results turn out to be in expected correspondence to those of the simple model by IO.

In §2, the cooling and heating mechanisms, the basic equations, and the initial conditions are given. In §3, the numerical results for collapsed and expanding clouds are presented. Here it is also shown that a cloud may be crushed by the external pressure in the case of the large bulk heating; it is violently compressed by the generated inward shock wave. In §4, our numerical results are compared with those of the simple treatment by IO and the observational data of the La absorption clouds and an HI cloud. The possibility of the formation of a massive halo is also discussed by means of the results of collapsed clouds.

§ 2. Fundamental data

2.1. Cooling mechanisms

The intergalactic cloud and the ambient IGM are assumed to have the primordial abundance, namely, H and He in the ratio of 9 to 1 in number. Cooling processes examined here are as follows (in CGS units):

(i) Compton cooling

\[ L^c = 5.4 \times 10^{-38} (1 + z)^4 n_e T, \quad \text{erg cm}^{-3} \text{s}^{-1} \]  \hspace{1cm} (2.1)

where \( z \) is the cosmological redshift, \( T \) is the gas temperature, and \( n_e \) is the number density of electrons.

The following radiative cooling processes are summarized in the paper by Sherman.\(^{11}\) The number density of species \( i \) is determined by the ionization equilibria,\(^{3}\) namely,

\[ (\Gamma^i + \Gamma^{ei}) n_i = a_{i+1} n_{i+1} n_e, \]  \hspace{1cm} (2.2)

where \( \Gamma^i \), \( \Gamma^{ei} \) and \( a_i \) are the rate coefficients of photo-ionization, electron impact, and radiative recombination, respectively,\(^{11,12}\) with the combination \((i, i+1) = (\text{HI, HII}), (\text{HeI, HeII}), \) or \((\text{HeII, HeIII}). \) The following numerical values are adopted from the paper by Sherman.\(^{11}\)

(ii) Thermal bremsstrahlung

\[ L^{TB} = 1.8 \times 10^{-27} n_e T^{1/2} (n_{\text{HII}} + n_{\text{HeII}} + 4 n_{\text{HeIII}}). \]  \hspace{1cm} (2.3)

(iii) Line emission

\[ L^i(i) = a^i(i) n_e n_i T^{-1/2} \exp(-b^i(i)/T), \]  \hspace{1cm} (2.4)

where \((a^i(i), b^i(i))\) are \((3.4 \times 10^{-16}, 1.18 \times 10^5), (2.3 \times 10^{-16}, 2.47 \times 10^6), \) and \((3.4 \times 10^{-16}, 4.74 \times 10^8)\), for \( i = \text{HI, HeI, and HeII, respectively.} \)

(iv) Collisional ionization

\[ L^i(i) = a^i(i) n_e n_i T^{1/2} \exp(-b^i(i)/T)(1 - \exp(-b^i(i)/T)) \]  \hspace{1cm} (2.5)

with \((a^i(\text{HI}), b^i(\text{HI})) = (2.6 \times 10^{-21}, 1.58 \times 10^8), (a^i(\text{HeI}), b^i(\text{HeI})) = (2.8 \times 10^{-21}, 2.85 \times 10^8), \)
and \((a'(\text{HeII}), b'(\text{HeII}))=(6.4\times10^{-22}, 6.31\times10^{5})\).

(v) Radiative recombination

\[
L^{rr}(i)=a^{rr}(i)n_{e}n_{i}T^{1/2}f(i),
\]

where \(a^{rr}(\text{HII})=2.91\times10^{-27}\), \(a^{rr}(\text{HeII})=2.87\times10^{-26}\), \(a^{rr}(\text{HeIII})=1.16\times10^{-28}\), \(f(\text{HII})=F(1.6\times10^{5}/T)\), \(f(\text{HeII})=T^{-0.17}\), and \(f(\text{HeIII})=F(6.4\times10^{5}/T)\) using the function \(F(Y)\) defined as

\[
F(Y)=\begin{cases}
1/2(1.7+\ln Y+1/6Y) & \text{if } Y\geq0.5,
Y(-0.3-1.2\ln Y)+Y^{2}(0.5-\ln Y) \\
+Y^{3}(0.47-0.5\ln Y)+Y^{4}(0.21-0.17\ln Y) & \text{if } Y<0.5.
\end{cases}
\]

(vi) Dielectronic recombination

\[
L^{dr}(i)=a^{dr}(i)T^{-3/2}n_{e}n_{i}\exp(-b^{dr}(i)/T)
\]

with \((a^{dr}(\text{HeII}), b^{dr}(\text{HeII}))=(2.0\times10^{-13}, 4.72\times10^{6})\).

2.2. Heating mechanisms

UV-photons from QSOs are taken as a heating source. This UV heating is expressed as (in CGS units)\(^{12}\)

\[
H(i)=h(i)J_{0}n_{i} \quad (\text{erg cm}^{-3}\text{s}^{-1})
\]

with \(h(\text{HII})=2.0\times10^{-3}\), \(h(\text{HeI})=5.5\times10^{-3}\), and \(h(\text{HeII})=5.0\times10^{-4}\). If the energy flux at the Lyman limit \(J_{0}\) is normalized as \(10^{-21}\) erg s\(^{-1}\) cm\(^{-2}\) Hz\(^{-1}\) at \(z=2.5\),\(^{2,12}\) then it changes as

\[
J_{0}=10^{-21}J_{21}((1+z)/3.5)^{4}.
\]

In some cases we adopt the bulk heating as another heating source as taken by IO. The bulk heating is thought to be the energy ejection from active galaxies including QSOs, and its rate is written in terms of the number distribution of QSOs\(^{13}\) as

\[
H_{SN}=h_{SN}(0)(1+z)^{3}\exp(\beta(t_{0}-t)/t_{0}), \quad (\text{erg cm}^{-3}\text{s}^{-1})
\]

where \(t\) is the age of the universe at the redshift \(z\) (\(t_{0}\) is the present value) and \(\beta\) is a constant of the order of 10. The value of \(h_{SN}(0)\) can be estimated as follows. If we use the efficiency of the energy liberation to the rest-mass energy, \(\epsilon=10^{-5}\), the fraction used for the bulk heating, \(f=0.07f_{0.07}\), the smoothed mass density of active galaxies, \(\rho_{0}(0)=10^{-33}\rho_{0}(0)_{-33}\text{g cm}^{-3}\), and the duration of activity on an average, \(\tau=10^{17}\tau_{17}\) sec, then we have

\[
h_{SN}(0)=6\times10^{-36}\epsilon_{-5}f_{0.07}\rho_{0}(0)_{-33}\tau_{17}^{-1}. \quad (\text{erg cm}^{-3}\text{s}^{-1})
\]

Absorbing the above to one parameter \(\alpha\), we use

\[
h_{SN}(0)=6\times10^{-36}\alpha. \quad (\text{erg cm}^{-3}\text{s}^{-1})
\]

2.3. Basic equations

Under the spherical symmetry, basic equations for gas motion are
\[ u = \frac{dr}{dt} , \]  \hspace{1cm} (2.13)

\[ \frac{du}{dt} = \frac{1}{\rho} \frac{\partial p}{\partial r} - \frac{GM_r}{r^2} , \]  \hspace{1cm} (2.14)

\[ \frac{dM_r}{dr} = 4\pi r^2 \rho , \]  \hspace{1cm} (2.15)

\[ \frac{d}{dt} \ln \left( \frac{T^{11(\gamma - 1)}}{\rho} \right) = \frac{H - L}{P} , \]  \hspace{1cm} (2.16)

\[ \frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho u) = 0 , \]  \hspace{1cm} (2.17)

\[ P = \frac{k_{\nu} o T}{\mu_H} . \]  \hspace{1cm} (2.18)

Here, \( H \) and \( L \) are the sum of heating functions and that of cooling functions, respectively. The adiabatic index \( \gamma \) is taken as 5/3, and \( \mu_H \) is the average mass of a particle. The other symbols in the above expressions have their usual meanings. To integrate these combined equations numerically, we used the second order difference scheme by MacCormack.\(^{14}\) The spatial meshes were applied to shells having the same thickness.

2.4. Initial conditions

In this paper we consider the model of the universe with the deceleration parameter \( q_0 = 0 \) or 0.5. According to the analysis by IO, it may be expected that the gas becomes thermally unstable when it cools to \( 10^5 \sim 10^6 \)K after heated up to a higher temperature than \( 10^6 \)K.\(^{19}\) Keeping the pressure balance, the thermal instability enforces the IGM to separate into two phases of the lower (\( \sim 5 \times 10^4 \)K) and the higher temperature (\( \sim 2 \times 10^6 \)K). The former is regarded as the initial stage of an intergalactic cloud (suffix \( C \)) and the latter is the ambient IGM (suffix \( I \)). The growth time of this instability is as short as one-tenth of the Hubble time.\(^{16,17}\) Moreover, the two temperatures above do not sensitively depend on the epoch of the thermal instability. Therefore, we take the initial conditions as \( T_{C} = 5 \times 10^4 \)K, \( T_{I} = 2 \times 10^6 \)K and assume the pressure balance between them, \( n_{C}T_{C} = n_{I}T_{I} \). The mass scale of the clouds expected from such instability is \( 10^{-4} M_{I} \sim M_{J} \) with the Jeans mass \( M_{J} \).\(^{39} \) If the thermal instability takes place in the IGM of the temperature \( T_i \) at the epoch \( z_i \), the Jeans mass is written as

\[ M_{J} = 1.3 \times 10^4 T_i^2 (1+z_i)^{-2/3} , \quad (M_6) \]  \hspace{1cm} (2.19)

where we assume the relation \( \bar{P} = n_i T_i = 10^{2}(1+z_i)^6 \text{ cm}^{-3}\text{K} \) which OI have estimated. Thus we can expect the intergalactic clouds with the mass \( 10^{7} \leq M_C \leq 10^{13} M_6 \) to be formed for \( z_i = 6 \sim 9 \) and \( 10^{5} \leq T_i \leq 10^{6} \)K. The cloud and the ambient IGM are assumed to be initially homogeneous. The expanding velocity is assumed to be \( sH_i r_i \) for the cloud and \( H_i r_i \) for the ambient IGM using the Hubble constant \( H_i \) and the radius at the initial epoch, where \( s \) is \( 0 \leq s \leq 5/3 \).\(^{39} \) Practically, the initial velocity of the boundary (\( R_{C} \leq r_i \leq 1.1 R_{C} \)) was determined by the interpolation with a linear function in terms of velocities at two ends.

The above initial conditions of the clouds are not fully founded by the numerical calculations of the thermal instability of a heated intergalactic gas. However, as is seen
Table I. Parameters for calculations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0$</td>
<td>Present Hubble constant ($=100$ km s$^{-1}$Mpc$^{-1}$)</td>
</tr>
<tr>
<td>$q_0$</td>
<td>Deceleration parameter ($=0.5$ or 0)</td>
</tr>
<tr>
<td>$z_I$</td>
<td>Initial epoch</td>
</tr>
<tr>
<td>$M_c$</td>
<td>Mass of an intergalactic cloud</td>
</tr>
<tr>
<td>$s$</td>
<td>Initial ratio of expanding velocity of a cloud to the Hubble velocity ($=u_c/H(t)$)</td>
</tr>
<tr>
<td>$T_c$</td>
<td>Initial temperature of a cloud ($=5\times10^4$ K)</td>
</tr>
<tr>
<td>$T_H$</td>
<td>Initial temperature of the ambient IGM ($=2\times10^4$ K)</td>
</tr>
<tr>
<td>$J_{11}$</td>
<td>UV heating rate ($=1$ in this paper)</td>
</tr>
<tr>
<td>$\alpha, \beta$</td>
<td>Bulk heating rate</td>
</tr>
</tbody>
</table>

in the later, the initial conditions do not much affect the following evolution if the cloud mass is large enough ($M_c \geq 10^{10} M_\odot$, the collapsed case), or if small enough ($M_c < 10^9 M_\odot$, the expanding case). The clouds between the collapsed and expanding ones, $10^8 \sim 10^{10} M_\odot$, will depend sensitively upon the initial condition, especially, upon the initial velocity and density distribution. Therefore, it is important to examine the evolution of the IGM from the thermal instability to the formation of clouds. Further, the bigger clouds with $M_c > 10^{11} M_\odot$ may not be formed within the Hubble time at the epoch $z_I > 6$. We start the calculations on the assumption that such bigger clouds are formed. Our model is one of the simplified extentsions of the suggestion by Rees and Ostriker.\cite{17} All the parameters are summarized in Table I.

§ 3. Numerical results

The conclusion of our numerical results is that the following evolution of intergalactic clouds after the thermal instability at the epoch $z_I$ depends substantially on their masses, and the critical mass is $M_{\text{crit}} = 10^9\sim10^4 M_\odot$, which weakly depends on the initial expanding velocity, but hardly on the epoch of the cloud formation $z_I$. That is to say, the clouds with $M_c \geq M_{\text{crit}}$ inevitably collapse at the epoch $0 \leq z \leq 4$, while those with $M_c \leq M_{\text{crit}}$ keep expanding till the present epoch. The cloud which collapses at the larger redshift becomes a more compact system as a whole. The critical mass is shown in Fig. 1, and it increases with increasing $s$ (or decreasing $z_c$).

The representative results of collapsed clouds and expanding ones are summarized,
Table II. Summary of collapsed clouds. The mass $M_c$, initial expansion parameter $s$ ($u_{ci} = sH_0r_i$), bulk heating parameter $(\alpha, \beta)$, and initial epoch $z_i$ are varied for each model with the deceleration parameter $q_0 = 0.5$. The values $z_e$, $T_{cr}$, $n_{cr}, R_{cr}$, and $R_{coll}$ are the final ones of the present calculations for the epoch of collapse, temperature and number density at the center, total radius, and radius of the collapsed region, respectively. Initial density parameter $\Omega_1$ and analytical estimate of the collapsed epoch $z_{ca}$ are also added.

<table>
<thead>
<tr>
<th>Model Name</th>
<th>$M_c(M_\odot)$</th>
<th>$s$</th>
<th>$(\alpha, \beta)$</th>
<th>$\Omega_1$</th>
<th>$z_{ca}$</th>
<th>$z_i$</th>
<th>$z_e$</th>
<th>$T_{cr}(10^4K)$</th>
<th>$n_{cr}(cm^{-3})$</th>
<th>$R_{cr}(kpc)$</th>
<th>$R_{coll}(kpc)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>II-1</td>
<td>$10^{12}$</td>
<td>0.5</td>
<td>0</td>
<td>1.1</td>
<td>1.1</td>
<td>6.0</td>
<td>1.1</td>
<td>0.86</td>
<td>0.91</td>
<td>170</td>
<td>35</td>
</tr>
<tr>
<td>II-2</td>
<td>$10^{11}$</td>
<td>0.5</td>
<td>0</td>
<td>1.1</td>
<td>1.1</td>
<td>6.0</td>
<td>1.1</td>
<td>0.71</td>
<td>1.41</td>
<td>41</td>
<td>130</td>
</tr>
<tr>
<td>II-3</td>
<td>$10^{10}$</td>
<td>0.5</td>
<td>0</td>
<td>1.1</td>
<td>1.1</td>
<td>6.0</td>
<td>1.1</td>
<td>0.63</td>
<td>$1.8 \times 10^4$</td>
<td>97</td>
<td>4.4</td>
</tr>
<tr>
<td>II-4</td>
<td>$10^{12}$</td>
<td>0.5</td>
<td>0</td>
<td>2.3</td>
<td>3.3</td>
<td>9.0</td>
<td>3.6</td>
<td>1.2</td>
<td>0.13</td>
<td>62</td>
<td>37</td>
</tr>
<tr>
<td>II-5</td>
<td>$10^{12}$</td>
<td>0</td>
<td>0</td>
<td>1.1</td>
<td>2.2</td>
<td>6.0</td>
<td>2.3</td>
<td>0.80</td>
<td>0.40</td>
<td>63</td>
<td>28</td>
</tr>
<tr>
<td>II-6</td>
<td>$10^{12}$</td>
<td>0</td>
<td>0</td>
<td>1.9</td>
<td>3.6</td>
<td>8.0</td>
<td>3.8</td>
<td>0.98</td>
<td>0.47</td>
<td>39</td>
<td>25</td>
</tr>
<tr>
<td>II-7</td>
<td>$10^{11}$</td>
<td>0.8</td>
<td>0</td>
<td>1.1</td>
<td>0.1</td>
<td>6.0</td>
<td>0.1</td>
<td>0.75</td>
<td>0.58</td>
<td>430</td>
<td>9.8</td>
</tr>
<tr>
<td>II-8</td>
<td>$10^{12}$</td>
<td>0.8</td>
<td>0</td>
<td>1.1</td>
<td>0.1</td>
<td>6.0</td>
<td>0.1</td>
<td>0.70</td>
<td>0.50</td>
<td>16</td>
<td>15</td>
</tr>
<tr>
<td>II-9</td>
<td>$10^{12}$</td>
<td>0.5</td>
<td>0</td>
<td>1.1</td>
<td>3.1</td>
<td>6.0</td>
<td>3.3</td>
<td>1.2</td>
<td>$5.0 \times 10^{-2}$</td>
<td>70</td>
<td>43</td>
</tr>
<tr>
<td>II-10</td>
<td>$10^{12}$</td>
<td>0.5</td>
<td>(1, 10)</td>
<td>1.1</td>
<td>1.1</td>
<td>6.0</td>
<td>1.4</td>
<td>1.2</td>
<td>$4.3 \times 10^{-3}$</td>
<td>96</td>
<td>95</td>
</tr>
</tbody>
</table>

*) $q_0 = 0$ universe.

Table III. Summary of expanding clouds. The meanings of parameters are the same as Table II. The value $z_f$ is final epoch of calculations.

<table>
<thead>
<tr>
<th>Model Name</th>
<th>$M_c(M_\odot)$</th>
<th>$s$</th>
<th>$(\alpha, \beta)$</th>
<th>$z_i$</th>
<th>$z_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>III-1</td>
<td>$10^8$</td>
<td>0.5</td>
<td>0</td>
<td>6.0</td>
<td>0</td>
</tr>
<tr>
<td>III-2</td>
<td>$10^8$</td>
<td>0.5</td>
<td>0</td>
<td>6.0</td>
<td>0</td>
</tr>
<tr>
<td>III-3</td>
<td>$10^7$</td>
<td>0.5</td>
<td>0</td>
<td>6.0</td>
<td>0</td>
</tr>
<tr>
<td>III-4</td>
<td>$10^9$</td>
<td>0.5</td>
<td>0</td>
<td>7.0</td>
<td>0</td>
</tr>
<tr>
<td>III-5</td>
<td>$10^9$</td>
<td>0.5</td>
<td>0</td>
<td>5.0</td>
<td>0</td>
</tr>
<tr>
<td>III-6</td>
<td>$10^9$</td>
<td>1.0</td>
<td>0</td>
<td>6.0</td>
<td>0</td>
</tr>
<tr>
<td>III-7</td>
<td>$10^9$</td>
<td>0.5</td>
<td>0</td>
<td>6.0</td>
<td>0</td>
</tr>
<tr>
<td>III-8</td>
<td>$10^9$</td>
<td>0.5</td>
<td>(0.1, 10)</td>
<td>6.0</td>
<td>0</td>
</tr>
<tr>
<td>III-9</td>
<td>$10^9$</td>
<td>0.5</td>
<td>(1, 10)</td>
<td>6.0</td>
<td>3.8</td>
</tr>
</tbody>
</table>

*) $q_0 = 0$ universe.

with the adopted parameters in Tables II and III, respectively. The resultant characteristics of collapsed and expanding clouds are described separately below.

3.1. Collapsed clouds

(a) Epoch of collapse

The cloud with $M_c \geq M_{crit}$ collapses without having the pressure gradient around its center. This result is confirmed by the following discussion. Using $T = 10^4 T_5 K$, $n = 10^{-3} n_3 cm^{-3}$, $M_c = 10^{10} M_{10} M_\odot$, the isothermal sound velocity $C_s$, and the initial radius $R_{ci}$, the dynamical time is estimated as

$$\tau_d = R_{ci}/C_s = 3.57 \times 10^9 M_{10}^{1/3} n_{3}^{-1/3} T_5^{-1/2} \text{ (yr)}$$

(3.1)
For instance, in the case of the model II-3 in Table II it has passed \(1.73 \times 10^9\) yr before the cloud collapses, while \(\tau_d = 1.69 \times 10^9\) yr from (3.1). Therefore, it is reasonably anticipated that the cloud having \(M_c \geq 10^{10}M_\odot\) will collapse before the influence of the boundary reaches its center. When the pressure gradient is neglected, the evolution of a collapsed cloud is dominated only by its self-gravity. The epoch of collapse can be analytically calculated as follows. The change of the radius at a fixed \(M_r\) (the mass within the radius \(r\)) is described as \(d^2r/dt^2 = -GM/r^2\), and this is integrated to \(u^2 = (\sigma H/\rho r)^2 - 2GM/(1/r - 1/r_0)\). Let \(r_m\) denote the turn-around radius, then we have \(r/r_m = 1 - s^2/\Omega_i\), where \(\Omega_i\) is the ratio of the initial cloud density \(\rho_{ci}\) to the critical one, \(\rho_{ci}/\rho_{crit,i} = 8\pi G\rho_{ci}/3H_i^2\). Thus the condition \(\Omega_i > s^2\) is necessary for the cloud to collapse. On this condition, \(\tau_c\), the time required to ultimately collapse, is obtained by integrating the equation of energy conservation as

\[
\tau_c = H_i^{-1}\Omega_i^2(\Omega_i - s^2)^{-3/2}(\pi/2 + \tan^{-1}(s^2/(\Omega_i - s^2)))^{1/2} + ((1 - s^2/\Omega_i)s^2\Omega_i)^{1/2}. \quad (3.2)
\]

\(\Omega_i\) is evaluated below by using the initial critical number density \(n_{crit,i} = 8.64 \times 10^{-6}h^2(1 + z_i)^3(1 + 2q_0z_i)\) cm\(^{-3}\) with \(H_0 = 100h\) km s\(^{-1}\)Mpc\(^{-1}\) and the initial number density of the cloud as \(n_{ci} = 10^{-2}(1 + z_i)^5/T_{ci}\),

\[
\Omega_i = n_{ci}/n_{crit,i} = 1.16 \times 10^{3/2}h^{-2}T_{ci}^{-1}(1 + z_i)^3(1 + 2q_0z_i). \quad (3.3)
\]

On the other hand, the epoch of collapse \(z_c\) is connected with \(\tau_c\) and \(z_i\) by the relation

\[
1 + z_c = ((1 + z_i)^{-1 + q_0} + (1 + q_0)H_0\tau_c)^{-1/(1 + q_0)}. \quad (3.4)
\]

From Eqs. (3.2)\textendash}(3.4), we can obtain the analytically estimated value \(z_{ca}\) of the epoch of collapse when we assign the parameters, \(q_0\), \(h(=1\) in our analysis), \(z_i\), \(s\) and \(T_{ci}\). In Table II, we added \(\Omega_i\) and \(z_{ca}\), where \(T_{ci}\) is fixed as \(5 \times 10^4\)K. For various values of \(T_{ci}\),

![Fig. 2](https://example.com/f2.png) **Fig. 2.** The epoch of collapse for various \(T_{ci}\) and \(z_i\) for the \(q_0 = 0.5\) case. Solid and dashed lines are analytical ones with the fixed \(T_{ci}\) and \(\Omega_i\), respectively. The results not specified by \(s\) are those of \(s = 0\). Calculations are done for the cases as \((a, \beta) = (0(\bigodot), (0.1, 10) (\triangle), and (1, 10) (\bigodot)\), respectively. Their analytical values are pointed by bars.

![Fig. 3](https://example.com/f3.png) **Fig. 3.** The relation among \(z_i\), \(z_c\), and \(s\) for \(q_0 = 0.5\). Solid lines are analytic estimates. Asterisks (*) represent numerical results with each model name of Table II.
Fig. 4. Evolution of a typical collapsed cloud (the model II-1 in Table II). $J$ represents the mesh number as the cloud for $J=1-100$ and the ambient IGM for the rest $J=101-150$. (a) Radius and (b) velocity are plotted to $(1+z)$. Dotted lines represent negative velocity. (c) Radial distributions of density and (d) temperature are plotted to radius for each epoch $z$. (e) Neutral hydrogen column density along the line of sight passing the $J$th mesh and average temperature, density, and fraction of neutral hydrogens $x_{ni}(=n_{ni}/n)$ are plotted to $(1+z)$. 
the appreciable correlation between $z_{ca}$ and $z_c$ from numerical calculations is shown in Fig. 2. For $T_{ci}=5 \times 10^4$K, the dependence of the epoch of collapse upon $z_i$ and $s$ is shown in Fig. 3. From Figs. 3 and 5, we can see that the clouds will collapse at $0 \leq z_c \leq 4$ if $6 \leq z_i \leq 9$ and $0 \leq s \leq 1.0$. In Fig. 4, the evolution of a typical collapsed cloud with $M_c = 10^{12} M_\odot$ and $s = 0.5$ is shown by several characteristic physical quantities. At the stage $z_i = 6$ to $z = 2.5$, the cloud expansion is decelerated by its self-gravity. Thereafter, the cloud changes to collapse, and at $z_c \approx 1.1$ the cloud density increases indefinitely.

Here we shall summarize the dependence of the epoch of collapse upon each parameter.

(i) $M_c$, $z_i$, $s$, $T_{ci}$ (or $Q_i$): With increasing $z_i$, or decreasing $s$ or $T_{ci}$ (i.e., increasing $Q_i$), the epoch of collapse becomes earlier while it does not depend upon $M_c$.

(ii) $q_0$: When the model II-1 is compared with the model II-9 in Table II, we find that $z_c$ is larger in the universe of $q_0 = 0$ than $q_0 = 0.5$. This comes from the fact that the universe of $q_0 = 0$ has the smaller Hubble velocity and the longer age than those of $q_0 = 0.5$ for the same $z$.

(iii) $(a, \beta)$: As is shown by the results of $(a, \beta) = 0$ in Fig. 2 and the model II-10 compared with the model II-1 in Table II, the bulk heating has little effect on $z_c$ for such a collapsing cloud. Since the bulk heating does compensate the adiabatic decrease of the pressure of the IGM, a dense region is formed near the boundary of the cloud (for example, at $z_c$ for the model II-10), but this perturbation never propagates onto the center within a free-fall time.

From the above results, we can conclude that when the IGM was reheated and underwent the thermal instability at $z \leq 9$, the clouds with $M_c \geq M_{crit}$ almost always collapsed at $0 \leq z \leq 4$ depending upon the value of $s$.

(b) Thermal history

As is seen in Fig. 4(e), the temperature of the cloud falls quickly to the equilibrium temperature $T_{eq}$ determined from $H = L$, $\sim 2.4 \times 10^4$K, and thereafter it roughly keeps constant with a slight decrease. With proceeding the collapse, its temperature

![Fig. 5. Evolution of the Lagrangian coordinates $r(i)$]

![Table IV. The relation among collapsed mass $M_{coll}$, $M_c$, and $s$. Other parameters are fixed as $q_0 = 0.5$, $z_i = 6$, and $(a, \beta) = 0$. The fraction $\eta$ and the index $p$ are defined as $\eta = (M_c - M_{coll})/M_c$ and $p = -(d\ln \eta/d \ln r)$, respectively.]

<table>
<thead>
<tr>
<th>$M_c(M_\odot)$</th>
<th>$s$</th>
<th>$M_{coll}(M_\odot)$</th>
<th>$\eta(%)$</th>
<th>$p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{10}$</td>
<td>0</td>
<td>$4.2 \times 10^5$</td>
<td>58</td>
<td>2.9</td>
</tr>
<tr>
<td>$10^{10}$</td>
<td>0.5</td>
<td>$9.0 \times 10^5$</td>
<td>91</td>
<td>2.4</td>
</tr>
<tr>
<td>$10^{10}$</td>
<td>0.5</td>
<td>$5.6 \times 10^9$</td>
<td>44</td>
<td>3.0</td>
</tr>
<tr>
<td>$10^{10}$</td>
<td>0.8</td>
<td>$2.8 \times 10^9$</td>
<td>72</td>
<td>2.9</td>
</tr>
<tr>
<td>$10^{12}$</td>
<td>0.5</td>
<td>$7.9 \times 10^{11}$</td>
<td>21</td>
<td>3.9</td>
</tr>
</tbody>
</table>
drops suddenly to less than $10^4 K$ because of the rapid increase of the cooling rate. The neutral hydrogen column density $N_H$ increases mainly by the increase of the neutral fraction and in part by a little change of the density before the collapse.

(c) Motion of boundary

According as the cloud contracts, its pressure becomes far greater than that of the ambient IGM. Therefore, the vicinity of the boundary between the cloud and the ambient IGM is exposed to an outward force. Finally, as is seen in Fig. 4(c), the density distribution exhibits a steep gradient. The ratio of gases left uncollapsed to the total cloud mass $M_C$ increases with decreasing the cloud mass. Numerical results for $\eta = (M_C - M_{\text{coll}})/M_C$ are given in Fig. 6, $M_{\text{coll}}$ being the mass of finally collapsed gases. Typical results are tabulated in Table IV. Actually, as is seen in Fig. 6, $\eta$ increases with decreasing $M_C$ and with increasing $s$. For a fixed $s$, $\eta$ may be fitted in terms of a power law of $M_C$ like $\eta = \eta_0 M_0^{s-k}$, with $M_0 = M_C/10^{10} M_\odot$. According to the present results, the values of $(\eta, k)$ are estimated as $(0.55, 0.38)$, $(0.90, 0.32)$ and $(1.32, 0.27)$ for $s = 0$, 0.5 and 0.8, respectively. The critical mass $M_{\text{crit}}$ is estimated from the condition $\eta = 1$ as $M_{\text{crit}} = 2.1 \times 10^9 M_\odot (s = 0)$, $7.2 \times 10^9 M_\odot (s = 0.5)$ and $2.8 \times 10^{10} M_\odot (s = 0.8)$. The fact that the motion of the boundary is sensitive to the value of $s$ is seen in Fig. 7 for the cloud with the mass $10^{11} M_\odot$. The mass $M_{\text{coll}}$ is considered as that of a bound system formed in the core, and the rest $M_C - M_{\text{coll}}$ is left surrounding the bound system, having over ten times higher density than the ambient IGM. This result is discussed in §4 in relation to the formation of a massive halo.

3.2. Expanding clouds

(a) Expansion and pressure equilibrium

The models III-1~3 in Table III have the same parameters as those of the models II-1~3 except for the mass $M_C$. However the evolution with $M_C \leq M_{\text{crit}} (= 10^{9.1} M_\odot)$ is distinctly different from that with $M_C \geq M_{\text{crit}}$. The evolution of a typical expanding cloud is shown by several characteristic physical quantities in Fig. 8. The pressure equilibrium
is restored in a comparatively short time, even if it is once broken. Using Eq. (3.1), the dynamical time $\tau_d$ is $3.8 \times 10^8$ yr in this case ($M_{10}=10^{-3}$, $n_3 \sim 3$, and $T_4 \sim 2$) which corresponds to the time passed until $z \sim 4$ from $z_4=6$. In the central part with the mass $M_r(r=0.1 \; R_c)$, we have $K_r/M_r=3u^2/10=3.6 \times 10^9$ erg/g, $W_r/M_r=-3GM_r/5r=-3.2$
\( \times 10^{16}\text{erg/g}, \) and \( U_r/M_r=3C_s^2/2=2.8 \times 10^{12}\text{erg/g} \) at the epoch \( z=4, \) where \( K_r, \) \( W_r, \) and \( U_r \) are the kinetic, the gravitational, and the thermal energy within the radius \( r, \) respectively. Thus we obtain the total energy \( E_r=K_r+W_r+U_r \geq 0, \) so that this cloud does not collapse but expands. We note that this is due to the energy input from UV-flux. After all, we can regard the cloud with \( M_c \leq M_{\text{crit}} \) as a pressure confined system.

(b) Dependence on parameters

(i) \( M_c: \) The cloud mass plays a significant role in attaining the pressure equilibrium, since the larger \( M_c \) requires more time before the cloud is subject to the overall influence of the ambient IGM. In Fig. 9, the evolution of expanding clouds with various masses \( M_c \) is plotted as well as a collapsed case with \( M_c=10^{10}M_\odot \) for comparison. Once the pressure equilibrium with the ambient IGM undergoing the adiabatic expansion is realized, the cloud radius varies in proportion to \( (1+z)^{-4/3} \) on account of the constant temperature, \( T_{\text{eq}} \sim 2 \times 10^4\text{K} \) (the broken line in Fig. 9). In fact, as is seen in Fig. 9, the cloud with a smaller mass undergoes the isothermal expansion at an earlier epoch.

(ii) \( z_t \) and \( s: \) Since we choose the initial density of the cloud as \( n_c \propto (1+z_t)^5, \) \( R_c \) is finally independent of \( z_t \) if the expansion is isothermal with keeping \( P_c=P_t. \) This fact is shown in Fig. 10, and it is confirmed that other physical quantities also do not depend upon \( z_t \) at \( z=1 \sim 2. \) Moreover we find that the final properties of expanding clouds are independent of \( s \) as is seen in Fig. 8(b), where the dashed lines represent the case \( s=1.0 \) (the model III-6 in Table III). It results from the large energy input by UV-photons.

(c) Effect of bulk heating and cloud crush

The results with \( (\alpha, \beta)=(0.1, \) 10) is also illustrated by means of the dash-dotted lines in Fig. 8(b), which shows a great increase of the temperature \( T_t. \) In contrast with this, \( T_c \) is raised slightly. A considerably large bulk heating brings about a drastic change of evolution, cloud crush. Because of a rapid increase of the ambient pressure, the critical mass for a pressure confined, isothermal cloud decreases (10). This results in a strong inward flow generating a shock wave. Finally, the cloud is crushed owing to the exceed-
Fig. 11. A crushed cloud (the model III-8). (a) Evolution of radius and (b) radial change of the Mach number \((u/c_s)\) are plotted.

.. figure:: image.png
   :alt: Diagram of cloud evolution

.. figure:: image2.png
   :alt: Mach number change

Evidently intense compression compared with the normal gravitational collapse as shown in Fig. 11. Such a crush may yield a very compact bound system with the mass less than \(10^6 M_\odot\). By contrast, the cloud with \(M_c \geq 10^{10} M_\odot\) is not crushed but collapses by its self-gravity.

§ 4. Discussion

4.1. Comparison with a simple treatment

The present numerical results agree well with those from the virial argument by IO, especially in an important result that whether a cloud collapses or expands depends sensitively upon the mass and its critical mass is \(M_{\text{crit}} = 10^8 \sim 10^{10} M_\odot\). Further, there has not existed definitive difference between the two numerical analyses also in other physical properties. But we must note that the numerical analysis in this paper has made clear that the epoch of collapse is essentially dominated by two factors, namely the formation epoch of intergalactic clouds and the initial expanding velocity. Moreover we can discuss the formation of a massive halo and other relevant problems in the following subsections.

4.2. Massive halo

As already described in §3.1 (c), the collapsed clouds will be ultimately composed of the collapsed core \((n \gtrsim 10^{-2} \text{cm}^{-3})\) and the adjacent low density envelope \((10^{-2} \gtrsim n \gtrsim 10^{-5} \text{cm}^{-3})\). The latter may be regarded as a galactic halo, which is left around a luminous body. The halo mass \(M_{\text{Halo}}\) is in the order of \(M_c - M_{\text{coll}}\), and the ratio of the halo mass to the total mass is summarized as \(\eta\) in Table IV and plotted to \(M_c\) in Fig. 6. From the results of the final radii, \(R_{\text{cf}}\) in Table II, we may say that more compact systems are formed at large \(z_c\), while the bound system has a more extended halo at smaller \(z_c\). As
Fig. 12. Density distribution of a collapsed cloud (II-3). The time for collapse at each radius and $M_r/M_C$ are also indicated. The power $p$ at the final stage $z_e=1.1$ of calculations is defined as $p = -(d \ln n/d \ln r)$.

4.3. Observations

(a) $La$ absorption clouds

Sargent et al. evaluated physical quantities of $La$ absorption clouds by the analysis of the $La$ absorption (1.62 $\leq z_{abs} \leq 3.28$) from the same viewpoint as OL. On the other hand, Oke and Korycansky have estimated the Doppler width of $La$ absorption lines as $\Delta U_C \sim 33$ km s$^{-1}$. These observed quantities are compared with our computations at $z = 2$ in Table V. As for the neutral hydrogen column density $N_{HI}$ and the Doppler width $\Delta U_C$, we adopted here as the average the values along the line of sight passing the Lagrangian coordinate $0.6R_{Ci}$ ($R_{Ci}$ being the initial cloud radius). In conclusion, as is seen in Table II, the extension of the halo is from some dozen to a hundred kiloparsecs. Particularly, for $M_C = 10^{12} M_\odot$ and $z_e = 0.1$ (the model II-8), the size of a halo amounts to some hundred kiloparsecs while the collapsed region is about 15 kpc in size and about $6 \times 10^{11} M_\odot$ in mass. The density distribution for a cloud with $10^{10} M_\odot$ at the final stage $z_e = 1.1$ is shown in Fig. 12, as well as the mass fraction $M_r/M_C$ and the collapse time $r$. The index $p$, defined as $n \propto r^{-p}$, is also plotted, which in this case lies between 2.0 and 2.9 with the average value 2.4 in the envelope. In other cases, the average values of $p$ are added in the last column of Table IV. Generally, $p$ tends to increase with increasing the mass. As a result, we have obtained more extended halo having a density distribution in close agreement with Hubble's law for massive elliptical galaxies, $\rho \propto r^{-3.19}$.

### Table V. Comparison of expanding clouds at $z = 2$ with the typical $La$ cloud. The model name corresponds to that of Table III.

<table>
<thead>
<tr>
<th>Model Name</th>
<th>$N_{HI}(\text{cm}^{-2})$</th>
<th>$T_e(10^4 \text{K})$</th>
<th>$T_r(10^4 \text{K})$</th>
<th>$R_C(\text{kpc})$</th>
<th>$n_C(10^{-4} \text{cm}^{-3})$</th>
<th>$\Delta U_C(\text{km/s})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>III-1</td>
<td>$9.5 \times 10^{14}$</td>
<td>2</td>
<td>2</td>
<td>23</td>
<td>0.6</td>
<td>45</td>
</tr>
<tr>
<td>III-2</td>
<td>$6.0 \times 10^{14}$</td>
<td>2</td>
<td>2</td>
<td>35</td>
<td>3</td>
<td>69</td>
</tr>
<tr>
<td>III-3</td>
<td>$4.8 \times 10^{14}$</td>
<td>2</td>
<td>2</td>
<td>10</td>
<td>0.7</td>
<td>18</td>
</tr>
<tr>
<td>III-4</td>
<td>$5.6 \times 10^{14}$</td>
<td>1.5</td>
<td>1</td>
<td>26</td>
<td>0.4</td>
<td>43</td>
</tr>
<tr>
<td>III-5</td>
<td>$2.1 \times 10^{15}$</td>
<td>2</td>
<td>4</td>
<td>20</td>
<td>1</td>
<td>40</td>
</tr>
<tr>
<td>III-6</td>
<td>$1.1 \times 10^{15}$</td>
<td>2</td>
<td>2</td>
<td>22</td>
<td>0.7</td>
<td>42</td>
</tr>
<tr>
<td>III-7</td>
<td>$2.4 \times 10^{15}$</td>
<td>2</td>
<td>3</td>
<td>18</td>
<td>1</td>
<td>16</td>
</tr>
<tr>
<td>III-8</td>
<td>$3.2 \times 10^{15}$</td>
<td>2</td>
<td>8</td>
<td>17</td>
<td>1</td>
<td>27</td>
</tr>
<tr>
<td>Typical $La$ cloud</td>
<td>$\sim 10^{14}$</td>
<td>$\leq 15$</td>
<td>$\leq 160$</td>
<td>$\sim 1$</td>
<td>$33$</td>
<td></td>
</tr>
</tbody>
</table>
shown in Fig. 13, the expanding clouds calculated here have an appreciable coincidence in physical properties with $L_a$ absorption clouds. The feature of expanding clouds at $z=0$ is characterized by $T_C$ and $N_{HI}$, whose time variations are shown in Figs. 8(b) and (d) for the model III-1. In this model, $T_C$ decreases to less than $10^4K$ and $N_{HI}$ drops to about $3\times10^{13}cm^{-2}$ at $z=0$. Hence there may be a lot of expanding clouds with a low temperature $T_C<10^4K$ and a low neutral hydrogen column density $N_{HI}\sim10^{13}cm^{-2}$.

(b) HI clouds

A large intergalactic HI cloud has been discovered by Schneider et al. in the western part of the Leo group of galaxies.

4.4. Mass spectrum of intergalactic clouds

The rapid increase of the $L_a$ absorption line density with increasing $z$ has been pointed out by Peterson. Young et al. have revealed a number density of $L_a$ absorption lines per unit redshift to be $N(z)=N_0(1+z)^\nu$ with $\nu=1.81\pm0.84$ and $N(3.5)=57\pm4$. Here, we propose a version that the above tendency arises from the decrease of the neutral hydrogen column density due to the cloud expansion. If we take an initial mass spectrum of intergalactic clouds as $N_C(M_C)=N_0M_C^{-m}Mpc^{-3}(10^{10}M_\odot)^{-1}$, then $N(z)$ is written as

$$N(z)=\int_{M(z)}^{M_{1}(z)} cH_0^{-1}\langle\sigma\rangle(1+z_i)^{-3}N_0M_C^{-m}(1+z)(1+2q_0z)^{-12}dM_C,$$

where $\langle\sigma\rangle$ is the average cloud cross-section defined as $\pi(3R_C/5)^2$, $M_i(z)$ is the maximum mass of intergalactic clouds, and $M(z)$ is the lower mass limit determined in terms of $N_{HI}=10^{14}cm^{-2}$ enough to permit detection within the present technique. If expanding clouds are assumed to undergo the ideal isothermal evolution keeping the pressure balance with the ambient IGM (valid at $z\leq2\sim3$ for $z_i=6$), then we obtain for $q_0=0.5$, $z_i=6$, $T_C=5\times10^4K$, $m>5/3$, and $M_1\gg M_2$,

$$N(z)=5.10N_0(1+z)^{-17/6}(M_2/10^{10}M_\odot)^{5/3-m}/(m-5/3).$$

Since our calculations have provided $M_2(N_{HI}=10^{14}cm^{-2})=M_0(1+z)^{-8}$ with $\delta=10.9$ and $M_0$
\[ N_c(z_i = 6) = 0.79 M_{10}^{-2.1}, \quad (\text{Mpc}^{-3}(10^{10} M_\odot)^{-1}) \]  
(4.3)

with \( M_{10} = M_c / 10^{10} M_\odot \). The same consideration for \( z_i = 7 \) gives

\[ N_c(z_i = 7) = 1.24 M_{10}^{-2.1}, \quad (\text{Mpc}^{-3}(10^{10} M_\odot)^{-1}) \]  
(4.4)

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