Are there one-to-one relationships between magnitude, moment, intensity and ground acceleration?

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Summary. The most important parameter that seismologists observe for engineering applications is earthquake-induced ground acceleration. Unfortunately, this parameter is a recent observation in only a few locations and for few events. Estimates of its value and its return period are usually related to other earthquake parameters which have been observed more widely and frequently also in the past such as intensity and magnitude. It is shown here that there is no evidence of a one-to-one relationship between any two of the following parameters: Magnitude \( M \), moment \( M_0 \), maximum intensity \( I \), and root-mean-square ground acceleration \( \alpha \) and peak ground acceleration \( a_p \) at a given distance. With observational data we will prove that the relationship for \( a_p \) and \( I \), \( M \) and \( M_0 \), \( M \) and \( I \) is not one-to-one. With theoretical considerations we will conclude that there is no one-to-one relationship also for any two of the parameters \( M \), \( a \), \( a_p \) and \( M_0 \). The observed data are too sparse for a discussion of the relationship between and \( I \) and \( M_0 \).

In conclusion there is little or no basis for estimates of the value of the return period for any of those parameters based upon any other parameter. If the parameters may be realistically associated with each other we shall prove that, given a value for one parameter one may estimate the range of values for the other parameter and the probability of occurrence in that range. This is discussed for \( M_0 \), \( a \) and \( a \), \( M \).

Introduction

Predicting the ground-acceleration vector which may occur in a seismic region is one of the most important problems of applied seismology. Compared to the parameters previously used, the ground acceleration vector has many advantages. However, it depends upon the energy released, the depth, the fault-plane solution, the size of the fault and the stress drop for each earthquake, all of which occur with different frequency distributions within a region. The ground acceleration vector is also dependent upon the \( Q \), the rise time, the rupture velocity and the soil properties of the area. All of these variables make prediction of the ground acceleration vector difficult.

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Estimations of the peak value $a_p$ of the ground acceleration were previously made by assuming that it is related to other observed earthquake parameters. It was also assumed that the estimate of the return period for $a_p$ would be the same as that of the corresponding parameter used, which was usually a parameter designed to estimate the energy of the earthquake. Most of the scales used to estimate earthquake energy were designed with limits on their validity, which were sometimes clearly established; but frequently these scales are used beyond their applicable ranges. Also it is often assumed that these scales have a one-to-one correspondence and moreover it is assumed that this correspondence is linear.

Here the empirical relationships between ground acceleration, maximum intensity, scalar seismic moment and local magnitude are analysed and shown to have no one-to-one relationship. Then it will be verified that the observed data show no evidence of a one-to-one relationship between $M_L$ and $I$, between $M_L$ and $m_B$, or $M_L$ and $I$. Therefore, it should be wise to observed systematically not only the ground acceleration but also all the other parameters whose statistical distribution determine that of the ground acceleration (Caputo 1981).

### The relationship between ground acceleration and intensity

The first relationship between peak ground acceleration $a_p$ and another parameter recorded in the seismological literature was introduced by Cancani (1904). It relates $a_p$ with the maximum intensity $I$ observed after an earthquake,

$$\log a_p = \frac{I}{3} - \frac{7}{6}.$$  

Once instrumentation improved to permit accurate estimates of the value of $a_p$, it was possible to check the validity of (1). The result, shown in Fig. 1(a), was considered a satisfactory proof of (1) because the scatter was believed to be from instrumental errors. When more data became available, as one may verify in Fig. 1(b), it was clear that the scatter observed in Fig. 1(a) was not due to instrumental errors; from this follows the inconsistency of formula (1) and of other similar formulae.

### The relationship between magnitude and ground acceleration

Attempts to correlate $I$ and $a_p$ seemed reasonable because both parameters are related to the damage suffered by buildings; but also a correlation between $a_p$ and $M$ seems reasonable because both parameters are related, although in a difference way, to the amplitude of the ground displacement.

Several empirical formulae relating $a_p$ to $M$ have been tentatively given. Examples are

$$\log a_p = 0.46 - 1.1 \log R + 0.63 M_L, \text{ Europe (Ambraseys 1975)},$$

$$\log a_p = 3.09 - 2 \log (R + 25) + 0.35 M_L, \text{ West USA (Esteva 1970)},$$

$$\log a_p = 3.75 - 2 \log (R + 40) + 0.35 M_L, \text{ West USA (Esteva & Villaverde 1973)},$$

$$\log a_p = 3.03 - 1.32 \log (R + 25) + 0.22 M_L, \text{ World (Donovan 1973)}.$$  

Either the analytical expressions for the formula obtained to relate $a_p$ to $M_L$ are different; or, when the formulae are similar, the constants appearing in them are different. Obviously then the values obtained for $\log a_p$ will be very different.

If, for instance, it is assumed that $M_L = 6$ and $R = 10$ km, the values of $a_p$ are: 1380 (Europe), 199 (World), 122 (West USA, Esteva 1970), 273 (West USA, Esteva & Villaverde
Relationships with ground acceleration

This confirms that there is too much scatter in the data from which the formulae were obtained. It can be shown that this scatter is not only from observational errors and regional effects, but also related to the inconsistency of the formulae for the relationship between $a_p$ and $M_L$.

On the basis of empirical data, Bernreuter (1979) has given the following relation between $a_p$, the stress drop $p$ and the linear dimension $l$ for a fault as

$$a_p = \frac{0.19}{\rho} \frac{p}{R} \left( \frac{l}{R} \right)^{0.7}$$

where $R$ is the epicentral distance, and $\rho$ is the density of the crust. $M$ may also be expressed as a function of $l$ and $p$ by considering the following relation (Caputo & Console 1980) for the energy $E$ released in California earthquakes

$$E = 10^{11.1 + 1.46M}.$$  \hspace{1cm} (4)

$E$ may also be expressed by the following relation (Keilis Borok 1959)

$$E = \frac{8}{7} \frac{l^3 p^2}{\mu}$$

where $\mu$ is the rigidity of the crust.

Using formulae (5), (4) and (3) we draw the lines of constant $M$ and $a_p$ in the plane log $l$, log $p$; they are straight, as seen in Fig. 2. One may also verify that, for $l > 1$ km and $p > 1$ bar, the values of $a_p R^{1.7}$ corresponding to the same value of $M$ vary in the interval

$$0.34 M + 7.28 < \log a_p R^{1.7} < 0.73 M + 6.08, \quad M > 3.1$$

(6)
where it is assumed $\rho = 2.8$, $\mu = 3 \times 10^{11}$. The same reasoning applies to the relationship between $M$ and the root-mean-square ground acceleration. The rms ground acceleration $a$ can be obtained using the Parseval theorem and by integrating the spectrum obtained by Brune (1970). Integrating between zero and the upper limit $\omega_M = 4Q/R$, which takes into account the effect of the $Q$, the integration is limited to the interval where the reduction from $Q$ is less than $e^{-2}$. Averaging the integral of the recorded signal over the interval $0-f_c$, where $f_c$ is the spectral corner frequency, we obtain

$$a = \frac{0.29\sqrt{Q}}{R} \int R \left(1 - \frac{3}{2}\tan^{-1}x + \frac{1}{2} \frac{1}{1+x^2}\right); \quad x = \frac{4Q}{2.34R}. \quad (7)$$

For $Q > 300$ and $R < 25l$, the bracketed term is essentially 1, and (7) coincides with the formula obtained by Caputo (1981) using a formula given by Hanks (1979).

Fig. 2 shows that the lines of constant $a$ and the lines of constant $M$ are not parallel, verifying that there is no one-to-one correspondence between $a$ and $M$. For $l > 1$ km and $p > 1$ bar the values of $a$ corresponding to the same value of $M$ are in the range

$$8.01 + 0.24M < \log aR^{1.5} < 6.51 + 0.73M, \quad M > 3.1. \quad (8)$$

The density distribution of $a$ in the range defined by (8) is obtained from the density distribution of $l$ and $p$ (Caputo 1976, 1981) $n(l, p) d\rho = D^{-\gamma} p^{-1+\alpha} dldp$ by changing the variables $l$ and $p$ with $a$ and $M$; we obtain

$$n(a, M) d\rho M = Dk_1^{-\nu-\frac{3}{2}\alpha+1} A^{-\nu+\alpha/4-\frac{1}{2}} d\nu-2+\frac{3}{2}\alpha 16^\nu (11.1 + 1.46M) d\rho M \quad (9)$$

where $A = 8/7M$. From (9) we see that for a fixed $M$, $n$ is a decreasing function of $a$, if $\alpha < -2/3$, because in general $\nu < 3$ (Caputo 1976, 1981); in this case the most probable values of $a$ in the interval defined by (9) are the small ones. The ratio of the periods of return for values of $a$ at the extremes of the interval defined by (8) is $10^{(0.49M-1.5)[\nu-2+(3/2\alpha)]}$. In California $\nu = 2.8$, $\alpha = -1$ (Caputo 1981) then $a = 10^{-0.34M+1.05}$. Analogous relationships can be easily found for $a_p$. 

Figure 2. Isolines of magnitude and peak acceleration of the ground and rma of the ground in the plane of linear dimensions of faults $l$ and stress drops $p$. The lines $a_p R^{1.7}$ = constant have been computed from equation (3), the lines $M$ = constant have been computed from equation (6) with $\log E = 11.1 + 1.46M$. The lines $a R^{1/2}$ = constant have been computed from equation (8).
The relationship between seismic moment and ground acceleration

The previous reasoning can be applied to the relationships between $a_p$, $a$, and $M_0$; it is shown (Fig. 3) that there is no one-to-one correspondence between any two of those variables.

One also finds that for $l \geq 1$ km and $p > 1$ bar the values of $a$ and $a_p$ corresponding to the same value of $\log M_0$ vary in the ranges

$$
\log M_0 + 3.43 < \log a_p R^{1.7} < \log M_0 - 12.67, \quad \log M_0 > 21.
$$

$$
\log M_0 + 5.25 < 10 \log a R^{1.5} < \log M_0 - 12.24, \quad \log M_0 > 21
$$

$M_0 = l^3 p.$

The density distribution of $a$ in the range defined by (11) is obtained from the density distribution of $l$ and $p$ by changing the variables $l$ and $p$ with $a$ and $M$

$$
n(a, M_0) \, d a \, d M_0 = \frac{2D}{5} K_1^{-(2\nu + 4\alpha)/5} C^{(-2\nu + 2 + \alpha)/5} M_0^{(2\nu - \alpha - 3)/5} a^{(2\nu + 4\alpha - 5)/5} \, d a \, d M_0.
$$

From (13) we see that for a fixed $M_0$, $n$ is a decreasing function of $a$ because in general $\alpha < -0.25$, $\nu < 3$ (Caputo 1976, 1981); therefore the most probable values of $a$ in the interval defined by (11) are the small ones. The ratio of the periods of return for the values of $a$ at the extremes of the interval defined by (11) is $(10^{-17.5} M_0^{5/6})^{(2\nu + 4\alpha - 5)/5}$. In California $\nu = 2.8$, $\alpha = -1$ (Caputo 1981), and therefore $a = 10^{11.9 - 0.57 M_0}$. Analogous relationships can be easily found for $a_p$.

The relationships between the various magnitude scales

Most magnitude scales are based upon the maximum amplitudes measured on seismograms. Examples of magnitude scales are $M_L$, local magnitude (Richter 1935), $M_s$, surface wave magnitude (Gutenberg & Richter 1936), and $m_b$, body wave magnitude (Gutenberg 1955). Recently another surface wave magnitude has been introduced (e.g. Vanek et al. 1962) in which the period of the measured wave is taken into account.

![Figure 3](https://academic.oup.com/gji/article-abstract/72/1/83/666820)

*Figure 3*. Isolines of moment, peak and rms acceleration of the ground in the plane of the linear dimension of faults $l$ and stress drop $p$. The lines $a_p R^{1.7} = constant$ have been computed from equation (3), the lines $a R^{1.5} = constant$ have been computed from equation (7), the lines $M_s = constant$ have been computed from equation (12).
Various authors have assumed that the relationship between $m_b$, $M_s$ and $M_L$ are linear and have empirically determined them. Most of the relationships vary from one region to another, but the major problem is the scatter of data, which is introduced because the definitions of the magnitudes do not have any clear indication what their relationships are with respect to other relevant parameters; among the hidden variables are depth, fault plane solution, fault dimensions, $Q$, stress drop, rise time and rupture velocity.

Fig. 4 shows the scatter of data in the $m_b$, $M_s$ relationship for the western United States. Notice that, as in the case of the relationship between $I$ and $a_p$, the scatter for one parameter is larger when the number of data points available for the same value of the other parameter is larger. This scatter is approximately 1.5 in both $m_b$ and $M_s$, too large to be acceptable. Fig. 5 shows the data concerning the relationship between $m_b$ and $M_L$ for the western United States which has the same scatter as that of the relationship between $m_b$ and $M_s$. The same applies to the data on $m_b$ and $M_s$ for 1041 events scattered around the world (Prozorov & Hudson 1974) represented in Fig. 6.

The data sets considered in this paragraph are the most comprehensive (Prozorov & Hudson 1974) or are those considered in the most recent studies for the application of seismology to engineering problems (Chung & Bernreuter 1981).

We may therefore infer that the relations between $m_b$, $M_L$, $m_b$ and $M_s$ are not one-to-one. Using formula (4) and the definition of the scalar seismic moment (Caputo & Console 1980) obtained

$$\log M_0 = 1.46 M + \log (2 \times 10^{11} \mu) - \log \rho$$

which suggests that the relation of $M$ to $M_0$ is not one-to-one. This is verified in Fig. 7 where the data of a set of California earthquakes (Thatcher & Hanks 1973) are shown.

Another relevant phenomenon is that the values of Log $M_0$ derived from higher values of $\rho$ and corresponding to the same value of $M$, should be lower than those derived from lower values of $\rho$, in agreement with formula (14), but this is not always verified.

The relationships between intensity and magnitude

This relationship was one of the first evaluated by seismologists and engineers, because magnitude was the first parameter introduced in seismology with some direct physical meaning.

![Figure 4. Distribution of $m_b$ and $M_s$ for western US earthquakes listed by Nuttli, Bollinger & Griffiths (1979).](https://academic.oup.com/gji/article-abstract/72/1/83/666820)
Figure 5. Distribution of $m_b$ and $M_L$ for western US earthquakes between 1963 and 1977 ($M_L > 4.5$). The solid circles are the NEIS values. The triangles are due to Nuttli et al. (1979), who recomputed values from the NEIS data tapes. From Chung & Bernreuter (1981).

Figure 6. Distribution of $m_b$ and $M_S$ for 1041 earthquakes scattered around the world (Prozorov & Hudson 1974). Points 1 denote single events, points 2 denote 2–4 events, points 3 denote 5–9 events, points 4 denote 10 events or more.
First we consider Fig. 8, which shows the relationship obtained by Peronaci (1982) for the Italian territory. Again there is a scatter of data which is too large to be caused by instrumental errors. This can be verified for all the data sets assembled to obtain the relationships between $I$ and $M_L$ for other regions of the world (e.g. Karnik 1969). As in the data displayed...
in Fig. 1(a and b), the scatter of the values of $I$, corresponding to the same value of $M_L$, increases with the number of data points.

**Conclusions**

It is shown that the data used by various authors to obtain linear relationships between the ground acceleration induced by earthquakes, the maximum intensity, the magnitude and the scalar seismic moment do not support a one-to-one correspondence between these parameters. For ground acceleration, magnitude and seismic moment, it is also shown that the definition of these parameters, and/or the relationships derived by empirical methods, imply that there is no one-to-one correspondence between them.

Any estimate of the value or of the return period, for any of these parameters that is based upon any other parameter does not have sound foundation. Given a value for one of the parameters, one may estimate the range of values for another parameter, if it may be realistically associated to the given one and the probability of occurrence in that range.

The results of this paper are not surprising if one considers that magnitudes are often measured for waves at different frequencies and that earthquakes with the same magnitude may have very different corner frequencies and/or spectra; $Q$ plays a relevant role (Caputo 1982) which is not properly accounted for in the empirical terms introduced to take into account depth and distance. Results concerning intensity are even less surprising if one takes into account the very low resolution of the scale and the fact that the values of $I$ depend on the quality of the construction material, on the age of the constructions and on the great variety of the uppermost layers of sediment.

The results of this paper indicate once more that the best parameter to use for measuring the energy of earthquakes is $M_b$; it should be calculated for all earthquakes at all stations, eventually using simple and/or empirical methods. Observations of $I$ and $M$ should be continued for continuity of earthquake catalogue data. Also, the values of $a$ and $a_p$ and their return period should be obtained from direct observation and/or from the statistical analysis of $I$ and $p$ (Caputo 1981).

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**References**


