Study of Supersymmetric Particles in Decays of $W$ Bosons

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(Received April 16, 1984)

We present phenomenological studies of light supersymmetric (SUSY) particles in $W^\pm$ decay. Our study shows that signals from hadronic decays of the SUSY particles will be distinguishable from their backgrounds. Decays of a sequential heavy lepton are also discussed.

§ 1. Introduction

Supersymmetry (SUSY) is a beautiful symmetry between bosons and fermions, although there is no evidence of it in nature. Apart from the beauty we have two motivations for this symmetry. One is the hierarchy problem. In GUTs we have a small ratio $M_w/M_x \sim 10^{-13}$, where $M_w$ and $M_x$ are masses of $W$ bosons and $X$ bosons. SUSY guarantees that this ratio persists to any orders if we set it small at the tree level; radiative corrections do not break the smallness of the ratio. Another motivation is that SUSY provides a natural framework to introduce gravity because of its close connection with the geometry of space-time.

SUSY theories of weak, electromagnetic and strong interactions involve many new particles as fermionic partners of gauge bosons, which are called gauginos, and as bosonic partners of quarks and leptons, which are called squarks and sleptons respectively. There are also charged and neutral higgsinos associated with spin-0 Higgs bosons. It should be noted that in most SUSY models colorless gauginos and higgsinos could mix with each other since they get the same quantum numbers after the symmetry of $SU(2) \times U(1)$ is broken.

Masses of these SUSY particles depend strongly on the SUSY breaking mechanism employed in a model. In some models they could be heavier than 100 TeV! However, some authors presented an interesting argument which indicates that some SUSY particles can be as light as, or even lighter than, weak gauge bosons in a wide class of broken SUSY models. This suggests that $W^\pm$ or $Z^0$ factory should be a nice place to test SUSY theories.

Mixing angles for diagonalization of mass matrices also depend on details of a model. Several authors have made systematic studies on them. They showed that branching ratios from weak gauge bosons to light SUSY particles are seizable in some mixing patterns.

In this note, with these features in mind, we explore phenomenologies of light SUSY particles in $\bar{p}p$ colliders. We study decays of $W^\pm$ bosons assuming that there exist two light mass eigenstates, one of which is charged and the other neutral. We will see that hadronic decays of the SUSY particles will give us distinguishable signals. Leptonic events seem, on the other hand, to suffer from much more serious backgrounds.

This note is organized as follows. Section 2 contains a brief review of mixing

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patterns in a simple SUSY model. In §3 we present possible decay modes of $W^\pm$ bosons into SUSY particles and discuss possible background events. In §4 hadronic decays of a SUSY particle are discussed in a quantitative manner. We also investigate decays of a sequential heavy lepton (a lepton in the fourth generation), which might generate events having the same topology as those from the SUSY particle, and how to discriminate the former from the latter. Finally §5 is devoted to discussions.

§ 2. Mixing between colorless gauginos and higgsinos

We consider a minimal SUSY model which has two light doublets of Higgs chiral superfields $H_1$ and $H_2$ with weak hypercharge $+$ and $-$, respectively. Let $W$ denote $SU(2)$ gauge field and $B$ denote $U(1)$ gauge field. The model involves fermionic partners of $H_1$, $H_2$, $W$ and $B$ denoted by $\tilde{H}_1$, $\tilde{H}_2$, $\tilde{W}$ and $\tilde{B}$, respectively. Mass terms in the original lagrangian for these gauginos and higgsinos are

$$\mathcal{L}_M = \epsilon \epsilon_i j \tilde{H}_i \tilde{H}_j - M_a \tilde{W}^a - M_{1\tilde{B}} \tilde{B}^j,$$

(2-1)

where $i, j$ are $SU(2)$ doublet indices while the index $a$ denotes $SU(2)$ triplet.

After breaking $SU(2) \times U(1)$, these mass terms read

$$\mathcal{L}_M = -\frac{1}{2} (\phi^0)^T M^n \phi^0 - (\phi^-)^T M^c \phi^+ + \text{h.c.,}$$

(2-2)

$$\phi^0 = (B, \tilde{W}^3, \tilde{H}_1^0, \tilde{H}_2^0)^T, \quad \phi^+ = (\tilde{W}^+, \tilde{H}_1^+)^T, \quad \phi^- = (\tilde{W}^-, \tilde{H}_2^-)^T,$$

$$M^n = \begin{pmatrix} M_1 & 0 & M_2 \sin \theta_w \sin \theta_H & -M_2 \cos \theta_w \sin \theta_H \\ 0 & M_2 & -M_2 \sin \theta_w \cos \theta_H & M_2 \cos \theta_w \cos \theta_H \\ M_2 \sin \theta_w \sin \theta_H & -M_2 \sin \theta_w \cos \theta_H & 0 & -\epsilon \\ -M_2 \cos \theta_w \sin \theta_H & M_2 \cos \theta_w \cos \theta_H & 0 & -\epsilon \end{pmatrix}$$

$$M^c = \begin{pmatrix} M_2 & -\sqrt{2} M_w \cos \theta_H \\ -\sqrt{2} M_w \sin \theta_H & -\epsilon \end{pmatrix}, \quad \tan \theta_H = \frac{\langle 0| H_1^0 |0 \rangle}{\langle 0| H_2^0 |0 \rangle}$$

where $M_w, M_2$ and $\theta_w$ are masses of $W^\pm$ and $Z^0$ bosons and Weinberg angle, respectively. Note that here we have four parameters $M_1, M_2, \epsilon$ and $\theta_H$ to which no constraint from experiments is known.

To discuss mass eigenstates we need to diagonalize these mass matrices. The charged "ino" mass matrix, $M^c$, is diagonalized by two $2 \times 2$ unitary matrices $U$ and $V$:

$$UM^c V^T = \begin{pmatrix} \tilde{m}_1^c & 0 \\ 0 & \tilde{m}_2^c \end{pmatrix} \equiv M^c.$$  

(2-3)

and the neutral one, $M^n$, by a $4 \times 4$ matrix $X$:

$$XM^n X^T = \begin{pmatrix} \tilde{m}_1^n & 0 & 0 & 0 \\ 0 & \tilde{m}_2^n & 0 & 0 \\ 0 & 0 & \tilde{m}_3^n & 0 \\ 0 & 0 & 0 & \tilde{m}_4^n \end{pmatrix} \equiv M^n.$$  

(2-4)

Explicit forms of $U$, $V$, $X$ and mass eigenvalues in terms of $M_1, M_2, \epsilon$ and $\theta_H$ are given.
It is easy to rewrite interaction terms using the charged mass eigenstate Dirac spinors
\[ \chi_i^+ = (V_{ij} \phi_j^+, U_{ij}^0 (\phi_j^-)^*)^T, \]
and the neutral mass eigenstate Majorana spinors
\[ \chi_i^0 = (X_{ij} \phi_j^0, X_{ij} (\phi_j^0)^*)^T. \]
For example, the charged weak current coupled with \( W^- \) boson is given by
\[ J_\mu^+ = g \bar{\chi}^0 \gamma_\mu \left[ O_{Li} \frac{1-\gamma_5}{2} + O_{Ri} \frac{1+\gamma_5}{2} \right] \chi_i^+, \]
where
\[ O_{Li} = \frac{1}{\sqrt{2}} X_{ij} V_{j2} - X_{ij} V_{j1}, \]
\[ O_{Ri} = -\frac{1}{\sqrt{2}} X_{ij} U_{j2} - X_{ij} U_{j1}. \]

Other useful descriptions are given in Appendix A.

§ 3. SUSY particles in \( W^\pm \) decays

In this section we discuss productions and subsequent decays of SUSY particles in \( W^\pm \) decays. We assume that one light and stable neutral mass eigenstate comes from a mixing between neutral colorless gauginos and neutral higgsinos. (Note that \( R \) invariance in SUSY theories guarantees the existence of one stable SUSY particle.) We refer to this state as \( \chi_i^0 \). We also assume there exists one charged mass eigenstate, \( \chi_i^\pm \), which is lighter than \( W^\pm \) bosons and heavier than \( \chi_i^0 \). Then we will have such sequential decay processes as
\[ \bar{p} p \rightarrow W^\pm + X \rightarrow \chi_i^\pm + \chi_i^0 \rightarrow \chi_i^0 + l^\pm + \nu_l. \]  
\[ (3.1) \]

Can we pick up signals from these processes in various backgrounds? In process (3.1) we will observe one charged lepton with small energy accompanied with large missing energy. This lepton will be easily distinguished from a lepton coming from direct decays, \( W^\pm \rightarrow l^\pm + \nu_l \), since the latter is much more energetic than the former. However, decays of
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Fig. 1.(a) A possible background event to process (3·1) coming from \( \tau \) lepton decays.

(b) A possible background event to process (3·2) coming from \( \tau \) lepton decays.

(c) A possible background event to process (3·2) coming from heavy quark jet events.

\[ \tau \] leptons like Fig. 1(a) will provide a serious background to process (3·1). Therefore it seems quite difficult to identify this process.

In process (3·2), on the other hand, several cuts are useful to eliminate background events from \( \tau \) decays associated with jets (Fig. 1(b)). In this case the final state of the event consists of two jets and missing \( E_T \) carried by two \( \chi_1^0 \) particles. Studies on a sequential heavy lepton \( \gamma \) show that one can suppress contamination from \( \tau \) events by requiring such cuts as \( E_T \gtrsim 10 \text{ GeV} \) or \( N^{\text{ch}} \gtrsim 5 \), where \( N^{\text{ch}} \) denotes the number of charged hadrons in each jet. These discussions apply to \( \chi_1^\pm \) events as well. A background coming from heavy quark jet events like Fig. 1(c) will also be separable because events from process (3·2) have no charged leptons. Decays of a sequential heavy lepton, however, would also provide events having two hadron jets and missing \( E_T \), if it is produced in \( W^\pm \) decays. Suppose we have a process

\[
\bar{p}p \rightarrow W^\pm + X \\
\Downarrow \{ \nu_L \} \\
\nu_L + q + q', \quad (3·3)
\]

where \( L \) denotes the heavy lepton. Then, we cannot discriminate between processes (3·2) and (3·3) by a mere topological consideration. This subject will be discussed in §4.
§ 4. Hadronic decays of a light charged SUSY particle

Let us make a quantitative study on the decays of $\chi_i^\pm$ using the lowest-order perturbation. We calculated $d\Gamma^S/d\mu^2$, $q\bar{q}'$ invariant mass $(\mu^2=(p_q+p_{q'})^2)$ distribution of the decay width ($\Gamma^S$), from Fig. 2(a) and (b) assuming that squarks $(\tilde{q})$ are heavier than $W^\pm$ bosons. Interactions including SUSY particles are also given in Fig. 2, which correspond to a limit $M_1, M_2$ and $\epsilon \to 0$. (More general cases are studied in Appendix B.)

![Fig. 2. Feynman diagrams for process (3-2), with Feynman rules corresponding to the limit $M_1, M_2$ and $\epsilon \to 0$.]

The result is

$$\frac{d\Gamma^S}{d\mu^2} = G_I + G_H + G_M,$$

$$G_I = \frac{e^2 g^2}{4\pi} \frac{1}{64\pi^2 M_{\chi_i^\pm}} T_B \alpha(2\beta^2 - 6\beta x + x),$$

$$G_H = \frac{e^2 g^2}{4\pi} \frac{1}{12\pi^2 M_{\chi_i^\pm}} \left\{ -\alpha - \left( \frac{b+\frac{f}{2}}{c} \right) T_L - \frac{b^2 - c^2 + bf}{c} T_A \right\},$$

$$G_M = \frac{e^2 g^2}{4\pi} \frac{1}{4\pi^2 M_{\chi_i^\pm}} T_B \left[ \frac{d}{2} a\beta + (b+f)d + e \right] \alpha$$

$$+ \frac{1}{2} \left[ d(bf + b^2 - c^2) + e(f+2b) \right] T_L + \frac{1}{c} \left[ be(b+f) - c^2(2bd + fd + e) \right] T_A,$$

where

$$b = \frac{1}{4} - \lambda, \quad c = \sqrt{\lambda\tau}, \quad d = x - \rho, \quad e = \sqrt{\lambda\tau\epsilon\rho}, \quad f = -\left( \frac{1}{4} - \delta \right),$$

$$\beta = \frac{1}{4} + x - \delta, \quad a = \sqrt{\beta^2 - x},$$

$$x = \mu^2/4M_{\chi_i^\pm}^2, \quad \rho = M_{W^\pm}/4M_{\chi_i^\pm}, \quad \epsilon = \Gamma_{W^\pm}/4M_{\chi_i^\pm},$$

$$\delta = M_{\tilde{q},0}/4M_{\chi_i^\pm}, \quad \lambda = M_{\tilde{q}}^2/4M_{\chi_i^\pm}, \quad \tau = \Gamma_{\tilde{q}}/4M_{\chi_i^\pm},$$

$$T_B = \frac{1}{(x-\rho)^2 + \rho\epsilon}, \quad T_L = \ln \left\{ \frac{(\beta + a)^2 - 2b}{(\beta - a)^2 - 2b} \right\} \left[ \frac{4c^2}{(x-4b\beta + 4(b^2 + c^2)} \right],$$

$$T_A = \arctan \frac{4ca}{x-4b\beta + 4(b^2 + c^2)}, \quad g = \frac{\epsilon}{\sin \theta_w},$$

and $M_{\chi_i^\pm}, M_{\tilde{q},0}, M_{\tilde{q}} (\Gamma_{\tilde{q}})$ and $M_W (\Gamma_W)$ are the masses (widths) of $\chi_i^\pm, \chi_i^0$, squark $(\tilde{q})$ and
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- The $\chi_{1\pm}$ invariant mass distribution of the decay width of $\chi_{1\pm}$ for $M_w=85$ GeV, $\Gamma_w=2$ GeV, $M_{\chi_{1\pm}}=100$ GeV, $\Gamma_{\chi_{1\pm}}=1$ GeV, $M_{\chi_{1\pm}}=0$ and $M_{\chi_{1\pm}}=30$, 40, 50, 60 and 70 GeV. (Hereafter we refer to the total width as $\Gamma^{\pm}$; $\Gamma^{\pm} = \int (d\Gamma^{\pm}/d\mu^2) d\mu^2$.)

- The maximum value of $\mu^2$ is equal to $M_{\chi_{1\pm}}^2$, here, since $\chi_{1\pm}$ is massless. We see that the shape of the distribution does not depend much on $M_{\chi_{1\pm}}$. Figure 4 shows $(1/\Gamma^{\pm})(d\Gamma^{\pm}/d\mu^2)$ with $M_{\chi_{1\pm}}=0$, $M_{\chi_{1\pm}}=60$ GeV and $M_q=100$, 200 and 500 GeV, which indicates that the distribution does not change much in this region of $M_q$. In Fig. 5 $(1/\Gamma^{\pm})(d\Gamma^{\pm}/d\mu^2)$ is plotted for $M_{\chi_{1\pm}}=0$, 5, 10, 15 and 20 GeV while $M_{\chi_{1\pm}}=60$ GeV and $M_q=100$ GeV. Note that the maximum value of $\mu^2$ gives a value of $(M_{\chi_{1\pm}}-M_{\chi_{1\pm}})^2$. We see that $M_{\chi_{1\pm}}$ has no significant effect on the shape of the distribution except when $\chi_{1\pm}$ is massless and $\mu^2$ is close to $M_{\chi_{1\pm}}^2$.

- Distribution from a sequential heavy lepton can be calculated from Fig. 6. The result reads

$$
\frac{d\Gamma^{\pm}}{d\mu^2} = \frac{e^2}{4\pi g^2} \frac{1}{192\pi^2} \frac{1}{(\rho' - x')^2 + \rho' e} a' (2\beta'^2 - 6\beta' x' + x'),
$$

where

$$
x' = \frac{\mu^2}{4M_L^2}, \quad \rho' = \frac{M_w^2}{4M_L^2}, \quad \epsilon' = \frac{\Gamma_w^2}{4M_L^2},
\beta' = \frac{1}{4} + x', \quad a' = \frac{1}{4} - x'
$$

and $M_L$ denotes the mass of the heavy lepton. In Fig. 7 $(1/\Gamma_{L})(d\Gamma_{L}/d\mu^2)$ is plotted for $M_L=30$, 40, 50, 60 and 70 GeV, where $\Gamma_{L}=\int (d\Gamma_{L}/d\mu^2) d\mu^2$. We see that Fig. 7 is hardly different from Fig. 3.

- Let us discuss how to distinguish events from $\chi_{1\pm}$ and those from $L$. Figure 8 shows that measurement of the shape of $(1/\Gamma)(d\Gamma/d\mu^2)$ is helpful if $\chi_{1\pm}$ is massive. We see that 10 percent of accuracy will be enough for $M_{\chi_{1\pm}} \approx 10$ GeV. If $\chi_{1\pm}$ is lighter than $\sim 10$ GeV, on the other hand, ambiguity will remain. In this case the best way would be to measure the production rate and compare it with prediction for $L$,

$$
R_L = \frac{B(W^\pm \rightarrow L^\pm \nu_L)}{B(W^\pm \rightarrow e^\pm \nu_e)} = \frac{1}{2M_w^2} (M_w^2-M_L^2)^2 (2M_w^2+M_L^2).
$$

$W^\pm$, respectively.

Figures 3～5 show numerical results for several values of mass parameters. In Fig. 3 we plot $(1/\Gamma^{\pm})(d\Gamma^{\pm}/d\mu^2)$ when $M_{\chi_{1\pm}} = 0$, $M_q=100$ GeV and $M_{\chi_{1\pm}}=30$, 40, 50, 60 and 70 GeV. (Hereafter we refer to the total width as $\Gamma^{\pm}$; $\Gamma^{\pm} = \int (d\Gamma^{\pm}/d\mu^2) d\mu^2$.) The maximum value of $\mu^2$ is equal to $M_{\chi_{1\pm}}^2$, here, since $\chi_{1\pm}$ is massless. We see that the shape of the distribution does not depend much on $M_{\chi_{1\pm}}$. Figure 4 shows $(1/\Gamma^{\pm})(d\Gamma^{\pm}/d\mu^2)$ with $M_{\chi_{1\pm}}=0$, $M_{\chi_{1\pm}}=60$ GeV and $M_q=100, 200$ and 500 GeV, which indicates that the distribution does not change much in this region of $M_q$. In Fig. 5 $(1/\Gamma^{\pm})(d\Gamma^{\pm}/d\mu^2)$ is plotted for $M_{\chi_{1\pm}}=0, 5, 10, 15$ and 20 GeV while $M_{\chi_{1\pm}}=60$ GeV and $M_q=100$ GeV. Note that the maximum value of $\mu^2$ gives a value of $(M_{\chi_{1\pm}}-M_{\chi_{1\pm}})^2$. We see that $M_{\chi_{1\pm}}$ has no significant effect on the shape of the distribution except when $\chi_{1\pm}$ is massless and $\mu^2$ is close to $M_{\chi_{1\pm}}^2$.

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$$

where

$$
x' = \frac{\mu^2}{4M_L^2}, \quad \rho' = \frac{M_w^2}{4M_L^2}, \quad \epsilon' = \frac{\Gamma_w^2}{4M_L^2},
\beta' = \frac{1}{4} + x', \quad a' = \frac{1}{4} - x'
$$

and $M_L$ denotes the mass of the heavy lepton. In Fig. 7 $(1/\Gamma_{L})(d\Gamma_{L}/d\mu^2)$ is plotted for $M_L=30$, 40, 50, 60 and 70 GeV, where $\Gamma_{L}=\int (d\Gamma_{L}/d\mu^2) d\mu^2$. We see that Fig. 7 is hardly different from Fig. 3.

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$$
R_L = \frac{B(W^\pm \rightarrow L^\pm \nu_L)}{B(W^\pm \rightarrow e^\pm \nu_e)} = \frac{1}{2M_w^2} (M_w^2-M_L^2)^2 (2M_w^2+M_L^2).
$$

$W^\pm$, respectively.
Fig. 4. The $q\bar{q}'$ invariant mass distribution of the decay width of $\chi_{1}^\pm$ for $M_W=85$ GeV, $\Gamma_W=2$ GeV, $\Gamma_\phi=1$ GeV, $M_{\chi_{1}^0}=0$, $M_{\chi_{2}^0}=60$ GeV and $M_\phi=100$, 200 and 500 GeV.

Fig. 5. The $q\bar{q}'$ invariant mass distribution of the decay width of $\chi_{1}^\pm$ for $M_W=85$ GeV, $\Gamma_W=2$ GeV, $M_\phi=100$ GeV, $\Gamma_\phi=1$ GeV, $M_{\chi_{1}^0}=60$ GeV and $M_\phi=0, 5, 10, 15$ and 20 GeV.

Fig. 6. Feynman diagram for process (3'-3).
Fig. 7. The $q\bar{q}'$ invariant mass distribution of the decay width of $L^\pm$ for $M_L=30, 40, 50, 60 \text{ and } 70$ GeV.

Fig. 8. The $q\bar{q}'$ invariant mass distribution of the decay width of $X_\pm$ or $L^\pm$ for (a) $M_L=M_{\pm}^0-M_{\pm}^0=40$ GeV, (b) $M_L=M_{\pm}^0-M_{\pm}^0=50$ GeV and (c) $M_L=M_{\pm}^0-M_{\pm}^0=60$ GeV.
From (4·3) we see that $R_{L^+}$ depends only on one parameter, $M_L$, which can be determined by measuring the maximum value of $\mu^2$ in $dT/d\mu^2$ distribution. (Note that production rate of $\chi_{1}^{\pm}$ depends on two unknown parameters even if we set values of $M_{X_{1}}$ and $M_{X_{1}^0}$.) If observed production rate is quite different from $R_{L^+}$, it is very likely that $\chi_{1}^{\pm}$ decays take place.

§ 5. Discussion

In previous sections we concentrated only on the study of $\chi_{1}^{\pm}$. Here we would like to make a few remarks on other SUSY particles.

Among all SUSY particles gluinos seem to be the most prospective to search for. Gluino mass can be light; in supergravity GUT models it is supposed to be about a few tens of GeV. \(1\) If it is true, gluinos would be produced even in SPS collider. If gluino mass is heavier but is less than a few hundred GeV, we will still have ample gluino production in \(\bar{p}p\) collisions since gluinos couple to gluons with strength $a_s$. Main difficulty in the search for gluinos is the backgrounds coming from jet events of heavy quarks.\(10\)

As for squarks and sleptons, it is likely that they are heavy. If some of them are light enough, they will be easily detected in $e^+e^-$ experiments because they are charged and background should not be so serious. So far, however, experimental results show no sign that such a threshold opens up.\(11\) In the case in which neutral slepton ($\tilde{\nu}$) mass is $\lesssim 10$ GeV and charged slepton ($\tilde{e}$) mass is $\lesssim 60$ GeV, we would be able to look for sleptons in \(\bar{p}p\) collider better than in $e^+e^-$ collider, because $W^\pm \rightarrow \ell^\pm \tilde{\nu}$ decays are possible.\(12\)

In this note we discussed productions and decays of a light SUSY particle, $\chi_{1}^{\pm}$, and pointed out an interesting possibility to observe them in hadronic decays of $W^\pm$ bosons. Because SUSY theories are of great interest in recent theoretical particle physics and cosmology, we believe it is worth searching for SUSY particles in experiments no matter how difficult it may be. We hope \(\bar{p}p\) experiments will improve our understandings of SUSY theories.

Acknowledgements

We would like to thank the CDF group and the Theoretical Division at Fermilab for their friendly hospitality. It is a pleasure to thank C. N. Leung for reading the manuscript.

Appendix A

Here we present matrices which diagonalize mass matrices of colorless gauginos and higgsinos.

As for charged “ino”s the diagonalized mass matrix is

$$
M_d = U M_c V^{-1} = \begin{pmatrix}
\tilde{m}_1^c & 0 \\
0 & \tilde{m}_2^c
\end{pmatrix},
$$

(A·1)
where

\[
U = \begin{pmatrix} \cos \theta_+ & -\sin \theta_+ \\ \sin \theta_+ & \cos \theta_+ \end{pmatrix}, \quad V = \begin{pmatrix} \cos \theta_- & -\sin \theta_- \\ \sin \theta_- & \cos \theta_- \end{pmatrix},
\]

\[
\tan \theta_\pm = \frac{b_\pm + \sqrt{b_\pm^2 + 4a_\pm^2}}{2a_\pm},
\]

\[
a_+ = \sqrt{2} M_w (-M_2 \sin \theta_H + \epsilon \cos \theta_H),
\]

\[
a_- = \sqrt{2} M_w (-M_2 \cos \theta_H + \epsilon \sin \theta_H),
\]

\[
b_\pm = M_2^2 - \epsilon^2 \pm 2 M_w^2 (\cos^2 \theta_H - \sin^2 \theta_H)
\]

and

\[
\tilde{m}_1 = M_2 \cos \theta_+ \cos \theta_- - \epsilon \sin \theta_+ \sin \theta_- \\
+ \sqrt{2} M_w (\cos \theta_H \cos \theta_+ \sin \theta_- + \sin \theta_H \sin \theta_+ \cos \theta_-),
\]

\[
\tilde{m}_2 = M_2 \sin \theta_+ \sin \theta_- - \epsilon \cos \theta_+ \cos \theta_- \\
- \sqrt{2} M_w (\cos \theta_H \sin \theta_+ \cos \theta_- + \sin \theta_H \cos \theta_+ \sin \theta_-).
\]

In order to present the diagonalization of the mass matrix of neutral "ino"s, we need quite tedious calculations. Here we confine ourselves to a simple case in which \( M_1, M_2 \) and \( \epsilon \to 0 \). Then we get

\[
M_d^c = X M^a X^{-1} = \begin{pmatrix}
M_1 \cos^2 \theta_w + M_2 \sin^2 \theta_w & 0 & 0 & 0 \\
0 & M_z & 0 & 0 \\
0 & 0 & -M_z & 0 \\
0 & 0 & 0 & 2\epsilon \sin \theta_H \cos \theta_H
\end{pmatrix}, \tag{A·2}
\]

while \( X \) is

\[
X = \begin{pmatrix}
\cos \theta_w & \sin \theta_w & 0 & 0 \\
\frac{1}{\sqrt{2}} \sin \theta_w & -\frac{1}{\sqrt{2}} \cos \theta_w & \frac{1}{\sqrt{2}} \cos \theta_H & -\frac{1}{\sqrt{2}} \sin \theta_H \\
\frac{1}{\sqrt{2}} \sin \theta_w & -\frac{1}{\sqrt{2}} \cos \theta_w & -\frac{1}{\sqrt{2}} \cos \theta_H & \frac{1}{\sqrt{2}} \sin \theta_H \\
0 & 0 & \sin \theta_H & \cos \theta_H
\end{pmatrix}.
\]

In this case \( M_d^c \) becomes

\[
M_d^c = \begin{pmatrix}
\sqrt{2} M_w \sin \theta_H & 0 \\
0 & -\sqrt{2} M_w \cos \theta_H
\end{pmatrix}, \tag{A·3}
\]

with

\[
U = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad V = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.
\]

Interaction terms between \( W^\pm, \chi_i^0 \) and \( \chi_i^\pm \) are given by
\[ J_{\mu}^+ = g\chi_0^+ \gamma_{\mu} \left[ O_{L1} \frac{1-\gamma_5}{2} + O_{R1} \frac{1+\gamma_5}{2} \right] \chi_j^+ , \]  
\[ O_{L11} = -\sin \theta_W, \quad O_{R11} = 0, \]
\[ O_{L12} = 0, \quad O_{R12} = -\sin \theta_W \]
in the same limit. Branching ratios of these particles from \( W^\pm \) bosons become
\[
\frac{R_{ij}(W^\pm \rightarrow \chi_0^\pm \chi_i^0)}{B(W^\pm \rightarrow e^\pm \nu_e)} = -\frac{6}{M_w^3} \left[ (O_{Li}^2 + O_{Ri}^2) \left( E_+ E_0 + \frac{1}{3} \rho^2 \right) + 2O_{Lij} O_{Rij} m_{j \pm} m_{0i} \right],
\]
where
\[ \rho = \frac{1}{2} \left[ M_w^2 - 2m^2_\pm - 2m^2_0 + (m^2_\pm - m^2_0)^2 / M_w^2 \right]^{1/2} \]
and
\[ m_{1\pm} = \sqrt{2} M_w \sin \theta_W, \quad m_{2\pm} = \sqrt{2} M_w \cos \theta_W, \]
\[ m_{01} = M_1 \cos^2 \theta_W + M_2 \sin^2 \theta_W. \]

Appendix B

We present analytic expression of \( d\Gamma^s / d\mu^2 \), where \( \mu^2 \) is the invariant mass of \( q \bar{q}' \) and \( \Gamma^s \) denotes the decay width from \( \chi_1^\pm \) to \( \chi_0^0 q \bar{q}' \), with interactions shown in Fig. 9. The result reads
\[
\frac{d\Gamma^s}{d\mu^2} = G_{i'} + G_{ii} + G_{i'm} + G_{i'm'}, \quad (B.1)
\]
where
\[ G_{i'} = 8(|a'|^2 + |b'|^2)(|c'|^2 + |d'|^2)G_1, \]
\[ G_{ii} = \frac{9}{2} (|l'|^2 + |m'|^2)(|n'|^2 + |t'|^2)G_{ii}, \]
\[ G_{i'm} = 3 \left( (a'^* n' - b'^* t')(c'^* l' - d'^* m') + (a'^* t' - b'^* n')(d'^* l' - c'^* m') \right) G_{i'm}, \]
\[ G_{i'm'} = \left( (a'^* n' + b'^* t')(c'^* l' - d'^* m') + (a'^* t' + b'^* n')(d'^* l' - c'^* m') \right) \]
\[ \times \frac{e^2}{4\pi} \frac{3}{8\pi^2 M_{\chi_0^0}} T_b \sqrt{\delta x} (d T_L + b \sqrt{\rho} - T_L) \]
and other quantities are defined in (4.1).
In the limit considered in \( \S 4, \)
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\[ a' = -\frac{1}{2}, \quad b' = -\frac{1}{2}, \quad c' = -\frac{\sqrt{2}}{4}, \quad d' = +\frac{\sqrt{2}}{4}, \]

\[ l' = +\frac{1}{2}, \quad m' = +\frac{1}{2}, \quad n' = +\frac{\sqrt{2}}{3}, \quad t' = -\frac{\sqrt{2}}{3} \] (B.2)

for which $G''_{\mu}$ vanishes.

Fig. 9. Feynman diagrams for process (3'-2), with Feynman rules corresponding to the general case.

References

5) P. Fayet, LPTENS 83/35 (1983).