New Sum Rules from the Current Anticommutator on the Null-Plane

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The divergent sum rules derived from the current anticommutator on the null-plane are regularized by the analytical continuation from the non-forward direction. The finite part of the sum rule is shown to have one ambiguity which depends on the dynamics. The Gottfried sum rule for \((F_{\pi^-}-F_{\pi^0})\) becomes free of this ambiguity if we sacrifice the consistency with the leading logarithmic approximation at the two loops in QCD. Then by the same approximation as above we obtain new sum rules from the finite parts of the sum rules.

§ 1. Introduction

Recently, it is found that the residue of the pomeron is closely related to the spontaneous chiral symmetry breaking of the vacuum.\(^1\)\(^-\)\(^3\) Originally, in the context of the chiral dynamics, a close connection between them was suggested by Weinberg with the use of the pion-coupling \(\chi_a\) matrix.\(^4\) The works in Refs. 1)\(^-\)3) essentially inherit this idea with the use of the light-like chiral charge which plays practically the same role as the pion-coupling \(\chi_a\) matrix.\(^5\),\(^6\) However, since the physical idea which underlines these works may be obscure, we explain it first. We consider, in the high-energy limit, there is a symmetry which is closely connected with the low-energy theorem,\(^4\) and that this symmetry is realized by the light-like chiral charge algebra. In case of the renormalizable linear \(\sigma\) model, we can consider the situation as follows. When the Nambu-Goldstone mechanism comes into play, the displayed \(\sigma\) field \(\phi(x)=\sigma(x)-\langle\sigma\rangle\) and \(\pi\) field transform as a \((1/2,1/2)\) representation under the light-like charge \((\tilde{Q}_a\pm\tilde{Q}_a^\dagger)/2\), and not under the ordinary charge \((Q_a\pm Q_a^\dagger)/2\).\(^5\),\(^6\) On the other hand, we can regard the non-linear \(\sigma\) model which compactly expresses the low-energy theorem as the limit of the large \(\sigma\) mass in the linear \(\sigma\) model.\(^7\),\(^8\) Thus, in the high-energy region where the mass of the \(\sigma\) particle cannot be neglected, the symmetry of the linear \(\sigma\) model, i.e., the one generated by the light-like charge algebra becomes clear. Now in QCD, we do not know the correct theory which expresses the spontaneous chiral symmetry breaking of the vacuum. However we know the low-energy theorem, and the success of the Adler-Weisberger sum rule\(^9\) which is based on PCAC and light-like chiral algebra.\(^5\) It should be noted that, through this sum rule, Weinberg found the algebra of pion-coupling matrix.\(^4\) Thus in QCD, though we do not know how the dynamical breakdown of the chiral symmetry occurs, we consider there is a symmetry which becomes clear in the high-energy region, and regard it as the one generated by the light-like chiral charge algebra. Now the most important point of the symmetry breakdown lies in the property of the vacuum. On the other hand, we know the pomeron should reflect a vacuum property in the high-energy region. Then, if our view is correct, the pomeron should be closely related to the spontaneous chiral symmetry breaking of the vacuum. To investigate this problem quantitatively the light-like chiral charge algebra or its local version is extended to the anticommutator on the null-plane in Ref. 2). However the sum rules derived by this extended version are divergent when the
intercept of the pomeron, $\alpha_p(0)\), is equal to 1. Thus in Refs. 1)~3), we regularize them by assuming $\alpha_p(0)<1$, and this point is the drawback of these works. In this paper we make these sum rules convergent even when $\alpha_p(0)=1$ by assuming the pomeron being a moving pole;\(^1\) we derive the sum rules in the nonforward direction, and continue these analytically to the forward direction.\(^{10}\) Then we find the sum rule has one ambiguity which depends on the dynamics. We discuss these points in §2. In §3, we derive the modified Gottfried sum rules.\(^{11}\) Among these, the one for $(F_2^\text{ep} - F_2^\text{en})$ becomes free of the ambiguity, if we sacrifice the consistency with the leading logarithmic approximation at the two loops in QCD.\(^{12}\) The experiment\(^{13}\) suggests the non-perturbative effect is important in this sum rule. Then, in §4, by the same approximation as the above, new sum rules are obtained from the finite parts of the sum rules. Conclusions are given in §5, and the theoretical bases of the sum rules are discussed in Appendixes A and B.

§ 2. Regularization of the sum rules

In Refs. 1) and 2), the sum rules

$$g_A^2(0)+\frac{2f^2}{\pi}\int_0^\infty \frac{d\nu}{\nu}\{\sigma^+(\nu)+\sigma^-(\nu)\}$$

$$= \frac{1}{2\pi} P \int_{-\infty}^\infty \frac{da}{a} \left\{ \frac{2\sqrt{6}}{3} A_0(a,0)+\frac{2\sqrt{3}}{3} A_8(a,0) \right\} \quad (2-1)$$

and

$$\frac{1}{2\pi} P \int_{-\infty}^\infty \frac{da}{a} \left\{ \frac{2\sqrt{6}}{3} A_0(a,0)+\frac{2\sqrt{3}}{3} A_8(a,0) \right\}, \quad (2-2)$$

are derived, where $A_c(p\cdot x, 0)$ is defined as

$$G_{\alpha\beta}(p\cdot x, 0) = d_{\alpha\beta\gamma} A_c(p\cdot x, 0) + f_{\alpha\beta\gamma} S_c(p\cdot x, 0), \quad (2-3)$$

and $G_{\alpha\beta}(p\cdot x, 0)$ is defined by Eq. (A-13). In Eq. (2-1), $g_A(0)$ is the axial-vector nucleon coupling constant, $f_\pi$ is the pion decay constant, $\sigma^+(\nu)$ is the total cross section of the $\pi^+P$ scattering at $q^2=0$, and $\nu = p\cdot q$. In Eq. (2-2) $q^2 = -Q^2, x = Q^2/2\nu$, flavor symmetry is assumed to be $SU(3)$ and Cabibbo angle is set to be zero. Equations (2-1) and (2-2) correspond to the Adler-Weisberger relation and the Adler sum rule respectively, and give us the relation\(^{**}\)

$$g_A^2(0)+\frac{2f^2}{\pi}\int_0^\infty \frac{d\nu}{\nu}\{\sigma^+(\nu)+\sigma^-(\nu)\} = \int_0^1 \frac{dx}{2x}\{F_2^\text{ep}(x, Q^2)+F_2^\text{en}(x, Q^2)\}. \quad (2-4)$$

Note that in Eqs. (2-1)~(2-3), the nonlocal quantity $G_{\alpha\beta}(p\cdot x, 0)$ or $A_c(p\cdot x, 0)$ is defined only in case of the stable particle and corresponds to the residue of a $c$-number singularity in the one-particle connected matrix element of the product of two local operators.\(^2\) We
discuss this point more in Appendixes A and B. Now the sum rules (2.1), (2.2) and (2.4) are expected to hold only when the intercept of the pomeron satisfies the condition \(a_\rho(0) < 1\). Thus we give the discussion which can be applied when \(a_\rho(0)\) is one by the method in Ref. 10). Now consider the non-forward hadronic tensor defined by*3

\[
W_{\alpha\beta}^{\mu\nu}(p, K, \Delta) = \int d^4x \exp[iK \cdot x] \langle p_1 | \left( J_\alpha^\mu \left( \frac{x}{2} \right), J_\beta^{\nu\mu} \left( -\frac{x}{2} \right) \right) | p_2 \rangle c
\]

with the tensor decomposition

\[
W_{\alpha\beta}^{\mu\nu}(P, K, \Delta) = P^{\alpha\beta}(q_1) P^{\nu\sigma}(q_2) [ -g_{\alpha\sigma} W_{1}^{ab} + P_1 P_\sigma W_{2}^{ab} \\
+ (P_1 \Delta_\sigma + P_\sigma \Delta_\lambda) W_{3}^{ab} + (P_1 \Delta_\sigma - P_\sigma \Delta_\lambda) W_{4}^{ab} + \Delta_\lambda \Delta_\sigma W_{5}^{ab} ]
\]

where

\[
P^{\alpha\beta}(q) = g^{\alpha\beta} - \frac{q^\alpha q^\beta}{q^2}, \quad W_{1}^{ab}(K^2, \nu, t, \delta), \quad P^{\mu} = \frac{1}{2}(p_{1}^{\mu} + p_{2}^{\mu}),
\]

\[
K^n = \frac{1}{2}(q_1^n + q_2^n), \quad \Delta^n = q_2^n - q_1^n, \quad t = \Delta^2, \quad \delta = K \cdot \Delta, \quad \nu = P \cdot K.
\]

The structure function \(W_i\) has the property

\[
W_{i}^{ab}(K^2, \nu, t, \delta) = W_{i}^{ba}(K^2, -\nu, t, -\delta), \quad i \neq 4,
\]

\[
W_{4}^{ab}(K^2, \nu, t, \delta) = -W_{4}^{ba}(K^2, -\nu, t, -\delta).
\]

Note that, compared with the structure functions, the opposite property under the crossing is defined by the commutator of the currents. Now the discussion in Appendix B shows that the same discussion as the one in Appendix A or in Ref. 2) can be repeated in the non-forward case. After all we can simply set

\[
\langle p_1 | \left( J_\alpha^\mu \left( \frac{x}{2} \right), J_\beta^{\nu\mu} \left( -\frac{x}{2} \right) \right) | p_2 \rangle c \delta(x^+) = \langle p_1 | \left( J_\alpha^{\nu+} \left( \frac{x}{2} \right), J_\beta^{\mu+} \left( -\frac{x}{2} \right) \right) | p_2 \rangle c \delta(x^+)
\]

\[
= \frac{1}{\pi} P \frac{1}{x - \delta^2(x^+)} \delta(x^+)
\]

\[
\times \{ d_{abc} A_c (P \cdot x, t, x^2 = 0, x \cdot \Delta = 0) + f_{abc} S_c (P \cdot x, t, x^2 = 0, x \cdot \Delta = 0) \} P^+.
\]

Then, by setting \(q_{1}^{+} + q_{2}^{+} = 0\) in addition to \(q_{1}^{+} = q_{2}^{+} = 0\), we obtain the sum rule

\[
G(t) + \frac{2 f_{\pi}^2}{\pi} \int_{\nu_0}^{\infty} d\nu \frac{m_{\pi}^4}{\nu^2 - (\frac{t}{4})^2} \{ \sigma^+(\nu, t, K^2 = 0, \delta = 0) + \sigma^- (\nu, t, K^2 = 0, \delta = 0) \}
\]

\[
= \frac{1}{2\pi} P \int_{-\infty}^{\infty} da \left\{ \frac{2 \sqrt{6}}{3} A_0(a, t, 0, 0) + \frac{2\sqrt{3}}{3} A_0(a, t, 0, 0) \right\},
\]

where \(\nu_0(\nu, t, K^2 = 0, \delta = 0)\) in the imaginary part of the amplitude for the off-shell pion nucleon scattering, \(\pi^+(q_1) + N(p_1) \rightarrow \pi^-(q_2) + N(p_2)\), with \(q_{1}^2 = q_{2}^2 = t/4 < 0\), \(G(t)\) shows the Born term which becomes \(g_{\pi}^2(0)\) as \(t \to 0\), and \(\nu_0 = m_{\pi} m_{\pi} + (m_{\pi}^2 + t/4)/2\). On the other hand, by setting \(q_{1}^2 = q_{2}^2 = t/4 < 0\) in addition to \(q_{1}^+ = q_{2}^+ = 0\), we obtain the sum rule

*3 Here we consider the spin independent part, i.e., spin averaged part.
\[ \frac{1}{2} \int_{\nu_1}^{\infty} d\nu \{ W_2^{\nu}(\nu, K^2=K^{12}, t, \delta=0) + W_2^{\nu}(\nu, K^2=-K^{12}, t, \delta=0) \} \]

where we define \( W_2^{ab} = W_2^{ab}/4\pi, \nu W_2^{ab} = F_2^{ab} \) in each reaction specified by \( ab \), and \( \nu_1 = (t/4-K^2)/2 \). The sum rules (2·10) and (2·11) are convergent when \( \sigma_0(0) \) is one as far as \( \sigma_0'(0) \) is positive, where \( \sigma_0'(0) \) is defined as \( \sigma_0(t) = 1 + \sigma_0'(0)t \), and give the relation

\[ G(t) + \frac{2f^2}{\pi} \int_{\nu_0}^{\infty} d\nu \frac{m_{\pi}^4}{m_{\pi}^2-t/4} \{ \sigma^+(\nu, t, K^2=0, \delta=0) + \sigma^-(\nu, t, K^2=0, \delta=0) \} \]

\[ = \frac{1}{2} \int_{\nu_1}^{\infty} d\nu \{ W_2^{\nu}(\nu, K^2=K^{12}, t, \delta=0) + W_2^{\nu}(\nu, K^2=-K^{12}, t, \delta=0) \}. \] (2·12)

We assume the leading high energy behavior of \( \sigma^+ \) and \( W_2 \) near \( t=0 \) as

\[ \frac{m_{\pi}^4}{(m_{\pi}^2-t/4)^2} \{ \sigma^+(\nu, t, K^2=0, \delta=0) + \sigma^-(\nu, t, K^2=0, \delta=0) \} \sim A_0 \left( \frac{\nu}{a} \right)^{\sigma_0(t)-1} \exp[bt], \] (2·13)

\[ \nu \{ W_2^{\nu}(\nu, K^2=K^{12}, t, \delta=0) + W_2^{\nu}(\nu, K^2=-K^{12}, t, \delta=0) \] 

\[ \sim \left( \frac{K_0^2}{K^2} \right)^{\sigma_0(t)-1} A_1(-K^2, t) \left( \frac{\nu}{a} \right)^{\sigma_0(t)-1} \exp[bt], \] (2·14)

where \( K_0^2 = -1(\text{GeV})^2 \). Before going into details we explain why we take the asymptotic behavior of \( \nu W_2 \) as in Eq. (2·14). First we consider the case \( t=0 \). In this case, we know the form in Eq. (2·14) contradicts the leading double logarithmic approximation in QCD. \(^{14}\)

However, what we need in the following discussion is the behavior in the region \( \nu \gg Q^2 \). It is well known, in this region, the leading double logarithmic approximation breaks down. \(^{15}\) This happens because a simple ladder diagram does not dominate even if we take \( Q^2 \gg m_{\pi}^2 \). This point is clearly explained in Ref. 15). As a first deviation from a simple ladder diagram, a multiladder “fan” diagram is considered, and it is found that the increase of \( \nu W_2 \) in small \( x \) is screened by this diagram, and that there is a tendency to restore the unitarity. Then it is concluded that, if we want to know the behavior of \( \nu W_2 \) in very small \( x \), we must sum up all the possible multiladder diagram, and is suggested that the unitarity is satisfied ultimately. This point is also anticipated in Ref. 14). Thus we can expect that the unitarity is restored in the limit \( x \to 0 \). However it is not yet clear whether the leading behavior in this limit is dominated by a moving pole or by a fixed pole. In case of a fixed pole, though the mathematical method in Refs. 1) and 2) can be used, the one in this paper cannot be applied. Then we assume the leading behavior is dominated by a moving pole, since the physical method makes clear the physical meaning of the condition obtained by the sum rule. Next, the assumed form \( (K_0^2/K^2)^{\sigma_0(t)-1} \) does not affect the following discussion. All the ambiguities with respect to the behavior in \( K^2 \).

\(^{14}\) If certain conditions are satisfied, more complicated singularities can be easily taken into account (see the discussion later in this section and in §4).
and $t$ are included in $A_1$. Therefore no assumption is made with respect to the behavior in $K^2$ and $t$ except the analyticity near $t=0$. Now we rewrite the sum rule (2·12) as

$$G(t) + \frac{2f_\pi^2}{\pi} \int_{v_0}^{\infty} dv \left[ \frac{m_\pi^4}{(m_\pi^2 - t/4)^3} (\sigma^+ + \sigma^-) - A_0 \left( \frac{v}{a} \right)^{a_{p(t)}-1} \exp[bo t] \right]$$

$$+ \frac{2f_\pi^2 A_0 \exp[bo t]}{\pi} \int_{v_0}^{\infty} dv \left( \frac{v}{a} \right)^{a_{p(t)}-1}$$

$$= \frac{1}{2} \int_{v_1}^{\infty} dv \left[ W_2^{vp} + W_2^{vp} - \frac{1}{\nu} A_1 \exp[bo t] \left( \frac{-v}{aK^2} \right)^{a_{p(t)}-1} \right]$$

$$+ \frac{1}{2} A_1 \exp[bo t] \left( \frac{-1}{K^2} \right)^{a_{p(t)}-1} \int_{v_1}^{\infty} dv \left( \frac{v}{a} \right)^{a_{p(t)}-1}. \quad (2·15)$$

By setting $-a_{p'}(0)t = \epsilon > 0$ and by assuming the analyticity of $A_1$ near $t=0$ as $A_1(-K^2, t) = A_0(-K^2) + tA_1'(-K^2) + O(t^2)$, we obtain

$$\frac{2f_\pi^2 A_0 \exp[bo t]}{\pi} \int_{v_0}^{\infty} dv \left( \frac{v}{a} \right)^{a_{p(t)}-1} = \frac{2f_\pi^2 A_0}{\pi} \frac{1}{\epsilon} + \frac{2f_\pi^2 A_0}{\pi} \left\{ -\frac{b_0}{a_{p'}(0)} \ln a - \ln \nu_0 \right\} + O(\epsilon),$$

$$\frac{1}{2} A_1 \exp[bo t] \left( \frac{-1}{K^2} \right)^{a_{p(t)}-1} \int_{v_1}^{\infty} dv \left( \frac{v}{a} \right)^{a_{p(t)}-1}$$

$$= \frac{1}{2} A_1 \left\{ \frac{1}{\epsilon} + 2 - \frac{b_0}{a_{p'}(0)} \ln 2a \right\} - \frac{A_1'}{2a_{p'}(0)} + O(\epsilon), \quad (2·16)$$

where $\nu_0' = \nu_0 + m_\pi^2 + m_\pi^2/2$. Then as we let $\epsilon \to 0$, since the pole with respect to $\epsilon$ must cancel each other from both sides of Eq. (2·15), we obtain

$$A_1 \left( \frac{Q^2}{a} \right) = \pi A_0^2(Q^2), \quad (2·18)$$

and under the condition (2·18) we obtain

$$g_\pi^2(0) + \frac{2f_\pi^2}{\pi} \int_{\nu_0}^{\infty} dv \left\{ \sigma^+(\nu) + \sigma^-(\nu) - A_0 \right\} + \frac{2f_\pi^2 A_0}{\pi} \ln \left( \frac{1}{2\nu_0} \right)$$

$$= \int_0^{\infty} \frac{dx}{2x} \left\{ F_2^{vp}(x, Q^2) + F_2^{vp}(x, Q^2) - A_1 \right\} - \frac{A_1'(Q^2)}{2a_{p'}(0)}, \quad (2·19)$$

by assuming analyticity in $t$ near $t=0$. Now we assume the smooth extrapolation of the off-shell pion-nucleon scattering amplitude to the on-shell one. Then Eq. (2·18) together with the experimental value $f_\pi \approx 0.094$ (GeV) and $A_0 \approx 109^{18}$ determine $A_1'(Q^2)$ to be 1.22, and this value corresponds to the behavior of the sea quark distribution near $x=0$ in Refs. 1) and 2). Note that the scale factor $a$ cancels out from both sides of Eq. (2·15) because of the condition (2·18). Thus we are left with the term $A_1'$ as a regularization dependent term in the sum rule (2·19). Without the method to calculate it, the sum rule (2·19) is contentless. In principle, this quantity can be determined by the dynamics. Instead of

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*) Strictly speaking, when $\sigma_p(0) = 1$, the condition corresponding to Eq. (2·18) in Refs. 1)~3) was not derived as a necessary one to make the sum rule (2·4) valid. Our discussion here clearly shows the condition (2·18) is a necessary one.
solving this problem dynamically, however, we give one method to determine it by considering the Gottfried sum rule in the following two sections. Now we consider the assumptions (2·13) and (2·14) more. First we note that a pomeron pole dominance in either reaction \((\sigma^+ + \sigma^-)\) or \(\nu(W_{ep} + W_{en})\) is related to the other one through the sum rule (2·12). This happens because the pole structure with respect to \(\epsilon\) must take the same structure on both sides of Eq. (2·12). For example, in case of Eqs. (2·13) and (2·14), the pole structure is \(1/\epsilon\). In case of \((ln\nu)^n(\nu/\epsilon)^{a_0} + \ldots + a_n/\epsilon\), and each coefficients of \(\epsilon^{-k}\) for \(1 \leq k \leq n + 1\), must be the same on both sides of Eq. (2·12). It is a trivial exercise of Laplace transform to formulate this fact rigorously. Then the discussion that follows Eq. (2·14) shows that the high energy behavior of \(\{\sigma^+(\nu, q^2=0) + \sigma^-(\nu, q^2=0)\}\) satisfies unitarity. Conversely, if we assume the smooth continuation of the off-shell quantity to the on-shell one such that the unitarity is kept, the small \(x\) behavior of \(\nu W_2\) must satisfy unitarity. Because of this property, we obtain the view that the small \(x\) behavior of \(\nu W_2\) is determined by the spontaneous chiral symmetry breaking of the vacuum.

§ 3. Modified Gottfried sum rules

By the method in §2, we obtain the sum rule

\[
\frac{d}{dQ^2} \left[ \int_0^1 \frac{dx}{x} \left( F_2^{ep}(x, Q^2) - F_2^{en}(x, Q^2) \right) - A_1^{ep}(Q^2) \right] = 0, \tag{3·1}
\]

and the condition

\[
\frac{d}{dQ^2} A_1^{ep}(Q^2) = 0, \tag{3·2}
\]

where \(A_1^{ep}\) and \(A_1^{en}\) are defined in a manner similar to that of those in Eq. (2·14). Further, the sum rule in case of the neutron target is given by changing the suffix \(ep\) in Eqs. (3·1) and (3·2) into \(en\). Then, subtracting the two sum rules in case of the proton target and the neutron one, we obtain

\[
\frac{d}{dQ^2} \left[ \int_0^1 \frac{dx}{x} \left( F_2^{ep}(x, Q^2) - F_2^{en}(x, Q^2) \right) \right] = \frac{1}{a_p(0)} \frac{d}{dQ^2} \left[ A_1^{ep}(Q^2) - A_1^{en}(Q^2) \right], \tag{3·3}
\]

where we take \(A_1^{en} - A_1^{ep}\) since the pomeron is flavor singlet at \(t=0\). Now, if we assume the asymptotic form (2·14) comes only from a flavor singlet piece, the right-hand side of Eq. (3·3) becomes 0. Then the ambiguity in the sum rule (3·3) disappears and the Gottfried sum rule becomes \(Q^2\) independent. Now consider in what sense this assumption is justified. It is well known that in the leading logarithmic approximation at the two loops in QCD we obtain

\[
\int_0^1 \frac{dx}{x} \left[ F_2^{ep}(x, Q^2) - F_2^{en}(x, Q^2) \right] = \int_0^1 \frac{dx}{x} \left[ F_2^{en}(x, Q_0^2) - F_2^{en}(x, Q_0^2) \right] + 0.01 \{ a_s(Q^2) - a_s(Q_0^2) \}, \tag{3·4}
\]

where \(a_s(Q^2)\) is the running coupling constant. In Eq. (3·4), the origin of the \(SU(2)\) symmetry breaking of the sea quarks in the proton lies only in the second term on the
right-hand side. The recent experimental value is \(^{13)}\)
\[
\int_0^1 \frac{dx}{x} \left[ F_2^{ep}(x, Q^2) - F_2^{en}(x, Q^2) \right] = 0.24 \pm 0.02. \quad (\text{statistical}) \tag{3.5}
\]

Though the systematical error is large, this value is about the same magnitude as the one obtained by SLAC-MIT experiment.\(^{11)}\) Thus the experimental value seems to suggest a large deviation from the symmetrical value 1/3. Then, since the symmetry breaking term in Eq. (3·4) is very small, it is unreasonable to expect the perturbation can explain this large difference. Rather it is natural to expect the non-perturbative effects can explain it.\(^{12)},^{17)}\) In this case, the perturbatively predicted \(Q^2\) dependence may be shielded by the large non-perturbative effects or, more practically, it can be regarded as negligible compared with the non-perturbative contribution. It is in this sense we can regard the sum rule (3·3) with the flavor singlet assumption at \(t\neq 0\) physically meaningful.

§ 4. New sum rules

Here we assume the residue of the pomeron at \(t\neq 0\) is flavor singlet. Thus it comes from \(A_0(a, t, 0, 0)\) in Eq. (2·11) in case of Eq. (2·14). Then, since
\[
\int_{\nu_1}^\infty d\nu W_2^{ep}(\nu, K^2 = -K'^2, t, \delta = 0)
= \frac{1}{6\pi} P \int_{-\infty}^\infty \frac{da}{a} \left\{ \frac{2\sqrt{6}}{3} A_0(a, t, 0, 0) + A_s(a, t, 0, 0) + \frac{\sqrt{3}}{3} A_8(a, t, 0, 0) \right\}, \tag{4·1}
\]
the residues of the pomeron are related as
\[
A_1(-K^2, t) = 6 A_1^{ep}(-K^2, t). \tag{4·2}
\]

Especially we obtain
\[
A_1^0 = 6 A_1^{ep}, \quad A_1^1 = 6 A_1^{1ep}. \tag{4·3}
\]

Then we differentiate Eq. (2·19) as
\[
\frac{d}{dQ^2} \left[ \int_0^1 \frac{dx}{2x} \left( F_2^{ep}(x, Q^2) + F_2^{en}(x, Q^2) - A_1^0 \right) \right] = -\frac{1}{2a_0'(0)} \frac{d}{dQ^2} A_1^1(Q^2). \tag{4·4}
\]

By comparing Eqs. (3·1), (4·3) and (4·4), we obtain
\[
\frac{d}{dQ^2} \left[ \int_0^1 \frac{dx}{2x} \left( F_2^{ep}(x, Q^2) + F_2^{en}(x, Q^2) - A_1^0 \right) \right] = 6 \frac{d}{dQ^2} \left[ \int_0^1 \frac{dx}{2x} \left( F_2^{ep}(x, Q^2) - \frac{1}{6} A_1^0 \right) \right]. \tag{4·5}
\]

Then, \(A_1^0\) cancels from both sides of Eq. (4·5), we obtain
\[
\frac{d}{dQ^2} \left[ \int_0^1 \frac{dx}{2x} \left( F_2^{ep}(x, Q^2) + F_2^{en}(x, Q^2) - 6 F_2^{en}(x, Q^2) \right) \right] = 0. \tag{4·6}
\]

The integrated form of the sum rule (4·6) becomes
\[ \int_0^1 \frac{dx}{2x} \left\{ F_2^{ep}(x, Q^2) + F_2^{en}(x, Q^2) - 6F_2^{ep}(x, Q^2) \right\} = g_A^2(0) + \frac{2f_\pi^2}{\pi} \int_0^\infty \frac{d\nu}{\nu} \left\{ \sigma^+(\nu) + \sigma^-(\nu) - A_0 \right\} + \frac{2f_\pi^2 A_0}{\pi} \ln \left( \frac{1}{2\nu_{0'}} \right) - 3C_p, \quad (4.7) \]

where \( C_p \) is the integration constant of Eq. (3.1). A similar relation holds in case of the neutron target as

\[ \int_0^1 \frac{dx}{2x} \left\{ F_2^{np}(x, Q^2) + F_2^{en}(x, Q^2) - 6F_2^{en}(x, Q^2) \right\} = g_A^2(0) + \frac{2f_\pi^2}{\pi} \int_0^\infty \frac{d\nu}{\nu} \left\{ \sigma^+(\nu) + \sigma^-(\nu) - A_0 \right\} + \frac{2f_\pi^2 A_0}{\pi} \ln \left( \frac{1}{2\nu_{0'}} \right) - 3C_n, \quad (4.8) \]

where \( C_n \) is defined in a manner similar to that of \( C_p \) in case of the neutron target. The difference of Eqs. (4.7) and (4.8) gives the Gottfried sum rule as

\[ \int_0^1 \frac{dx}{x} \left\{ F_2^{ep}(x, Q^2) - F_2^{en}(x, Q^2) \right\} = C_p - C_n. \quad (4.9) \]

In parton language, Eq. (4.6) constrains the \( Q^2 \) dependence of the flavor symmetry breaking of the sea quarks. Compared with the Gottfried sum rule, it offers us new information of strange quark in the proton. Sum rules (4.7) and (4.8) contain unknown constants \( C_p \) and \( C_n \) respectively. However, since they hold in any \( Q^2 \), we can determine them at one \( Q^2 \) and use them as input at other \( Q^2 \). Then these sum rules also constrain the strange sea quark in the proton. Finally it should be noted that the method can be easily applied to the case of the non-leading pole with respect to \( \epsilon \) and the finite part as \( \epsilon \rightarrow 0 \) in case of more complicated singularities as far as these singularities are given as \( \int f(\nu)\nu^{\mu(t-1)} \) and \( \lim_{\nu\rightarrow\infty} f(\nu)/\nu^{\delta}=0 \) for any \( \delta > 0 \) with respect to the behavior in \( \nu \) in Eq. (2.13) or (2.14).

§ 5. Conclusion

Regularization of the divergent sum rules derived from the current anticommutator on the null-plane is discussed. It is shown that the Gottfried sum rule for \( (F_2^{ep} - F_2^{en}) \), in general, depends on \( Q^2 \). In this case, however, we must consider that the residue of the pomeron at \( t \neq 0 \) has a flavor non-singlet piece. In this paper, we do not take this view. The leading logarithmic approximation at the two loops in QCD shows the \( Q^2 \) dependence in the Gottfried sum rule is very small even if it exists. Further, the experiment suggests that the non-perturbative effects are important in this sum rule. Then we assume the residue of the pomeron at \( t \neq 0 \) is flavor singlet. In this case, we obtain new sum rules (4.6)~(4.8) in addition to the Gottfried sum rule. These new sum rules give us information of the flavor symmetry breaking of the sea quarks in the proton, especially of a heavy quark. In case of an \( SU(3) \) model with Cabibbo angle being zero explicitly given in this paper, \( SU(2) \) symmetry breaking part of the sea quarks is related to the \( SU(3) \) symmetry breaking part of the sea quarks.
Appendix A

Here we give a far more general discussion than the one given in Ref. 2) to make sense the non-local quantity in the text. Now we consider the hadronic tensor defined by

\[ C_{\alpha \beta}^{\mu \nu} = \int d^4 x \exp[i q \cdot x] \langle \rho | [J_\alpha^{\mu}(x), J_\beta^{\nu}(0)] | \rho \rangle_c, \tag{A.1} \]

and its DGS representation\(^{18,19}\)

\[ C_{\alpha \beta}^{\mu \nu} = \int d^4 x \exp[i q \cdot x] \int_0^\infty d\lambda^2 \int_{-1}^1 d\beta \left[ (\partial^\alpha \partial^\nu - \Box g^{\mu \nu}) \{ h_1^{ab}(\lambda^2, \beta) + i p \cdot \partial g_1^{ab}(\lambda^2, \beta) \} \right. \]
\[ + \left. \{- \Box p^\mu p^\nu + p \cdot \partial (p^\mu \partial^\nu + p^\nu \partial^\mu) - g^{\mu \nu} (p \cdot \partial)^2 \} h_2^{ab}(\lambda^2, \beta) \} \exp[i \beta p \cdot x] i d(x, \lambda^2). \tag{A.2} \]

As is well known,\(^{18,19}\) in case of the stable particle, the representation (A.2) can be generalized as

\[ W_{\alpha \beta}^{\mu \nu} = \int d^4 x \exp[i q \cdot x] \langle \rho | [J_\alpha^{\mu}(x), J_\beta^{\nu}(0)] | \rho \rangle_c, \tag{A.3} \]
\[ = \int d^4 x \exp[i q \cdot x] \int_0^\infty d\lambda^2 \int_{-1}^1 d\beta \left[ (\partial^\alpha \partial^\nu - \Box g^{\mu \nu}) \{ h_1^{ab}(\lambda^2, \beta) + i p \cdot \partial g_1^{ab}(\lambda^2, \beta) \} \right. \]
\[ + \left. \{- \Box p^\mu p^\nu + p \cdot \partial (p^\mu \partial^\nu + p^\nu \partial^\mu) - g^{\mu \nu} (p \cdot \partial)^2 \} h_2^{ab}(\lambda^2, \beta) \} \exp[i \beta p \cdot x] i d(x, \lambda^2), \tag{A.4} \]

where \( i d(x, \lambda^2) \) is defined as

\[ i d(x, \lambda^2) = \frac{1}{(2\pi)^3} \int d^4 k \exp[-ik \cdot x] \delta(k^2 - \lambda^2). \tag{A.5} \]

Now we consider restricting \( C_{\alpha \beta}^{\mu \nu} \) and \( W_{\alpha \beta}^{\mu \nu} \) to the null-plane. Here the light-cone variables are defined as \( x^\pm = (x^0 \pm x^3)/\sqrt{2} \), \( g^{++} = g^{--} = 0 \), \( g^{+} = 1 \). Intuitively, the restriction to the null-plane, \( x^+ = 0 \), can be done by the integration over \( q^- \). To assure this intuitive method, we need some conditions on the weight functions \( h_1^{ab}(\lambda^2, \beta) \) and \( g_1^{ab}(\lambda^2, \beta) \). Thus we consider the quantity \( \{ \lim_{\Lambda \to \infty} \int_0^\Lambda q^- \exp[-(q^-)^2/\Lambda^2] C_{\alpha \beta}^{\mu \nu} \} \) or \( \{ \lim_{\Lambda \to \infty} \int_0^\Lambda q^- \exp[-(q^-)^2/\Lambda^2] W_{\alpha \beta}^{\mu \nu} \} \) and extract these conditions. Then we obtain that the current commutator and the current anticommutator are restricted to the null-plane under the same conditions.\(^2\) After all we obtain

\[ \int_0^{\Lambda} dq^- C_{\alpha \beta}^{\mu \nu}(p, q) = 2\pi \int_0^{\Lambda} dq^- \int_0^{\Lambda} d\lambda^2 \int_{-1}^1 d\beta \left[ (\beta^2 h_1^{ab} + \lambda^2 h_2^{ab}) - \beta^2 m^2 (h_2^{ab} - \beta g_1^{ab}) \right] (p^+)^2 \]
\[ - 2p^+ (h_2^{ab} - \beta g_1^{ab}) (p \cdot q + \beta m^2) (q^+ + \beta p^+). \]
Now we assume the commutation relation

\[ [I_{a^+}(x), J_{b^-}(0)]\delta(x) = if_{abc} J_c(0) \delta(x). \]  

(A·8)

By substituting Eq. (A·8) on the left-hand side of Eq. (A·6), we obtain

\[ p^+ \int_0^\infty d\lambda^2 \int_{-1}^1 d\beta \{h_{1ab} + m^2(h_{2ab} - \beta g_{1ab})\} = if_{abc} \langle p | J_c^+(0) | p \rangle \]  

(A·9)

and

\[ 2\pi \int_0^\infty dq^- \int_0^\infty d\lambda^2 \int_{-1}^1 d\beta \{(\beta^2 h_{1ab} + \lambda^2 h_{2ab}) - \beta^2 m^2(h_{2ab} - \beta g_{1ab})\}(p^+)^2 \]
\[ - 2p^+(h_{2ab} - \beta g_{1ab})(p \cdot q + \beta m^2)(q^+ + p^+)\delta((q + \beta p)^2 - \lambda^2)e(q^+ + p^+) = 0. \]  

(A·10)

Now, since Eq. (A·10) holds for the arbitrary parameter \( p^+ > 0, q^+ \), we take \( (q^+/p^+)^2 > 1 \), then the integration path, \( (q + \beta p)^2 - \lambda^2 = 0 \), in the \((\beta, \sigma)\) plane, where \( \sigma - \beta^2 - \beta^4 \), pass the support of the weight functions only for \( p \cdot q > 0 \), and we can safely set \( \epsilon(q^+ + p^+) = 1. \)

Thus we obtain

\[ 2\pi \int_0^\infty dq^- \int_0^\infty d\lambda^2 \int_{-1}^1 d\beta \{(\beta^2 h_{1ab} + \lambda^2 h_{2ab}) - \beta^2 m^2(h_{2ab} - \beta g_{1ab})\}(p^+)^2 \]
\[ - 2p^+(h_{2ab} - \beta g_{1ab})(p \cdot q + \beta m^2)(q^+ + p^+)\delta((q + \beta p)^2 - \lambda^2)e(q^+ + p^+) = 0. \]  

(A·11)

for \( (q^+/p^+) > 1 \). The same kind of the discussion can be done for \( (q^+/p^+) < -1 \) and we obtain Eq. (A·11) also in this case. Further, by the restriction \( |q^+/p^+| > 1 \), the integration domain of \( \sigma \) and \( \beta \) is not restricted, since \( q^- \) changes from \(-\infty\) to \( +\infty\) and since the condition \( (q + \beta p)^2 - \lambda^2 = 0 \) is a linear equation with respect to \( q^- \). The situation is completely different from the case when we discuss the scaling property of the structure function.\(^{20}\) Now, since \( q^+ \) and \( p^+ > 0 \) are arbitrary parameters and independent of each other, the restriction \( |q^+/p^+| > 1 \) is unnecessary, and Eq. (A·11) holds at any \( q^+ \) except at \( q^+ = 0 \). Then if we understand the value at \( q^+ = 0 \) by the limiting value as \( q^+ \to 0 \), Eq.
(A·11) holds at any $q^+$. Thus, by substituting Eq. (A·11) into Eq. (A·7), we obtain

$$
\int_{-\infty}^{\infty} dq^+ W_{ab}^+(p, q^+) = 2p^+ \int_{-\infty}^{\infty} dx^+ \exp[iq^+x^-] P \frac{1}{x^-} G_{ab}(p^+, x^-, 0), \tag{A·12}
$$

where

$$
G_{ab}(p \cdot x, 0) = -i \int_0^\infty d\lambda^2 \int_1^1 d\beta \exp[i\beta (p \cdot x)] \beta \{h_1^{ab} + m^2(h_2^{ab} - \beta g_1^{ab})\}. \tag{A·13}
$$

The same kind of the discussion can be done for the axial-vector currents if we assume

$$
[J_{a}^{\pm}(x), J_{b}^{\pm}(0)]\delta(x^+) = if^{abc} J_{c}^{\pm}(0)\delta(x), \tag{A·14}
$$

and we obtain finally

$$
\int_{-\infty}^{\infty} dq^+ \{W_{a}^{++} - W_{b}^{++}\} = 0, \tag{A·15}
$$

where $W_{ab}^{++}$ is defined in a manner similar to that of $W_{ab}^{++}$ for the axial-vector currents. The sum rules in §2 are nothing but Eqs. (A·12) and (A·15). Since the process and the assumption necessary to derive the sum rules from Eq. (A·12) or Eq. (A·15) are well known, we do not repeat it.\textsuperscript{211} One drawback of the discussion in this appendix is the possible divergence from the integration over $\beta$ which is expected in the Regge theory. However this point is overcome by considering the analytical continuation from the non-forward direction. Thus we give the discussion of the DGS representation in the non-forward case in Appendix B.

**Appendix B**

Here we point out that the discussion in Appendix A can be repeated in the non-forward case. In order to avoid the inessential complication, we take the scalar currents and consider

$$
C_{ab} = \int d^4 x \exp[iK \cdot x] \langle p_1 \left[ J_a\left(\frac{x}{2}\right), J_b\left(-\frac{x}{2}\right) \right] p_2\rangle. \tag{B·1}
$$

It is straightforward to show

$$
C_{ab}(K^2, p \cdot K, A^2, A \cdot K)
= \int_0^\infty d\lambda^2 \int_0^1 d\beta \int_0^\infty d\gamma \epsilon(p \cdot K + \beta p^2 + \gamma A \cdot P) \delta((K + \beta p + \gamma A)^2 - \lambda^2) H_{ab}(\lambda^2, \beta, A^2, \gamma), \tag{B·2}
$$

where the Born term is not included in Eq. (B·2). We add it by hand after all the consideration is completed. Now we consider the case $q_1^2 = q_2^2$, $p_1^2 = p_2^2$ and $A^2 < 0$, which is really needed in this paper. In this case, $A \cdot K = A \cdot P = 0$, thus Eq. (B·2) takes the form

$$
C_{ab}(K^2, PK, A^2, 0)
= \int_0^\infty d\lambda^2 \int_0^1 d\beta \int_0^\infty d\gamma \epsilon(p \cdot K + \beta p^2) \delta(K^2 + 2\beta P \cdot K - \sigma) H_{ab}(\lambda^2, \beta, A^2, \gamma), \tag{B·3}
$$
where $\sigma = \lambda^2 - \beta^2 P^2 - \gamma^2 \Delta^2$. Since the essence of the discussion in Appendix A is the support property of $H_{ab}$ in the $(\beta, \sigma)$ plane, we consider it in the following. The physical vectors, $P$ and $K$, always satisfy the condition $(P \cdot K)^2 \geq P^2 K^2$. Then, for the fixed $\Delta^2 < 0$ and $(P \cdot K)^2 \geq P^2 K^2$, we call the union of the $s$ channel physical region and the $u$ channel one in the $(P \cdot K, K^2)$ plane as $\bar{R}$, and its complement as $R$. For any point in $R$, if the integration path exists in Eq. (B·3), $H_{ab}$ must be zero. This line of reasoning is identical to that in the forward case as far as $\min(M_s^2 - m_s^2, M_u^2 - m_u^2) \geq -\Delta^2 / 4$, where $M_s^2 (M_u^2)$ is the minimum of $(P + K)^2$ in the $s$ channel ($(P - K)^2$ in the $u$ channel). After all we obtain the support of $H_{ab}$ as in the figure, where $H_{ab}$ exists only in the shaded region. The important property of Eq. (B·3) is that the integration path always intersects or tangents to the parabola $\sigma = -\beta^2 P^2$, and that the sign function $\varepsilon(P \cdot K + \beta^2)$ changes it sign below the parabolla $\sigma = -\beta^2 P^2$. The property is the same as in the forward case and sufficient to show that the DGS representation of the current anticommutator exists and that the discussion in Appendix A can be repeated in the non-forward case.

Fig. 1. The support property of $H_{ab}$ in the $(\beta, \sigma)$ plane, where $H_{ab}$ exists only in the shaded region.

References

9) S. Adler and R. F. Dashen, Current Algebra and Application to Particle Physics (Benjamin, New York, 1968).

*) This condition is in general stronger than the condition where the $s$ channel and the $u$ channel are disconnected, however both conditions becomes identical when $M_s = M_u$. 

New Sum Rules from the Current Anticommutator on the Null-Plane


