Conclusions

1 The apparent time mean widening of the jet in the region immediately downstream of the nozzle exit is characterized by the formation and subsequent coalescence of vortices formed outside the nozzle exit for a wide range of Reynolds numbers.

2 Two classes of coalescence, nascent stage and mature stage, were observed to occur.

3 Vortex growth and coalescence may be symmetrical or asymmetrical with respect to the centerline of the jet nozzle exit. The symmetrical mode leads to varicose instability while the asymmetrical mode leads to sinuous instability.

4 The transport velocity of the vortices is on the order of 0.5 to 0.55 of the average velocity at the nozzle exit and varies with the distance from the nozzle exit.

5 Three typical dimensionless breakdown frequencies (in terms of Strouhal number) based on visualization of the vortex development were found to obey the relation $\text{St} = 0.012\,(\text{Re})^{0.5}$.

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DISCUSSION

O. K. Oseberg[2] and S. J. Kline[3]

Results closely connected to those of Rockwell and Niccolls have been reported recently by the discussers (Oseberg and Kline, 1971).[4] Nine cases were investigated covering three ratios of jet to external velocity and three initial conditions: suction, no suction-blowing, and blowing in the boundary layers of both surfaces shortly upstream from the nozzle lips.

The primary results of that study which relate to those of Rockwell and Niccolls are as follows.

1 In the zone close to the nozzle lips the vortices are convected downstream at a velocity approximating $U_c = (\bar{U} + U_\infty)/2$; where: $\bar{U}$ and U_c follow the nomenclature of Rockwell and Niccolls and U_∞ is the velocity of the external flow field.

2 An invariant Strouhal number, S, can be formed in the following way:

$$\text{S} = \frac{Bf}{U_c}$$

where B = total displacement (or blockage) at the start of the jet; that is

$B = \delta_j{}^* + \delta_\infty{}^* + t$

$\delta_j{}^*$ = displacement thickness of inner surface boundary layer

$\delta_\infty{}^*$ = displacement thickness of outer surface boundary layer

t = body base thickness at lip

For the cases of suction and no suction-blowing, S has the value:

$$0.205 \leq \text{S} \leq 0.280$$

The flat plate wake data of Sato and Kuruki (1961)[5] and the cylinder wake data of Roshko (1954)[6] also fall within this range.

3 The vortex arrays observed were primarily symmetric. However, in some instances an antisymmetric mode was observed. The causes underlying the appearance of these distinct modes appear to be subject to large uncertainty at this time.

4 Even for the very widely varying initial conditions studied, the effects of history vanish and similar normalized velocity profiles are observed for $x/w \geq 13$.

Relating these results to those of Rockwell and Niccolls, it is possible to make the following observations. With regard to 1 in the foregoing, we expect that the external velocity in their apparatus would lie in the range $0 \leq U_\infty/\bar{U} \leq 0.1$; this leads to

$$0.5 \leq \frac{U_c}{\bar{U}} \leq 0.55.$$

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[2] Assistant Professor, San Francisco State College, San Francisco, Calif.

[3] Professor and Department Head, Stanford University, Stanford, Calif.

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This is precisely what Rockwell and Niccolls report.

With regard to conclusion 2 in the foregoing, the invariant Strouhal number, S, can be rewritten in the nomenclature of Rockwell and Niccolls as follows:

$$S = \mathrm{St} \cdot \frac{\bar{U}}{U_e} \cdot \frac{B}{w}.$$

Given the Reynolds number and the strong acceleration for the nozzle employed by Rockwell and Niccolls one expects laminar boundary layers at the lip. Hence for fixed geometry, one expects to find a proportionality $\delta_j^*/w = \text{const } \mathrm{Re}^{-0.5}$. Also, $U_e = \text{const } \bar{U}$. Combining these results one obtains:

$$S = \frac{\delta_j^* f}{U_e} = C_2.$$

Since δ_j^* and t are not given by Rockwell and Niccolls, it is not clear how C_2 can be evaluated; perhaps the authors can supply this information as a further check on the result given in 2 previously.

P. Freymuth[7]

The paper by Rockwell and Niccolls adds an interesting method of flow visualization to the investigation of free jets and their figures show in remarkable clarity the development of vortex flow. Obviously the Reynolds number range investigated is low enough that the thickness of the shear layers bounding the jet is comparable to the jet width and thus "crosstalk" between the opposing shear layers is possible. The degree of coupling between the two shear layers should depend on the ratio of shear layer thickness to jet width which presumably is the only characteristic number on which the properly nondimensionalized flow field depends. Only in the limiting cases of fully developed jet flow (low Reynolds numbers) and of single free shear layers separated from each other by a large potential core (high Reynolds numbers) is this parameter unneeded. Although the ratio of shear layer thickness to jet width is controlled by the Reynolds number $\bar{\mathrm{R}}\mathrm{e}$ for a particular nozzle configuration information on this ratio for the figures shown would be very helpful.

It should be mentioned that the frequency halving effect has been noticed in plane jets also by Wille [21][8] and by Browand [22]. Kelly [23] explained this effect that also occurs in axisymmetric jets as a secondary instability of the already periodically disturbed shear layer. Michalke [24] reviews the state of the art up to 1970.

Rockwell and Niccolls find from their pictures that the symmetric mode of disturbances is prevalent at low Reynolds numbers. Calculations by Michalke and Schade [25] who approximate a symmetric jet profile by a sequence of straight lines indicate that the spatial amplification factor for disturbances should be larger for the asymmetric than for the symmetric mode. Also, recent calculations by Mattingly and Criminale [26] using a curved velocity profile and the theory of spatially growing disturbances developed by Gaster [27] show that the spatial amplification factor is higher for the asymmetric mode.

It remains an uninvestigated problem whether stabilization of the natural disturbance frequency near the frequency of maximum amplification is due to selective amplification of the random aerodynamic disturbances close to the nozzle lips or whether it is due to a feedback mechanism between the amplified disturbances and the flow near the nozzle lip. The latter mechanism would be similar to the one that stabilizes the shedding frequency of a Karman vortex street in the wake behind a cylinder. Only very recently a first attempt has been made by Morkovin and Paranjape [28] to understand the flow mechanisms close to the nozzle lip in the presence of an exciting sound field.

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Authors' Closure

The authors wish to thank Professors Oseberg, Kline, and Freymuth for their interesting discussions. The primary intent of our investigation was the detailed portrayal of the vortex growth and the attendant destruction of the jet potential core. However, quantitative characterization of the dimensionless breakdown frequency is also important in the transition phenomena. With regard to determination of an invariant Strouhal number, experimental $\frac{\delta^*}{W} = \frac{\delta_j^*}{W} = f(\bar{\mathrm{R}}\mathrm{e})$ is not available for this nozzle. Wille [30] cites the work of Schade and Michalke [29][9] for a vortex filament axisymmetric nozzle for which

$$\frac{\delta^*}{D} = \frac{1.2}{\sqrt{\bar{\mathrm{R}}\mathrm{e}}}$$

and Becker and Massaro [16], for an ASME axisymmetric nozzle, obtained

$$\frac{\delta^*}{D} = \frac{0.9}{\sqrt{\bar{\mathrm{R}}\mathrm{e}}}$$

For our nozzle, which had no parallel length at its exit, it is expected that $\frac{\delta^*}{W}$ is of the order of $\frac{1.0}{\sqrt{\bar{\mathrm{R}}\mathrm{e}}}$. (As Becker and Massaro demonstrate, an upper limit for the displacement thickness can be estimated using the Blasius flat plate relation.) Using $\frac{\delta^*}{W} \cong \frac{1.0}{\sqrt{\bar{\mathrm{R}}\mathrm{e}}}$ gives an estimate of the invariant Strouhal number based on average velocity at the nozzle exit of $\mathrm{St}' = \frac{f\delta^*}{\bar{U}} \cong 0.012$. Michalke and Wille [31] present the following data for a planar jet:

$$\frac{f\delta^*}{\bar{U}} = 0.016 \quad \text{for} \quad \bar{\mathrm{R}}\mathrm{e} > 10^4$$

and

$$0.01 \lesssim \frac{f\delta^*}{\bar{U}} \lesssim 0.016 \quad \text{for} \quad \bar{\mathrm{R}}\mathrm{e} < 10^4$$

Also, for an axisymmetric jet they report

$$\frac{f\delta^*}{\bar{U}} \cong 0.023 \quad \text{for} \quad \frac{\delta^*}{D} = \frac{1.2}{\sqrt{\bar{\mathrm{R}}\mathrm{e}}}$$

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[7] Department of Aerospace Engineering Sciences, University of Colorado, Boulder, Colo.

[8] Numbers in brackets designate Additional References at end of discussion.

[9] Numbers 29–31 in brackets designate Additional References at end of Closure.

For the independent investigation of Becker and Massaro the result is

$$\frac{f\delta^*}{\bar{U}} = 0.01 \quad \text{for} \quad 10^3 \leq \bar{\text{Re}} \leq 10^4$$

Sato [9] chose to employ momentum thickness (θ) as the characteristic length. His invariant Strouhal number, for symmetrical fluctuations in planar jets is

$$\text{St}' = \frac{f\theta}{\bar{U}} = 0.015 \quad \text{for} \quad 1500 \leq \bar{\text{Re}} \leq 8000$$

Concerning Professor Freymuth's comment that the shear layer thickness should be comparable to the jet width at the $\bar{\text{Re}}$ values of our study, use of $\frac{\delta^*}{W} \cong \frac{1.0}{\sqrt{\bar{\text{Re}}}}$ for the extreme values of Re for our investigation gives

$$0.011 \lesssim \frac{\delta^*}{W} \lesssim .024$$

for the lowest and highest values of $\bar{\text{Re}}$ of this study. The boundary layer thickness should be only about three times thicker than the displacement thickness. Inspection of Figs. 6 and 7 shows that this ratio is indeed small.

It is difficult to correlate our results as a limiting case of the dimensionless number of Professors Oseberg and Kline involving U_∞, $\bar{U}$, t, $\delta^*(= \delta_j^*)$, and δ_∞^*. The parameter t is not easily determined for our study since the inside wall and outside wall of the nozzle are not parallel near the exit. Also, U_∞ is a low magnitude entrainment velocity, so it is formidable to measure δ_∞^* and U_∞ for low speed large scale jet flows.

The authors agree with Professors Oseberg and Kline that the causes of symmetrical and asymmetrical modes of vortex growth are not adequately understood. Although references [25] and [26] show a predominance of the asymmetric mode as a result of theoretical determination of the spatial amplification factor, Sato's hot wire measurements [9] indicate that the shape of the symmetrical nozzle exit profile is important in determining whether the jet growth is in the symmetrical or asymmetrical mode. In the particularly revealing study of Morkovin and Paranjape [28], where a planar jet was excited locally near the nozzle exit, it was noted that although the jet was excited symmetrically, any small departures from the symmetry of the exciting field, of the jet itself, or of the jet nozzle exit lips tended to drive the jet into the asymmetric mode.

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