Plate motions and the geomagnetic field — I. Quaternary and late Tertiary

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Received 1982 September 17; in original form 1982 June 14

Summary. We have tested the hypothesis that the apparent increase in the northward offset of the axial dipole (i.e. the ratio \( g_2^0 / g_1^0 \)) with age for the last 25 Ma (Wilson & McElhinny) is due to the failure to correct for plate motions. Spherical harmonic analyses were performed on two types of data, palaeomagnetic poles from continents, islands and seamounts, and magnetic inclinations from deep sea cores, after returning the sampling sites to their predrift locations in fixed hotspots coordinates. The results show that the quadrupole term \( g_2^0 \) maintained a value of about 0.05 \( g_1^0 \) throughout the last 35 Ma, and that the axial octupole had a small value of about 0.02 \( g_1^0 \), for the last 5 Ma. Sea core inclinations analysed separately gave essentially the same results as continental palaeopoles for the last few million years. Independent data sets for the past 2 Ma and for the period 2–6 Ma gave nearly identical solutions, showing that the persistence of the axial terms is not a phenomenon of the more densely sampled Quaternary field alone. Nor is it an artefact of predominantly northward motion of the plates: returning all 0–5 Ma data to a 5 Ma reconstruction failed to eliminate the quadrupole. The non-zonal coefficients for the 0–5 Ma field are typically an order of magnitude less than the zonal terms, with the exception of \( h_3^1 \), and are probably insignificant, suggesting that longitudinal drift is generally effective in averaging out these components.

The present distribution of data is inadequate to determine coefficients of third and higher degree prior to 5 Ma, but there is evidence that \( g_2^0 \) may have changed little during the last 35 Ma. In addition, there has been very little relative motion between the palaeomagnetic dipole axis, as defined by the first degree terms, and the axis of the hotspots reference frame.

1 Introduction

There is good evidence that the geomagnetic field, averaged over the last few million years, contained persistent axial non-dipole components sufficient to introduce errors of several
degrees into palaeopole positions calculated using the axial geocentric dipole formula (Creer,
Georgi & Lowrie 1973; Wells 1973; Georgi 1974; Wilson & McElhinny 1974; Merrill &
McElhinny 1977; Coupland & Van der Voo 1980). Of these studies, only the latter
attempted to correct for the effects of plate motions in order to extend the analysis to times
earlier than 5 Ma. Thus the conclusion of Wilson & McElhinny that the quadrupole com-
ponent has declined progressively over the last 25 Ma must be treated with caution.

Coupland & Van der Voo found that the Gauss coefficients $g_2$ and $g_3$ were both important
during the last 7 Ma, each having the same sign as the axial dipole. They concluded that the
late Tertiary (7–26 Ma) field had a rather different configuration, however, with a near-zero
value of $g_2$ and a substantial $g_3$ term. The axial offset dipole model (Wilson 1970, 1971)
would therefore seem to be inappropriate, implying as it does a dominantly
$g_1 + g_2$ field.

In this study, a palaeomagnetic data set representing 0–35 Ma was used to re-examine the
average field configuration, with particular emphasis on the global distribution of sampling
sites. Spherical harmonic analysis was carried out for all coefficients to the third degree to
see how effective time-averaging is in reducing the non-zonal terms. If the bias toward
positive declinations observed by Wilson (1972) truly reflects the average field, then some
significant non-zonal components are to be expected and their omission will lead to errors in
the zonal coefficients.

Global coverage is improved by the inclusion of sea core palaeomagnetic inclinations in
the inversion. For the past 5 Ma, these cores are sufficient for an analysis to be performed
independently, allowing a check to be made on the results from the predominantly continental
data set.

2 Data base

Palaeopoles were drawn from all compilations published in the Geophysical Journal since
Cowley 1977–1980) with additions from the Ottawa catalogues (Hicken et al. 1972; Irving,
Tanczyk & Hastie 1976) where these did not duplicate other results. Only a very few poles
from the USSR were used because of the apparently rather inadequate cleaning and dating
of most samples (Harrison 1979; Harrison & Lindh 1982). As a result, the total number of
poles used is less than in some previous studies based on older sources.

The basic selection criteria are those of the above compilers and hence vary slightly, e.g.
in the minimum number of samples per study accepted. It was decided, however, to include
all poles from these sources initially, and then to select on the basis of their positions in the
rotated framework: those requiring exceptionally large non-dipole components being
deleted. For data in the range 0–15 Ma, poles which lay more than 15° from the pole of the
reconstructions were discarded. Thus for the period 0–5 Ma 23 poles, from a data set of 162
poles, were eliminated, and for 5–15 Ma the data set was reduced from 64 to 54 poles. The
poles which were discarded usually disagreed with results obtained from nearby formations.
For older data a limit of 30° was set in order to retain a sufficiently large number of poles,
but no data fell outside this limit. However, two poles, which satisfied the selection criteria
but disagreed strongly with poles derived from nearby localities, were removed from this
data set.

An additional set of palaeomagnetic inclination measurements was compiled from the
Quaternary and Recent piston core data of Opdyke & Henry (1969) and from the Pacific
cores of Hammond, Epp & Theyer (1979) spanning the interval 0–35 Ma. In the case of the
latter, the published latitude changes were converted back to palaeoinclinations by adding
the site latitudes and then using the dipole formula.
Geomagnetic field during the past 35 Ma

Each datum was assigned either a best age based on radiometric dating, with a correction for the revised decay constants of Dalrymple (1979), or an age range based on the stratigraphic age given in the source lists, and referred to the new time-scale of Harland et al. (1983). Data older than 5 Ma from regions for which reliable rotations are not yet available, such as the Mediterranean area and the so-called displaced terrains of North America, were eliminated.

Directions corresponding to both normal and reversed polarities were included, the latter after changing the sign of the magnetic inclination and shifting the declination by 180°. Implicit in this is the assumption that the non-dipole field is fully reversing, that is, all terms change sign at a field reversal. Any standing component will give rise to errors in the values of the computed coefficients. Merrill & McElhinny (1977) have indicated that the quadrupole may be larger during reversed polarity intervals, but since the distribution and age of normal and reversed data inevitably differ, and Merrill & McElhinny applied no corrections for plate motions, it is by no means clear that the small difference they observed is significant.

3 Analysis

Briden, Hurley & Smith (1981) have described a method for creating palaeographic maps based on published spreading histories such as those of Pitman & Talwani (1972) for the North Atlantic, and Norton & Sclater (1979) for the Indian Ocean. A similar procedure was adopted here. In outline, finite rotations obtained from fits of marine magnetic anomalies, fracture zones and continental margins are used to return a set of plates to their former positions with respect to a reference fragment, here taken as Africa. Only plates for which the history of relative motion is fairly well-known are included; future work will allow the inclusion of further plates, such as those in the Mediterranean region and south-east Asia. The rotations used here are shown in Table 1. Note that the magnetic anomalies used in making fits have been referred to the polarity time-scale of Lowrie & Alvarez (1981), so that slight differences occur between published rotations and those given here.

The entire reassembly must be rotated so that it is correctly positioned with respect to the palaeogeographic axis. For the purposes of this study, the fixed hotspot frame of Morgan (1981) was used to locate the geographic poles for each reconstruction by simply adding his Africa-hotspots rotations to the relative rotations given in Table 1. The advantage of this is that the palaeogeographic coordinates may be fixed without reference to the palaeomagnetic data. Pacific plate data could be repositioned via Pacific-Antarctic rotations such as given by Weissel, Hayes & Herron (1977) and then using the sequence of rotations: Antarctica-Africa-hotspots, but it was decided to rotate directly into the hotspot framework via the Pacific-hotspots rotation pole determined from the Hawaiian chain by Clague & Jarrard (1973) using the rate given in Gordon & Cape (1981). This assumes that there has been no relative motion between the hotspots, but avoids errors in the chain of rotations through Antarctica. The difference between the two methods over the last 30 Ma turned out to be of the order of a degree or so in the position of the reconstructed sites. Errors introduced by motion of the hotspots both in relation to each other and to the rotation axis are assumed to be small within the time-scale of interest here. The plate-hotspot parameters used are given in Table 2, and the reconstructions and data distributions illustrated in Figs 1–4.

Non-overlapping time-windows were established representing the intervals 0–5, 5–15 and 15–35 Ma. For a pole or core to be included in one of these, its radiometric age or the mean of its age range had to lie within the window. Data in the two older groups were then returned to their pre-drift locations at 10, 20 or 30 Ma, according to which was closest to their assigned ages, using the rotations in Table 2. No rotations were applied to 0–5 Ma data,
Figure 1. 0–5 Ma data distribution on the present world outline map. • – sampling site for which a palaeomagnetic pole has been determined, ▼ – deep-sea core sampling site (inclination only). Cylindrical equidistant projection.
Geomagnetic field during the past 35 Ma

Figure 2. 15 Ma data distribution on 10 Ma reconstruction, symbols as for Fig. 1.
Figure 3. 15–25 Ma data distribution on 20 Ma reconstruction, symbols as for Fig. 1.
Figure 4. 25–35 Ma data distribution on 30 Ma reconstruction, symbols as for Fig. 1.
Table 1. Finite rotations used in making plate reassemblies.

<table>
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<tr>
<th>Plate Age Pole Angle</th>
<th>Plate Age Pole Angle</th>
<th>Plate Age Pole Angle</th>
<th>Plate Age Pole Angle</th>
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<td>(Ma) lat. (°N) long. (°E)</td>
<td>Australia + India/Antarctica</td>
<td>Antarctica/Africa</td>
<td>Eurasia/North America</td>
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<tr>
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<td>4 9.70 36.50 -2.81</td>
<td>4 5.80 -37.20 0.73</td>
<td>4 68.00 137.00 -1.04</td>
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<td>10 5.80 -37.20 1.83</td>
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<td>Antarctica/Africa</td>
<td>Eurasia/North America</td>
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<tr>
<td></td>
<td>4 5.80 -37.20 0.73</td>
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<td>30 5.80 -37.20 5.48</td>
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</tbody>
</table>

*Positive angle represents anticlockwise rotation when viewed from outside the Earth.

so that results from all parts of the Earth's surface could be used for this interval. Thus, the maximum difference between the 'best age' of a sample and the rotations used to relocate it was 5 Ma. The repositioned palaeopoles were then transformed into equivalent angles of inclination and declination at the rotated sites. The repositioned data sets were then analysed by least-squares to find the ratio $g_n^m / g_1^0$ for all Gauss coefficients to degree 3 (15 in all). Since the data had been normalized to the present field polarity, $g_1^0$ was set to -1.0 and a starting model of $g_n^m / g_1^0 = 0$ was adopted for the remaining coefficients. The function minimized was:

$$
\Phi = \sum_{i=1}^{M} \theta_i^2 + \sum_{j=1}^{N} \Delta I_j^2
$$

where $\theta_i$ is the difference between the observed direction of remanent magnetization at site $i$ and the field direction computed from the current set of coefficients, and $\Delta I_j$ is the inclination misfit at the deep-sea core sampling site. Weighting schemes based on the size of
Table 2. Plate-hotspot rotations.

<table>
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<tr>
<th>Plate</th>
<th>Age (Ma)</th>
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<th>Pole long. (°E)</th>
<th>Angle (°)</th>
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<td>4.64</td>
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<td>30</td>
<td>59.14</td>
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<td>-4.22</td>
</tr>
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</table>

residuals and on the computed values of $F$, the scalar field strength, and $H$, the horizontal field, at individual sites were tried but none was found to be more effective than unit weighting in resolving test fields.

Adequate global coverage is essential since large data gaps will allow a poorly distributed set of measurements to be fitted by a field with large non-zonal components which may give unreasonable values in the unsampled regions. Thus it is more important to have sites
representing each of the major regions of the Earth's surface than it is to have the data individually weighted. As long as a palaeomagnetic result conformed to the selection criteria described, then it was given equal weight in this study.

A subroutine (EO4FCF) was chosen from the Numerical Algorithms Group library to perform a modified Gauss–Newton type minimization using numerical estimates of the derivatives. The program used was tested using the 20 regionally averaged points of Creer et al. (1973) and found to give a slightly better fit to the data than the linear technique employed by those authors, with a reduction in the rms angular misfit from 3.6° to 3.3° in the case of a second degree solution, and to 2.9° for a third degree solution. Including the 12 regionally averaged palaeoinclinations of Georgi (1974), a second degree solution was computed in which ΔI_{rms} was reduced from 2.7° obtained by that author to 2.6°, and a third degree solution reducing ΔI_{rms} from 2.5° to 2.3°. In every case, g_0^0 assumed a value close to 0.05 g_0^0, while the third degree solutions gave values of g_3^0 close to zero.

4 Results for 0–5 Ma

It is clear from the results shown in Table 3 that the geomagnetic field during the last 5 Ma is quite well represented by Gauss coefficients of low degree. In the solution for 139 palaeopoles (column A), only three non-dipole coefficients appear to persist in the time-averaged field: the zonal terms g_2^2 and g_3^3, plus h_3^1. The values obtained for the first two are in agreement with those obtained by Merrill & McElhinny (1977) and explain the tendency toward low palaeomagnetic inclinations observed in the data together with a slight hemispherical asymmetry. A persistent h_3^1 was also found in Creer et al.'s analysis of 12 Quaternary and late Tertiary regionally averaged points (Creer et al. 1973), and could explain, in particular, the bias toward positive (eastward) declinations found in Icelandic lavas (Wilson 1972).

Analysis of the 66 deep-sea cores alone gave the result shown in column B of table 3. Again, the zonal quadrupole is important, but this time g_3^3 and h_3^1 are smaller while the non-zonal terms are greater. Bearing in mind that only the inclinations were fitted, however, the agreement between solutions A and B is quite good. Combining the two types of data gave the result in column C, which is our best estimate of the 0–5 Ma field.

Analysis of sea cores alone was repeated having applied the inclination corrections calculated by Harrison (1974) for the Pacific data. Surprisingly, the results showed much larger non-zonal coefficients with a best-fit pole nearly 9° from the reconstructed pole. The original data were therefore preferred, since it is unlikely that such a low-degree solution as that in column B could be obtained by chance. This discrepancy is probably explained by the non-Fisherian distribution of data from certain cores (Harrison, private communication).

Although no plate motion corrections were applied to the 0–5 Ma data, some of the older samples will have moved through several degrees of latitude since magnetization. To establish that such motions really cannot explain the persistence of the axial non-dipole field, the data were relocated on a 5 Ma plate assembly and reanalysed. Since most data are, in fact, less than 2 Ma in age, this represents an overcorrection for plate motions. Even so, the result (column D) shows that neither g_0^0 nor g_3^0 are eliminated, while the larger rms residuals testify that the 5 Ma reconstruction is inferior to the present arrangement of the plates.

It appears that Wilson's model of a dipole offset along the rotation axis affords quite a good representation of the field for 0–5 Ma, although g_0^3 is slightly larger than predicted by that model (g_0^3/g_2^0 = -3r/2R, where r is the northward offset and R is the mean Earth radius). Using a value of 0.05 g_0^0 for g_0^0 and neglecting g_3^3, Wilson's approximation gives:

\[
\text{northward offset } r = \frac{Rg_0^3}{2g_3^0} = 159 \text{ km.}
\]
There is a strong likelihood that any differences in the field prior to 2 Ma will be swamped by the more numerous Quaternary results. To investigate this, two independent subgroups were created representing the intervals 0–2 and 2–6 Ma, the unequal lengths allowing comparable numbers of palaeopoles to fall in each, though most of the sea cores are less than 2 Ma in age. The younger group was analysed directly, while the 2–6 Ma set was returned to a 4 Ma reconstruction using the rotations in Table 2. Analysis gave results very similar to those above for 0–2 Ma (Table 2, column E), with a best-fit pole virtually coincident with the present geographic pole. The 0–2 Ma zonal coefficients agree with the

<table>
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<th>Age of reconstruction (Ma)</th>
<th>0–5 Ma</th>
<th>0–2 Ma</th>
<th>2–6 Ma</th>
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<td></td>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
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<td>$g_1^5$</td>
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</tbody>
</table>

$M$ is the number of palaeopoles used, $N$ the number of deep-sea core inclinations. $(\phi, \lambda)$ is the (latitude, longitude) of the pole of the best-fit tilted dipole calculated from:

$$\phi = \sin^{-1} \frac{g_1^5}{\sqrt{(g_1^5)^2 + (h_1^5)^2}}$$

$$\lambda = \tan^{-1} \frac{h_1^5}{g_1^5}.$$

$\Delta I_{rms}$ and $\Delta D_{rms}$ are the root mean square errors in the fit of $M+N$ inclinations and $M$ declinations respectively.

A: solution for 139 palaeopoles on present-day map.
B: solution for 66 deep-sea core inclinations on present-day map.
C: solution for 139 palaeopoles plus 66 deep-sea core inclinations on present-day map.
D: as C on 5 Ma reconstruction.
E: solution for 75 palaeopoles and 59 deep sea cores, representing only the last 2 Ma, on present-day map.
F: solution for 63 palaeopoles and seven deep sea cores representing the interval 2–6 Ma on 4 Ma reconstruction.
values computed by Merrill & McElhinny (1977) but disagree with the somewhat larger values of $g_2^0$ found by Wells (1973) and Coupland & Van der Voo (1980). For the older group, the field was mainly axial with $g_2^0$ very similar to its Quaternary value, and a best-fit pole about 2° from the reconstructed pole. It appears that both $g_2^0$ and $h_2^1$ were larger prior to 2 Ma and thus it is the older data which are contributing most to the values computed for these coefficients for 0–5 Ma. Apart from this, there is no substantial difference between the time-averaged field during the Quaternary and during the 4 Ma immediately preceding it.

Since the data seemed more than adequate to describe the average field to the third degree during this time, the analysis was extended to include $g_4^0$ in an attempt to test Cox’s hypothesis that the longitudinal drift averages out only non-axial field components, leaving the zonal terms with essentially their present-day values (Cox 1975). Results indicated that $g_4^0$ has tended to average out to much smaller values over periods of 2 Ma or more, the present value of $-0.067 g_4^0$ (Peddie 1982) comparing with $-0.010 g_4^0$ obtained for the last 5 Ma. Other coefficients in the analysis were virtually unchanged by the inclusion of the fourth degree term.

As mentioned previously, the number of palaeopoles from the USSR was limited in these analyses. Further inversions were performed with the inclusion of those Russian poles given a two-star rating in the Ottawa listings. Little difference was observed in the zonal coefficients, but most of the non-zonal terms increased and the overall fits were slightly degraded. This is thought to reflect the generally poorer quality of these data, as noted by Harrison (1979) and Harrison & Lindh (1982), and appears to justify their exclusion.

5  Pre-5 Ma results

There is a marked bias toward northern hemisphere sites for pre-5 Ma data, as may be seen from a plot of the reconstructed site distributions (Figs 2–4). Therefore, the large values of $g_3^0$ obtained by Coupland & Van der Voo (1980) are a reflection of the inclination measured at a few southern hemisphere locations only. Using their 5–15 Ma data on the present 10 Ma reconstruction, values of 0.011 $g_3^0$ and 0.041 $g_3^1$ were computed for $g_2^1$ and $g_3^1$ respectively, with a best-fit pole at 86.5°N, 147.6°E suggesting that $g_3^0$, not $g_2^0$, was the important coefficient, as Coupland & Van der Voo observed. On deleting the pole from New Zealand, however, $g_2^1$ and $g_3^1$ were transformed to 0.063 $g_1^0$ and 0.008 $g_0^0$, respectively, while the best-fit pole lay at 88.1°N, 98.6°E, illustrating the sensitivity of the solution to small changes in the sparse southern data.

There is no clear pattern observable in these data. Four poles from Australia (8/29, 11/20, 11/22, 14/111) numbers refer to compilations published in the Geophysical Journal show almost uniform positive inclination anomalies (i.e. observed inclination – dipole field inclination), $\delta I$, of about 5° and declinations of about 10° in rotated coordinates, suggesting that either their ages or the reconstruction may be in error. This is supported by the disagreement between the Australian and Indian Ocean APWPs noted by Klootwijk & Peirce (1979). A pole from South Island, New Zealand (12/29), and one from Chatham Island (16/23) also give positive values of $\delta I$ as do two Antarctica poles (12/36, 14/130). However, a result from Mauritius (11/13) and two from South America (9/34, 11–255 – from the Ottawa catalogue) give far-sided poles. In addition, all cores from the Indian Ocean and from the Pacific show negative inclination anomalies, although they do not extend much further south than 10°S.

Thus there is no definite indication of either persistently high or low inclinations south of the palaeoequator. A dipole plus quadrupole field would cause all inclinations to be more negative (except at the pole) than a pure dipole field, whilst a dipole plus octupole field, as...
Geomagnetic field during the past 35 Ma

Table 4. Results of analysis of 5–35 Ma data.

<table>
<thead>
<tr>
<th></th>
<th>5–15 Ma</th>
<th>15–35 Ma</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>$g_1^0$</td>
<td>-0.061</td>
<td>-0.007</td>
</tr>
<tr>
<td>$h_1^0$</td>
<td>0.013</td>
<td>-0.005</td>
</tr>
<tr>
<td>$g_2^0$</td>
<td>0.013</td>
<td>-0.062</td>
</tr>
<tr>
<td>$h_2^0$</td>
<td>-0.026</td>
<td>-0.027</td>
</tr>
<tr>
<td>$g_3^0$</td>
<td>-0.006</td>
<td>-0.001</td>
</tr>
<tr>
<td>$h_3^0$</td>
<td>0.008</td>
<td>-0.015</td>
</tr>
<tr>
<td>$h_3^2$</td>
<td>0.025</td>
<td>0.038</td>
</tr>
<tr>
<td>$M$</td>
<td>54</td>
<td>54</td>
</tr>
<tr>
<td>$N$</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>$\phi$</td>
<td>86.4</td>
<td>89.5</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>167.8</td>
<td>-141.7</td>
</tr>
<tr>
<td>$\Delta f_{\text{rms}}$</td>
<td>3.8</td>
<td>4.2</td>
</tr>
<tr>
<td>$\Delta D_{\text{rms}}$</td>
<td>7.6</td>
<td>7.3</td>
</tr>
</tbody>
</table>

See Table 3 for explanation of symbols.

B: solution for 54 palaeopoles plus seven deep sea cores representing 5–15 Ma.
C: solution for 41 palaeopoles representing 15–35 Ma.
D: solution for 41 palaeopoles plus 11 deep sea cores representing 15–35 Ma.

suggested by Coupland & Van der Voo, would give low inclinations in the north, high inclinations in the south. A combination of dipole plus quadrupole plus octupole could give negative values of $\delta I$ in the northern hemisphere and small or zero values in the south.

It might be noted that, since the Australian data are repositioned via Antarctica, any error in the Antarctica–Africa rotation pole or the rate of motion around it would affect both sets of data. A uniform rate since anomaly 16 time (~40 Ma) has been assumed (Norton & Sclater 1979), but it is conceivable that the instantaneous rate has fluctuated.

In view of these problems, it is considered premature to look beyond second degree solutions here. Thus, results of an analysis for the first eight coefficients only are given (Table 4) for 5–15 Ma and 15–35 Ma, both with and without deep-sea core data. For the interval 5–15 Ma, the addition of seven core inclinations increased $g_2^0$ substantially and brought the best-fit pole into agreement with the position of the rotation axis in fixed-hotspot coordinates. Deleting the South Island/Chatham Island poles in the analysis of poles alone had much the same effect: $g_2^0$ increased from 0.013 $g_1^0$ to 0.064 $g_1^0$, while the best-fit pole lay at 89.2°N, 98.8°W, confirming the combined data result. Thus, it is suggested, the $g_1^0 + g_3^0$ field derived by Coupland & Van der Voo for 7–26 Ma may be a result of the inadequacy of the data distribution and reconstruction parameters, and the persistent non-dipole field is better represented by $g_2^0$.

For the 15–35 Ma data set, there is again the problem of a paucity of southern hemisphere sites, so that the spherical harmonic series is once more truncated after $h_2^0$. The results in Table 4 (columns C and D) show that $g_2^0$ again persisted, being easily the largest of the second degree coefficients. The fit of this model is quite poor, however, reflecting the broader selection criteria, but perhaps also increased errors due to uncertainties in dating and reconstruction. Nevertheless, there is less than 1° between the best-fit pole for the combined data and the hotspot pole, supporting the joint assumptions that, (a) the time averaged field is largely zonal, and (b) the hotspot frame has been virtually fixed with respect to the rotation axis during the last 30 Ma. The present results, therefore, disagree with those of Hammond et al. (1979) and of Morgan (1981) who concluded that there has been significant relative motion between the two during this interval.
The northward dipole offsets corresponding to the values of $g_2^0$ obtained from the combined analyses of 5–15 Ma and 15–35 Ma data are 198 and 271 km respectively. These values suggest that much of the 555 km obtained by Wilson & McElhinny (1974) for 7–25 Ma is due to uncorrected plate motions.

6 Errors

The errors in palaeomagnetic direction data are difficult to assess since, as Merrill & McElhinny (1977) pointed out, tight grouping of individual directions may reflect only inadequate sampling of the secular variation so that linear propagation cannot be used realistically to estimate the errors in the determination of the Gauss coefficients.

The method of Wells (1973), involving the inversion of pseudo-palaeomagnetic fields sampled with random errors at the actual sites used in the analyses, was therefore used to gauge the significance of the solutions obtained. A model field, in which $g_1^0 = -1.0$, $g_2^0 = -0.05$ and all other coefficients were set to zero, was used to synthesize artificial field directions at the sites corresponding to each of the above age groups. To these directions were added errors comprised of two angles: $\theta$, the magnitude of the angular error, drawn from a Fisherian distribution, and $\beta$, the direction of the error on a unit sphere. The latter was obtained by multiplying $2\pi$ by a pseudo-random number on the interval $(0, 1)$, while the former required some assumption about the mean error in the observed directions. The assumed values of $\theta$ used in these calculations are shown in Table 5.

Ten such fields were synthesized for each data distribution and analysed by the same method as the real data. The rms error for each coefficient was then computed from the ten

<table>
<thead>
<tr>
<th>Age range (Ma)</th>
<th>$E(\theta)$</th>
<th>$\kappa$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–5</td>
<td>6.6</td>
<td>150</td>
</tr>
<tr>
<td>5–15</td>
<td>8.1</td>
<td>100</td>
</tr>
<tr>
<td>15–35</td>
<td>11.5</td>
<td>50</td>
</tr>
</tbody>
</table>

$E(\theta)$ is the expected value of the average error angle, $\kappa$ is the precision parameter for a Fisher distribution.

Table 5. Average angular errors assumed in the calculation of pseudo-palaeomagnetic data.

<table>
<thead>
<tr>
<th>Age range (Ma)</th>
<th>$g_1^0$</th>
<th>$g_2^0$</th>
<th>$g_3^0$</th>
<th>$h_1^1$</th>
<th>$h_2^1$</th>
<th>$h_3^1$</th>
<th>$h_2^2$</th>
<th>$h_3^2$</th>
<th>$h_4^2$</th>
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</thead>
<tbody>
<tr>
<td>0–5 Ma</td>
<td>0.016</td>
<td>0.008</td>
<td>0.007</td>
<td>0.011</td>
<td>0.015</td>
<td>0.017</td>
<td>0.004</td>
<td>0.006</td>
<td>0.004</td>
</tr>
<tr>
<td>5–15 Ma</td>
<td>0.011</td>
<td>0.008</td>
<td>0.007</td>
<td>0.025</td>
<td>0.015</td>
<td>0.017</td>
<td>0.004</td>
<td>0.006</td>
<td>0.004</td>
</tr>
<tr>
<td>15–35 Ma</td>
<td>0.014</td>
<td>0.025</td>
<td>0.018</td>
<td>0.029</td>
<td>0.046</td>
<td>0.017</td>
<td>0.030</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6. Rms errors in spherical harmonic coefficients computed from ten artificial fields based on pseudo-palaeomagnetic data.
Geomagnetic field during the past 35 Ma

Different values so obtained. These rms values are given in Table 6 as a guide to the possible errors in the Gauss coefficients. It appears that the present distributions are adequate for the resolution of $g_2^2$, $g_3^3$ and $h_2^2$ for 0–5 Ma, and that the values obtained for $g_2^2$ in the earlier intervals are significant.

Minster et al. (1974) in a least squares inversion of instantaneous plate motion data, used numbers known as data importances to assess the contribution of each piece of information used. These were obtained by taking the diagonal elements of the matrix $P$ where

$$ P = V^{-1/2} A (A^T V^{-1} A)^{-1} A^T V^{-1/2} $$

in which $V$ is the data error autocorrelation matrix and $A$ is the matrix of partial derivatives. In the present case of unit weighting, this becomes simply the information density matrix $AH$ where $H = (A^T A)^{-1} A^T$. This matrix was computed using the partial derivatives corresponding to the solutions given in Tables 3 and 4. In Figs 5 and 6, circles have been drawn around the sampling sites for the two older age groups, the diameters of which are proportional to the diagonal elements of the information density matrix. As with the data

Figure 5. Information contribution of 5–15 Ma data. The diameters of the circles are proportional to the importance of palaeomagnetic data, calculated as described in the text. Other symbols and projection as Fig. 1.

Figure 6. As Fig. 5 for 15–35 Ma data.
importances, the sum of these elements is the number of degrees of freedom in the model, i.e. 7.

These figures confirm our suspicions that some results are having a disproportionate effect on the inversions, particularly those at low latitudes and in the southern hemisphere. Many more accurate results are required from these regions before the values of the spherical harmonic coefficients can be computed with certainty. Some form of regional weighting is also desirable but one faces the problem that equal weight is given to directions representing the average of several reliable results and to directions based on only one, perhaps doubtful, determination. Other forms of weighting designed to compensate for differences in latitude, precision of dating and quality of the palaeomagnetic data, may prove useful in future work.

7 Conclusions

It is clear from these analyses that the geocentric axial dipole field (GADF) model is a rough approximation only to the average field since mid-Tertiary times. For accurate work, such as the determination of apparent polar wander paths, the persistent terms of second and third degree should also be considered and the following modified relation between inclination, $I$, and colatitude, $\theta$, employed:

$$\tan I = \frac{2g_1^0 \cos \theta + g_2^0 (\gamma_2 \cos^2 \theta - \frac{3}{2}) + g_3^0 (10 \cos^3 \theta - 6 \cos \theta)}{g_1^0 \sin \theta + g_2^0 (3 \sin \theta \cos \theta) + g_3^0 (15 \gamma_2 \sin \theta \cos^2 \theta - \frac{3}{2} \gamma_2 \sin \theta)} .$$

A value of $g_2^0 = 0.05 g_1^0$ would offer an improvement over the GADF model for the whole of the period 0–35 Ma, whilst a value of $g_3^0 = 0.02 g_1^0$ might be added for the last 5 Ma, though this value may be due, in part at least, to data errors as regionally averaged data give values close to zero. In addition, $h_1^0$ may be significant in the average field for 0–5 Ma.

We have established that the predominantly northward motion of the plates cannot explain the persistent quadrupole during the last few million years. Returning all data younger than 5 Ma to a 5 Ma reconstruction, thereby overcorrecting for the effect, does not eliminate either $g_2^0$ or $g_3^0$ from the solution. Thus these terms appear to be true features of the time-averaged field.

Cox's model of non-random distribution of core-surface field sources predicts that the zonal terms should maintain their instantaneous values in the average field while the non-zonal terms disappear. The decline of $g_2^0$ from its present value to around $-0.01 g_1^0$ in the average field indicates that this model cannot be strictly correct. Coupland & Van der Voo (1980) have suggested a way in which the hypothesis may be modified so as to produce a time averaged $g_1^0 + g_2^0 + g_3^0$ field.

Southern hemisphere data are at present too sparse to permit inversions for third degree coefficients prior to 5 Ma, but a second degree analysis of 5–15 Ma data gave a largely axial field with $g_2^0$ only slightly larger than for 0–5 Ma. Further palaeomagnetic work south of the palaeoequator together with improved reconstructions are required in order to determine whether $g_3^0$ persists in the mean field before 5 Ma. The 15–35 Ma field was similarly axial with a persistently positive ratio $g_2^0/g_1^0$. It may be that the field configuration has remained essentially constant since the mid-Tertiary therefore, being approximated, to second order, by the offset dipole model which should not yet be discarded. Further work will show whether coefficients such as $g_2^0$ and $h_1^0$ are, in fact, persistent as indicated here. The apparent increase in dipole offset with age observed by Wilson & McElhinny is largely explained by plate motions since mid-Tertiary time, although there may be a smaller change in the value of $g_2^0$ in the same sense.
Geomagnetic field during the past 35 Ma

There is little detectable relative motion between the African plate hotspots and the rotation axis as defined by palaeomagnetic data during the last 30 Ma, or between these hotspots and the Hawaiian hotspot.

Acknowledgments

We thank Professors A. Cox and K. M. Creer, Drs M. W. McElhinny, N. Hamilton, C. G. A. Harrison and F. J. Lowes for helpful discussions and Professor R. Van der Voo for kindly supplying a list of the data used by D. Coupland and himself in their study of long-term non-dipole components of the field.

References


**Appendix**

0-5 MA DATA

**PALAEOPOLeS**: NUMBERS REFER TO THE LISTS PUBLISHED IN THE GEOPHysical JOURNAL, EXCEPT THOSE PREFIXED BY 'OT' WHICH REFER TO THE OTTAWA CATALOGUES.

**NEW ZEALAND**: 11/9 AUSTRALIA: 18/1, 8/2, OT12-211 ANTARCTICA: 12/12, 8/3, 14/24, 14/25, 14/32 MADAGASCAR: 10/6, 11/10, 14/5, 14/23, 14/27, 15/9, 10/7, 10/8, 11/11

14/48, 14/49, 15/8 AFRICA: 8/1, 8/2, 10/4, 13/5, 14/39, 15/11, 16/9

16/11, OT12-214, OT12-201, 14/21, 14/4, 14/22, OT12-131, 13/9, 14/46

16/15, OT11-1617, OT11-320, 14/6A, 14/35, 14/90, 14/74, 15/16

MEDITERRANEAN: 10/11, 11/14, 14/6, 14/51, 14/55, 14/56, 14/57, 13/10, 14/73

SPAIN: 10/5 EURASIA: 11/12, 12/9, 14/7, 14/36, 14/37, 18/7, 14/45, 12/17

12/21, 12/32, 14/66, OT11-325, OT11-218, 8/13, 14/7, 16/8, 14/72-286, OT12-90, 15/210, ICELAND: 13/7, 13/9, 11/18, 12/20, 14/40, 13/13, 17/16, 18/19, 18/20, 15/22, 15/23, 14/24, 14/25, 14/6B, 14/75, OT11-846, OT11-101, 14/70, 14/72, OT11-530, 14/674, OT13-22, 13/17 NAZCA: 13/4, 13/14, 14/55, 14/56, 14/57, 13/10

14/73 SPAIN: 10/5 EURASIA: 11/12, 12/9, 14/36, 14/37, 18/7, 14/45, 12/17

12/21, 12/32, 14/66, OT11-325, OT11-218, 8/13, 14/7, 16/8, 14/72-286, OT12-90, 15/210, ICELAND: 13/7, 13/9, 11/18, 12/20, 14/40, 13/13, 17/16, 18/19, 18/20, 15/22, 15/23, 14/24, 14/25, 14/6B, 14/75, OT11-846, OT11-101, 14/70, 14/72, OT11-530, 14/674, OT13-22, 13/17 NAZCA: 13/4, 13/14, 14/55, 14/56, 14/57, 13/10

5-15 MA DATA

**PALAEOPOLeS**: NUMBERS REFER TO THE LISTS PUBLISHED IN THE GEOPHysical JOURNAL. EXCEPT THOSE PREFIXED BY 'OT' WHICH REFER TO THE OTTAWA CATALOGUES.

**NEW ZEALAND**: 16/23, 12/29 ANTARCTICA: 12/36, 15/219 MADAGASCAR: 11/13

19/1, 19/7, 9/14 AF'ICA: 8/18, 14/94, 3/15, 18/17, 15/20, 10/5-5, 5/24, 8/24, 14/83, 14/87, 10/32, 11/19 EURASIA: 14/88, 7/7, 7/8, 10/24, 11/16, 12/30, 14/77, 14/38, 14/61, 14/82,

14/72-212, 5/24, 11/21, 12/28, 12/36, 12/40, 12/42, 14/28

ICELAND: 14/26, 14/29, 14/27, 18/31, 14/32, 14/95, 14/98

NORTH AMERICA: 13/15, OT11-362, 11/7, 11/18, 15/24, OT-709, 11/10-708

SOUTH AMERICA: OT11-255, 9/34

**DEEP-SEA CORES**: PACIFIC: K76-37, K76-12, M70-17, K76-15, K76-18, M70-39, K72-46, K70-6, K76-7, K76-5

15-35 MA DATA


5-15 MA DATA

**PALAEOPOLeS**: NUMBERS REFER TO THE LISTS PUBLISHED IN THE GEOPHysical JOURNAL. EXCEPT THOSE PREFIXED BY 'OT' WHICH REFER TO THE OTTAWA CATALOGUES.

**NEW ZEALAND**: 16/23, 12/29 ANTARCTICA: 12/36, 15/219 MADAGASCAR: 11/13

AF'ICA: 9/13, 9/17, 9/14 AF'ICA: 8/18, 14/94, 3/15, 18/17, 15/20, 10/5-5, 5/24, 8/24, 14/83, 14/87, 10/32, 11/19 EURASIA: 14/88, 7/7, 7/8, 10/24, 11/16, 12/30, 14/77, 14/38, 14/61, 14/82,

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ICELAND: 14/26, 14/29, 14/27, 18/31, 14/32, 14/95, 14/98

NORTH AMERICA: 13/15, OT11-362, 11/7, 11/18, 15/24, OT-709, 11/10-708

SOUTH AMERICA: OT11-255, 9/34

**DEEP-SEA CORES**: PACIFIC: K72-37, K76-12, M70-17, K76-15, K76-18, M70-76, K76-6