Roles of Torsion Particles in the Very Early Universe

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Problem of thermalization in the very early universe is discussed by taking account of interactions among Grand Unified Theory (GUT) particles and torsion particles which are expected in a class of extended theory of gravity. It is shown that four thermal histories of the very early universe are possible, depending on the values of the coupling constant and the mass of torsion particles. Among them, there is a history in which GUT particles are thermalized for $10^{15-16}$GeV $\leq T \leq 10^{14}$GeV by torsion particles.

§ 1. Introduction

The Poincaré Gauge Theory (PGT)\(^1\) is an extended theory of General Relativity (GR). It is based on more general geometrical framework including curvature and torsion. The latter couples with spins of matter and this interaction becomes important at the early universe. Torsion particles arise from the weak field approximation of this theory.\(^2\)

Recently, cosmology of the early universe is prosperously discussed by many authors. In these researches, the problem of thermalization of the universe plays important roles.\(^3\)-\(^6\) In this paper, we investigate the thermal histories of the very early universe ($10^{19}$GeV $\geq T \geq 10^{15-14}$GeV, the era before the phase transition of GUT), which consists of torsion particles and GUT(SU(5)) particles (fermions $F$, gauge bosons $G$ and Higgs particles $H$).

It has been shown\(^3,4\) that for $10^{19}$GeV $\geq T \geq T_c=10^{15-14}$GeV a system consisting of GUT particles and gravitons cannot be in thermal equilibrium (Fig. 6(I)). While, we will show that in a system consisting of torsion particles and GUT particles, four thermal histories of the very early universe are possible, depending on the value of parameters of PGT (Fig. 6(I)~(IV)). In history (I), the universe is not thermalized for that era. In history (II), the universe is once thermalized at $T= T_e$ then become nonequilibrium at $T = T_d$ then thermalized again at $T= T_c$. In history (III), the universe is in thermal equilibrium at $T \leq 10^{17-16}$GeV. This history is desirable for the scenarios such as the inflationary\(^5\) or the new inflationary universe,\(^6\) because it is desirable for these theories that the universe is in thermal equilibrium at least just before the beginning of the de Sitter expansion (it begins at $T = T_c \sim 10^{15-14}$GeV\(^7\)). In history (IV), the strong gravitational interaction complicates a picture of the very early universe (the $N$-region, see §6). In this way, there are various thermal histories of the very early universe when torsion particles are involved.

In §2, a brief review about the nonequilibrium era in the system of GUT particles and gravitons for $10^{19}$GeV $\geq T \geq 10^{15-11}$GeV is given. In §3, simple introduction of PGT and its weak field approximation are given. We adopt in this paper the simplest case that includes only a massive $0^-$ torsion particle and at this time we introduce the mass of the torsion particle $m_t$ and the coupling constant $g_t$ as free parameters. Then, the interaction
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between torsion particles and GUT particles is stated. In §4, first using this interaction, the reaction rates \( \Gamma(g_t, m_t, T) \) of processes which involve both the torsion particle and GUT particles are estimated. Next, the thermalization condition

\[
\Gamma(g_t, m_t, T) \geq 3\dot{R}/R \tag{1.1}
\]

is examined as inequalities among \( g_t, m_t \) and \( T \). Here, \( R \) is the scale parameter of the Friedmann universe. We shall take \( g_t \) and \( m_t \) as free parameters. In §5, further restrictions of these parameters are discussed by considering several conditions such as the average matter density of the universe, the decoupling of the torsion particle from ordinary matter and the hyperfine splitting of the hydrogen atom. Lastly, by the rate equation (the evolution equations for particle number densities) the conditions for realization of thermal equilibrium within the time interval corresponding to \( 10^{17}\text{GeV} \gtrsim T \gtrsim 10^{15}\text{GeV} \) are given. In §6, under these conditions, it is shown that the four thermal histories of the very early universe are possible, depending on the value of \( g_t \). In §7, summary is given and related problems are listed. In the Appendix, validity of the flat spacetime background approximation is discussed.

§ 2. Nonequilibrium era in the very early universe

The reaction rate of an \( i \)-particle for a process \((ij \rightarrow kl)\) is given by

\[
\Gamma_i(T) = \sum_{j,k,l} n_j \langle \sigma(ij \rightarrow kl) \rangle_{TA}, \tag{2.1}
\]

where summation is over all particle species \((j, k, l)\) and their helicity states, \( n_j \) is the total number density of \( j \)-particles and \( \langle \cdots \rangle_{TA} \) means the thermal average over \( i \) and \( j \) particles. The thermalization condition is given by (1.1).

Ellis and Steigman have shown\(^3\) by order estimation of \( \Gamma_{\text{GUT}} \) that the system consisting of only GUT particles cannot be in equilibrium at \( T \gtrsim 10^{16}\text{GeV} \) in the radiation dominated very early universe. Because \( \Gamma = 3\dot{R}/R \) at \( T = 10^{15}\text{GeV} \), the thermal equilibrium seems to be effectively realized at roughly \( T \lesssim 10^{14}\text{GeV} \). It has also been shown\(^3,4\) that the system consisting of gravitons of GR and GUT particles cannot be in equilibrium for \( m_p \gtrsim T \gtrsim 10^{16}\text{GeV} \), where \( m_p \) is the Planck mass given by \( 1.2 \times 10^{19}\text{GeV} \). It is derived for \( T \lesssim m_p \)

\[
\frac{\Gamma_{\text{grav}}}{R/R} \approx \left[ \frac{T}{m_p} \right]^m \quad (m=1, 2, \ldots) \tag{2.2}
\]

based on dimensional analysis.

Consequently, there exists the nonequilibrium era for \( 10^{16}\text{GeV} \gtrsim T \gtrsim 10^{15-14}\text{GeV} \) as for the system consisting of GUT particles and gravitons.

§ 3. The interactions between torsion particles and GUT particles

From now on, the system consisting of torsion particles and GUT particles is discussed.

The Poincaré Gauge Theory\(^1\) is a gauge theory for extended gravity. Its gauge group is \( T \otimes L_{\text{internal}} \), where \( T \) is the translational gauge group and \( L_{\text{internal}} \) is the internal
Lorentz gauge group. It contains GR and the New General Relativity (NGR) as its special cases. The underlying spacetime manifold of this gravitational theory is the Riemann-Cartan spacetime characterized by curvature and torsion. The torsion couples with the intrinsic spin of matter. This interaction is expected to become important in the very early universe (see (3·6) and below).

In the weak field approximation in which two gauge fields are considered as weak fields on the flat spacetime background (see the Appendix), there emerge one graviton and six torsion particles which can be either massive or massless. It has been shown that there are four cases in which torsion particles can coexist as normal particles with positive mass and positive energy.

In the following discussion, we shall take the simplest case in which only a single kind of torsion particle exists. This particle is a pseudoscalar defined by

$$\phi = \frac{-3a_1}{\sqrt{2(\gamma + \frac{3}{2}a)}} \partial^\rho a_\rho , \quad (\text{dim } \phi = E^1) \quad (3·1)$$

where $a$ is the constant defined from Einstein's $G$ as

$$a = \frac{1}{16\pi G} , \quad (G = m_p^{-2}) \quad (3·2)$$

The field $a_\rho$ is one of the irreducible components of the translation gauge field strength $T_{ij\kappa}$ which is related to the torsion tensor $T_{\mu\nu\lambda}$ as

$$T_{ij\kappa} = e_i^\rho e_j^\sigma e_\kappa^\lambda T_{\mu\nu\lambda} , \quad (3·3)$$

where $e_i^\rho$ are the tetrad fields. The tensor $T_{\mu\nu\lambda}$ is related to the affine connection $\Gamma^\lambda_{\mu\nu}$ as

$$T_{\mu\nu\lambda} = \Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu} . \quad (3·4)$$

The parameters $a_1$ and $\gamma$ in (3·1) satisfy

$$\gamma + \frac{3}{2}a > 0 , \quad (3·5a)$$
$$a_1 < 0 . \quad (3·5b)$$

The linearized Lagrangian of $\phi$ is

$$L_\phi = L_\phi^0 + L_{\text{int}}$$
$$= \frac{1}{2} \phi (\Box - m_t^2) \phi - g_t \varepsilon_{\rho\lambda\mu\nu} \partial^\rho S^{\lambda\mu\nu} \phi , \quad (3·6)$$

where

$$m_t^2 = \frac{-2(\gamma + \frac{3}{2}a)}{3a_1} \quad (3·7a)$$
$$g_t = \frac{1}{4\sqrt{2} \sqrt{\gamma + \frac{3}{2}a}} \quad (3·7b)$$
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and $S_{\mu\nu}$ is the spin tensor of fermion fields.

Why must we consider this interaction in the very early universe? Firstly, torsion particles arise from a more general framework than that of GR. Therefore, if there is no special reason for $T_{\mu
u\lambda}=0$, it is natural to take into account roles of torsion particles in the very early universe. Secondly, torsion particles couple with the intrinsic spins of matter as shown above. Therefore, this interaction appears in the era in which the gravitational interaction becomes important in microscopic systems such as the very early universe. (For macroscopic systems, spin density is usually averaged out except for special cases such as neutron stars.)

In order to make our discussion specific we take the $SU(5)$ GUT assuming that the curvature can be neglected, that is, the background spacetime is flat. In the following, the 45 fermions (3 families) are collectively denoted by $F_{f}$ ($f=1, 2, 3$), the 24 gauge bosons by $G$ ($5\times 5$ matrix) and the 24 Higgs particles by $H$ (for simplicity Higgs of 5 are omitted). Then the $SU(5)$ Lagrangian is expressed as

$$L_{\text{SUS}} = -\frac{1}{4} \text{Tr}(G_{\mu\nu}G^{\mu\nu}) + \sum_{f=1}^{3} \bar{F}_{f} i \gamma^{\mu} F_{f} + L_{\text{Higgs}},$$

where

$$G_{\mu\nu} = \partial_{\mu} G_{\nu} - \partial_{\nu} G_{\mu} - i g [G_{\mu}, G_{\nu}],$$

$$D_{\mu}^{f} = \partial_{\mu} + \frac{i}{2} A_{\mu f} S_{hf} - i g G_{\mu}.$$  

In (3.10), $A_{h\mu}$ denotes the Lorentz gauge field, $S_{hf}$ is the Lorentz generator given by (3.12) and $g$ is the $SU(5)$ gauge coupling constant estimated to be around 0.5 at about $T=10^{18}$GeV by renormalization group analysis.

The spin tensor is defined by

$$S_{\mu\nu} = \frac{\partial L_{\text{SUS}}}{\partial (\partial_{\mu} \phi_{a})} (S_{\mu\nu})_{ab} \phi_{b},$$

where $\phi_{a}$ are components of independent fermion fields and $S_{\mu\nu}$ is given as

$$(S_{\mu\nu})_{ab} = \frac{1}{4} [\gamma_{\mu}, \gamma_{\nu}]_{ab}. \quad \text{(for } F_{f})$$

It is known that gauge bosons cannot interact with torsion particles at the tree (classical) level. However, we leave a room for interaction between gauge bosons and torsion particles in the following form:

$$L_{\phi-G} = \delta g^{2} g_{4} \phi_{a} \phi_{b}^{*} \text{Tr}(G_{\mu\lambda}G^{\mu\lambda}).$$

This form of Lagrangian does not exist in the minimal coupling Lagrangian, but is possibly induced from radiative corrections. In the present work, $\delta$ is a free parameter. However, it is expected to be $O(1)$, so we regard it as unity in the following.

From (3.8), (3.9) and (3.13), the interaction between torsion particles and GUT particles in the weak field approximation is expressed as

$$L_{\text{int}} = L_{\text{int}}^{(1)} + L_{\text{int}}^{(2)} + L_{\text{int}}^{(3)}$$

with
where \(i, j, k \), specify 24 species for gauge bosons and \(C_{ij}\) and \(D_{ijk}\) are group theoretical factors. The value of \(C_{ij}\) is 4 and the value of \(D_{ijk}\) is

\[
D_{ijk} = \begin{cases} 
-4\sqrt{2}i & \text{(for } i=j, j=k, k=i) \\
0 & \text{(otherwise)}
\end{cases}
\]

under a simplification of

\[
G_{ij} \rightarrow \frac{2}{\sqrt{30}}B G_{ij} \quad (i=1, \ldots, 24)
\]

that is, we omit the terms such as \(-(2/\sqrt{30})B\) and express \(G_1, G_2, \ldots, G_7\) as \(G_i (i=1, \ldots, 24)\). Interaction \((3\cdot14)\) in momentum space is given in Fig. 1.

§ 4. The thermalization condition for the system consisting of torsion particles and GUT particles

In this section, the reaction rates for various processes including torsion particles are calculated from the interactions derived in §3.

These calculations are performed in the flat spacetime background approximation. The condition for this approximation to be valid is discussed in the Appendix for the closed, open and flat universe, separately (see also Table II). In the following, we shall assume that this approximation is admitted.

We shall also assume that perturbation theory can be applied to the interactions of \((3\cdot14)\) and we shall treat the case \(T \gg m_t\) (see §5). For the order estimation of \(\sigma\) (the total cross sections of the processes in Fig. 2), we use propagators at \(T=0\) (see the end of this section) and regard energy and momentum transfer as of the same order of magnitude.

In Fig. 2, the two-body scattering processes, \((\phi 1), (G1), (G2), (G3), (H)\) and \((F)\) give the reaction rates of \(\phi, G, H\) and \(F\) respectively. The \((\phi 2), (\phi 3)\) and \((\phi 4)\) are the decaying processes of \(\phi\).

In the very early universe, the particle horizon sets a lower limit to energy of par-
Fig. 2. The processes including torsion particles. We take only graphs of the order \(-g_t, g_t g\) and \(g_r^2\). The process \(F\) is the inverse of the process \(\phi_1\). For \(\phi\) and \(F\), only the most significant processes are listed.

![Diagram of processes including torsion particles](image)

Fig. 3. The factor \(\Gamma(T)\) arising from the presence of the particle horizon (Eq. (4.1)). For \(T \leq 10^{18}\) GeV, \(\Gamma(T)\) is almost unity.

\[
E \geq E_{\text{min}} = \frac{1}{4t}. \quad (4.1)
\]

This makes \(\Gamma\) smaller than that of full distributions by the factor \(\Gamma(T)\). However, \(\Gamma(T)\) is nearly equal to one for \(T \approx 10^{18}\) GeV as is shown in Fig. 3 and therefore this effect is not significant. Before the phase transition (PT) of GUT, the summation in Eq. (2.1) is over the whole particle species involving torsion particles and GUT particles. After PT of GUT, some of GUT particles become massive and do not contribute to the reaction rates.

Now we can estimate the reaction rates \(\Gamma\) in the radiation dominated universe (RDU) before PT of GUT in the period \(10^{18}\) GeV \(\approx T \approx 10^{15}\) GeV. The value of \(\dot{R}/R\) in RDU is given as

\[
\frac{\dot{R}}{R} = \left[ \frac{4\pi^3}{45} N \right]^{1/2} \frac{T^3}{m_p} = 10^{-26.7}[T(\text{eV})]^2(\text{eV}),
\]

where \(N\) is the effective number of total helicity states and is about 160 for \(SU(5)\). From (4.2) and \(\Gamma\), we can calculate \(\Gamma\) vs \(\dot{R}/R\). The estimations of \(\Gamma\) and \(\Gamma\) vs \(\dot{R}/R\) are given

<table>
<thead>
<tr>
<th>(\Gamma)</th>
<th>(\phi_1)</th>
<th>(\phi_2)</th>
<th>(\phi_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Gamma_t)</td>
<td>(10^4 g_t^2 T^3)</td>
<td>(10g_t^4 m_t^2)</td>
<td>(10^{-37} g_t^6 g_r^2 T^3)</td>
</tr>
<tr>
<td>(\Gamma_t / \dot{R}/R)</td>
<td>(10^{-26.4+1+2n})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Gamma_1)</td>
<td>(0.1 g_t^4 T^3)</td>
<td>(g_t^4 T^3)</td>
<td>(10^{32} g_t^6 g_t^4 m_t^4 T)</td>
</tr>
<tr>
<td>(\Gamma_1 / \dot{R}/R)</td>
<td>(10^{-32.4+1+2n})</td>
<td>(10^{-30.4+1+2n})</td>
<td></td>
</tr>
<tr>
<td>(\Gamma_2)</td>
<td>(F)</td>
<td>(H)</td>
<td>(\phi_4)</td>
</tr>
<tr>
<td>(\Gamma_2 / \dot{R}/R)</td>
<td>(10^2 g_t^4 T^3)</td>
<td>(300 g_t^2 T^3)</td>
<td>0</td>
</tr>
<tr>
<td>(\Gamma_t / \dot{R}/R)</td>
<td>(10^{-28.4+1+2n})</td>
<td>(10^{-28.9+1+2n})</td>
<td>0</td>
</tr>
</tbody>
</table>
in Table I in the notations,

\[ T = 10^T \text{ (eV)}, \]  
\[ m_t = 10^m \text{ (eV)}, \]  
\[ g_t = \frac{10^n}{m_p} = 10^{n-28.1} \text{ (eV}^{-1}), \]  

where \( m \) and \( n \) are new free parameters introduced for convenience.

When the conditions \( T \gg m_t \) and \( g_t T \ll 1 \) are satisfied (the latter is a condition under which the power expansion in \( g_t \) is justified, see §5), the processes \((G3), (\phi2)\) and \((\phi3)\) are negligible compared with \((\phi1), (G1), (G2), (H)\) and \((F)\). For example,

\[ \frac{\Gamma_{G3}}{\Gamma_{\phi1}} \approx \frac{1}{g^2 (g_t T)^2} \left[ \frac{m_t}{T} \right]^4 \ll 1. \]  

The thermalization condition for the process \((F)\) before PT of GUT is

\[ \frac{\Gamma_{\phi1}}{3R/R} \sim 10^{4+2n-27.7} \gg 1, \]  

that is

\[ l \gtrsim -2n+27.7. \]  

When \( n \) is given, the thermalization condition for fermions is

\[ T \gtrsim 10^{-2n+27.7} \text{ (eV)}. \]  

Therefore, the decoupling temperature \( T_d \) is \( 10^{-2n+27.7} \text{(eV)}. \)

It is shown by an actual calculation that the correction to \( \Gamma_i \) by finite temperature effect of Green functions would not change the above order estimation for reaction rates.

§ 5. The restrictions of \( g_t \) and \( m_t \)

5.1. The restriction by the average matter density of the universe

The mass density of \( \phi \) at present time is given by

\[ M_{\phi} = m_t \cdot n(T_d) \cdot \text{[diluting factor]} \cdot \text{[decay factor]}, \]  

where \( T_d \) is the decoupling temperature of the torsion particles. The diluting factor is the one due to the expansion of the universe. Along the scenario of the new inflationary universe, we take the inflational factor of the reheating time \((10^8 \text{GeV} \equiv T_r \lesssim T \lesssim 10^{13} \text{GeV})\) as \( \sim 10^{28.6} \). Except for the reheating time we can set \( RT = \text{const} \). The present temperature is \( T_p = 2.7K = 2.3 \times 10^{-13} \text{GeV}, \) thus we get

\[ \frac{R^3(T_d)}{R^3(T_r)} \cdot [10^{-28}]^3, \frac{R^3(T_r)}{R^3(T_p)} \sim [T_d(\text{GeV})]^3, \]  

\( \text{(case a)} \)  

\[ \frac{R^3(T_d)}{R^3(T_p)} \sim \frac{10^{-37.9}}{[T_d(\text{GeV})]^3}, \]  

\( \text{(case b)} \)  

\[ \text{(5.2a)} \]  

\[ \text{(5.2b)} \]
Fig. 4. The restricted regions of $g_t$ and $m_r$. Shaded regions (I), (II), (III) and (IV) are forbidden by the average matter density of the universe for case b (See (5·2). The forbidden region for case a is out of this figure.), the decoupling of $\phi$ from ordinary matter, the experiment of the hyperfine splitting of the energy level of the hydrogen atom and the condition $m_r < m_P$ respectively. The value $n=2$ is desirable for the thermal equilibrium of the very early universe.

where case a is the one in which $\phi$ decouples before PT of GUT, case b is after PT.

There is another chance of enormous dilution, i.e., the PT of Weinberg-Salam. But this PT is shown to be second order or at most weakly first order, so this does not contribute significantly to the dilution factor.

The decay factor is given as

$$\text{[decay factor]} = \exp[-\Gamma_{g_2} (t_p - t_d)],$$

(5·3)

where $\Gamma_{g_2}$ is the decay width of $(\phi 2)$-process (Table I), $t_p$ is the age of the universe and $t_d$ is the time at which $\phi$ decouples. As the result,

$$t_p - t_d \approx t_p \approx 10^{32.8} \text{ (eV$^{-1}$)}$$

(5·4)

and we get

$$\text{[decay factor]} \approx 10^{-10^{[98.5 - 22.2]}}.$$  

(5·5)

The mass density $M_\phi$ should be smaller than the observed value of the present matter density. There are two kinds of observed value. The one is the galactic mass density of $\sim 3 \times 10^{-31}$ g/cm$^3$ and the other is the so-called “dark matter” density of $\sim 2 \times 10^{-29}$ g/cm$^3$. As yet we do not know of what the dark matter consists. Therefore the possibility that the dark matter consists of torsion particles is not excluded. Here, let us take the latter bound,

$$M_\phi \leq 2 \times 10^{-29} \text{ g/cm$^3$} \sim 10^{-10.1} \text{ (eV$^4$)}.$$  

(5·6)

From (5·1), (5·2), (5·5) and (5·6), we get the forbidden region (I) shown in Fig. 4.

5.2. The restriction from the decoupling of torsion particles from ordinary matter

Torsion particles must not change well-established scenarios of the standard cosmology such as the nuclear synthesis at $\sim 10^6$ eV and the photon decoupling at $\sim 1$ eV. Here, we take a very severe condition that torsion particles must not affect the phenomena at energies less than $\sim 10^{11}$ eV (the energy of the Weinberg-Salam PT). This condition is expressed as

$$\int_{t_1}^{t_p} \Gamma_\phi(t) dt \ll 1,$$

(5·7)

where $t_1$ is a time at which $T=10^{11}$ GeV. This yields

$$n \leq 7.5$$

(5·8)

and we get forbidden region (II) of Fig. 4.
5.3. The restriction from the experiment of the hyperfine splitting of the hydrogen atom

The torsion particle mediates the spin-spin interaction between Dirac particles. Let us calculate a shift in the energy level of the hydrogen atom $\Delta E_{\phi}(H)$, due to the spin-spin interaction between the proton and the electron. In this section, we almost follow the analysis of Hayashi and Shirafuji.\(^{11}\)

The torsion field $\phi$ is generated by the intrinsic spin of the proton

$$\Box - m_\epsilon^2)\phi = 3g_\epsilon \partial^a S_\rho,$$

where $S_\rho$ is the spin density of the proton at rest at the origin\(^{11}\)

$$S_\rho \begin{cases} S_0 = 0, \\ S = -2S_\rho \delta^3(x). \end{cases}$$

From (5.9) and (5.10) we get

$$\phi(r) = -\frac{3}{2\pi} g_\epsilon m_\epsilon e^{-m_\epsilon r} \frac{(S_\rho \cdot r)}{r^2}.$$  

The Dirac equation for the electron is

$$i \left( \gamma^\mu \partial_\mu + \frac{1}{4} g_\epsilon \gamma^\mu \epsilon_{\nu\rho\sigma\delta} \partial^\nu \phi + ie\gamma^\mu A_\mu - m_\epsilon \right) \psi = 0.$$  

Putting (5.11) into (5.12) and making the nonrelativistic approximation and taking the case $S_\rho = (0, 0, S)$, we get the following equation for the large component of the electron spinor wave function:

$$i \frac{\partial \psi}{\partial t} = (H_0 + H_{\text{int}}) \psi,$$

$$H_0 = \frac{1}{2m_\epsilon} (P + eA)^2 - \frac{e^2}{r},$$

$$H_{\text{int}} \sim g_\epsilon^2 m_\epsilon^2 e^{-m_\epsilon r} \frac{z(r \cdot \sigma)}{r^3},$$

where $z$ is the $z$-component of the coordinates. From this we get

$$\Delta E_{\phi}(H) = \langle 0 | H_{\text{int}} | 0 \rangle \sim \frac{g_\epsilon^2 m_\epsilon^2}{a_B^3} \frac{1}{(m_\epsilon + \frac{2}{a_B})^2},$$

where $a_B$ is the Bohr radius. On the other hand, the energy shift $\Delta E_{\phi}(H)$ should be restricted by the present limit of experiment\(^{11}\) as

$$\Delta E_{\phi}(H) < \Delta E_{\text{exp}}(H) - \Delta E_{\text{QED}}(H) \sim (1.5 \pm 2.4) \times 10^{-11}\text{eV}.$$  

This condition sets the forbidden region (III) of Fig. 4.

5.4. The condition for realization of thermal equilibrium

Using the rate equation, we get the conditions for realization of thermal equilibrium within a time interval that corresponds to $10^{17}\text{GeV} \gtrsim T \gtrsim 10^{15}\text{GeV}$. Let us pay attention to fermions, since fermions have the largest number of helicity states (the other particles can
be treated in the same way). Let \( n_F(t) \) be the number density of fermions per unit volume (not comoving volume) then the rate equation for \( n_F(t) \) is

\[
\frac{dn_F}{dt} = -3n_F \frac{R}{R} + n_F \Gamma^{(#1)} - n_F \Gamma^{(F)},
\]

(5.16)

where the first term on the R.H.S. is the one due to the expansion of the universe. When condition (1.1) is satisfied, this term is negligible. The second term is due to the process \((\phi 1)\) and the third is due to the process \((F)\) (Fig. 2).

Firstly, we take the case in which \( G, H \) and \( \phi \) are already in thermal equilibrium and \( \delta n_F = n_F - n_F^0 \ll n_F^0 \). Under this condition, we get a solution such as \( \delta n_F = \delta n_F(0) \cdot \exp[-t/\tau_F] \), where \( \tau_F \) is the relaxation time of \( \delta n_F \). Here we demand early realization of thermal equilibrium in the time interval \( \Delta t \) corresponding to \( 10^{17} \text{GeV} \lesssim T \lesssim 10^{18} \text{GeV} \),

\[
\tau_F \lesssim \frac{1}{100} \Delta t,
\]

(5.17)

\( \tau_F \) is given by

\[
\tau_F = \Gamma_F^{-1} \approx (10^2 g_s^2 g_t^2 T^3)^{-1} = 10^{54.8-2n-31} \text{ (eV}^{-1})
\]

At \( T = 10^{17} \text{GeV} \), \( \tau_F \) is

\[
\tau_F = 10^{-2n-23.2} \text{ (eV}^{-1})
\]

(5.18)

On the other hand, \( \Delta t \) is given by

\[
\Delta t = 10^{-21.8} \text{ (eV}^{-1})
\]

(5.19)

From (5.17), (5.18) and (5.19) we get

\[
n \gtrsim 0.3
\]

(5.20)

as the condition for realization of thermal equilibrium of \( F \). The same analysis can be carried out also for \( G, H \) and \( \phi \). The consequences are shown in Fig. 5(a)~(d) by the solid lines \((y)\). Secondly, in the case in which GUT particles and torsion particles start from a nonequilibrium state which satisfy \( \delta n_i \ll n_i \), we get \( \tau \) like (5.18) by the coupled rate equations among \( \phi, F, G \) and \( H \).

Lastly, in the case in which all particles start from a nonequilibrium state with

\[
\delta n \gg n^0,
\]

(5.21)

unfortunately we have no method to treat with this problem. In the following, we assume \( \delta n \ll n^0 \).

5.5. The other restrictions

Furthermore, there must be the following conditions. As we restrict our argument to \( m_t \ll T < 10^{18} \text{GeV} \), we should take

\[
m < t < 27.
\]

(5.22)

The condition for validity of the perturbation expansion with respect to the coupling constant \( g_t \) (dim \( g_t = E^{-1} \)) is
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\[ g_t T \ll 1, \quad (5\cdot23) \]

where \( T \) is the typical energy scale exchanged by particles. We get from (5·23) and (4·3c)

\[ l \leq -n + 28.1. \quad (5\cdot24) \]

Conditions (5·6), (5·8), (5·15) and (5·22) are shown in Fig. 4. Conditions (5·20), (5·24) and (1·1) are shown in Fig. 5.

§ 6. The roles of torsion particles

concerning thermal equilibrium in the very early universe

In this section, we show that in the system involving torsion particles and GUT particles the four thermal histories of the very early universe are possible, depending on the value of \( g_t \) in the allowed regions.

From Table I, we get equilibrium regions of the system of \( \phi, G, F \) and \( H \) as Fig. 5 (the shaded regions).

The \( N \)-region (nonperturbative region) of Figs. 5 and 6 corresponds to

\[ g_t T > 1 \quad (6\cdot1) \]

and hence perturbative calculations are not justified there as already mentioned. Furthermore, the weak field approximation cannot be admitted in the \( N \)-region for the following reason. The torsion field \( \phi \) can be expanded into power series of \( g_t T \) (dim \( g_t T = E^0 \)), regarding the second term of (3·6) as perturbation,

\[ \phi = \phi_0 + (g_t T)\phi_1 + (g_t T)^2\phi_2 + \cdots. \quad (6\cdot2) \]

Fig. 5. The ranges of temperature in which the thermal equilibrium is realized for \( \phi, F, G \) and \( H \) (shaded regions). Below \( T \leq 10^{13}\text{GeV} \), GUT particles thermalize the universe. The lines (a), (\( \beta \)) and (\( \gamma \)) express conditions (5·23), (1·1) and (5·17) respectively. Each line has an uncertain width of almost unity because of the order estimations. About the \( N \)-region, see (6·1) and below.
Fig. 6. The four thermal histories of (6·9). The shaded regions and the blank regions represent the equilibrium era and nonequilibrium era respectively.

In the case of (6·1), $\phi$ largely fluctuates far from $\phi_0$. Several people suggested\(^{17}\) that for $T \gtrsim m_\rho$ (for the N-region in this case) strong quantum gravitational effects make the system in "thermal equilibrium" like "hot soup". However in this case we must abandon such a simple picture that the particles move in continuous spacetime background. Therefore, a unified theory including gravity is required in order to study the N-region.

Four thermal histories of the very early universe in the period $10^{16}\text{GeV} \gtrsim T \gtrsim 10^{15}\text{GeV}$ are possible in the present scheme (Fig. 6), depending on the value of the coupling constant $g_t$.

History (I) of fermions (Fig. 6(a)) corresponds to the range of the coupling constant $g_t = 10^n/m_\rho$ with

$$n \lesssim 0.8. \quad (6\cdot3)$$

In this case, the universe is not in thermal equilibrium for the era under consideration. (In (6·3)~(6·6) and (6·9), we consider the cases for fermion $F$, the cases of $\phi$, $G$ and $H$ can be treated in the same way.)

History (II) of fermions (Fig. 6(b)) corresponds to the range of

$$0.8 \lesssim n \lesssim 1.9. \quad (6\cdot4)$$

In this case, the matter ($\phi$, $F$, $G$ and $H$) are once thermalized at $T = T_e$ ($\tau \lesssim 10^4\Delta t'$ is satisfied for $n \lesssim 0.2$, where $\Delta t'$ is the time interval corresponding to $T_e \gtrsim T \gtrsim T_d$) and decouples at $T = T_d$ and expands almost freely because of $3\dot{R}/R \gg \Gamma$ and masslessness of all particles. Then GUT particles are thermalized again at $T \sim 10^{15}\text{GeV}$, but torsion particles are not.

History (III) of fermions (Fig. 6(c)) corresponds to the range of

$$1.9 \lesssim n \lesssim 2.6. \quad (6\cdot5)$$

In this case, the universe is in thermal equilibrium throughout $T \lesssim 10^{17.5}\text{GeV}$ for $\phi$ and $F$. This history is desirable for the inflationary or the new inflationary universe scenario (see §1).

In history (IV) of fermions (Fig. 6(d)), the $N$-region extends beyond $T = 10^{15}\text{GeV}$ because of the strong gravitational interactions. It corresponds to the range

$$n \gtrsim 2.6. \quad (6\cdot6)$$

In order to understand the physical meaning of (6·3)~(6·6), we compare $g_t$ with $g_\phi$, the coupling constant associated with the usual gravitons of GR. In GR, the equation of graviton field $\phi_{ik}$ ($\dim \phi_{ik} = E^1$) is
\[ \psi_i^k = \frac{16 \pi}{m_p} \tau_i^k, \quad (6.7a) \]

where
\[ \psi_i^k = h_i^k - \frac{1}{2} \delta_i^k h, \quad h = h_i^i \quad (6.7b) \]

and
\[ g_{ik} = g_{ik}^{(0)} + \frac{1}{m_p} h_{ik} \quad (6.7c) \]

and \( \tau_i^k \) is the energy-momentum tensor of matter fields. From (6.7a) and (5.9) we should say
\[ g_\phi = \frac{16 \pi}{3m_p}. \quad (6.8) \]

Comparing \( g_t \) and \( g_\phi \), (6.3)~(6.6) are translated as follows:

- **History (I)** \( \rightarrow \ g_t \lesssim 0.4g_\phi \)
- **History (II)** \( \rightarrow \ 0.4g_\phi \lesssim g_t \lesssim 5g_\phi \)
- **History (III)** \( \rightarrow \ 5g_\phi \lesssim g_t \lesssim 25g_\phi \)
- **History (IV)** \( \rightarrow \ 25g_\phi \lesssim g_t \).

As for the Higgs particles, history (III) is realized for
\[ 40g_\phi \lesssim g_t \lesssim 100g_\phi. \quad (6.9) \]

In this case, the method of the finite temperature effective potential\(^{18}\) for the Higgs field is justified for \( T \lesssim 10^{18}\text{GeV} \).

Further, we refer to the mass of torsion particles. Corresponding to the four histories, the limits of \( m_t \) are roughly given from Fig. 4 as

1. **(I)** \( m_t \lesssim 60 \text{eV} \) or \( 100 \text{MeV} \lesssim m_t \lesssim 10^{15}\text{GeV} \)
2. **(II)** \( m_t \lesssim 60 \text{eV} \) or \( 10 \text{MeV} \lesssim m_t \lesssim 10^{15}\text{GeV} \)
3. **(III)** \( m_t \lesssim 60 \text{eV} \) or \( 1 \text{MeV} \lesssim m_t \lesssim 10^{15}\text{GeV} \)
4. **(IV)** \( m_t \lesssim 60 \text{eV} \) or \( 10^3 \text{MeV} \lesssim m_t \lesssim 10^{15}\text{GeV} \.

In the case \( \delta = 0 \), that is, there is no interaction between \( G \) and \( \phi \) (see §3), \( \phi \) and \( F \) are thermalized as before but \( G \) and \( H \) are not.

**§ 7. Conclusions**

In the present paper, we have considered the problem of thermalization of the very early universe for \( 10^{18}\text{GeV} \approx T \approx 10^{15}\text{GeV} \) by taking account of the interactions between torsion particles and GUT (\( SU_5 \)) particles. The former arises from the weak field approximation of PGT which is an extended theory of gravity. The interactions between
torsion particles are derived. Torsion particles interact with spins of matter and this interaction seems to become important at the very early universe. The order estimation of the reaction rates for the processes which involve torsion particles are carried out in the flat spacetime background approximation. Using these reaction rates, the thermalization conditions are given as the inequalities between $g_t$, $m_t$ and $T$. Further, using the rate equation, the conditions for realization of the thermal equilibrium are given as the inequalities between $g_t$ and $T$. The value of $g_t$ and $m_t$ are restricted by the several conditions, i.e., the average matter density of the universe, the decoupling of torsion particles from ordinary matter and the hyperfine splitting of the energy level of a hydrogen atom mediated by torsion particles.

There are four thermal histories of the very early universe depending on the value of $g_t$ in the allowed region. Among them, a desired history for the inflationary or the new inflationary universe, in which the very early universe is in thermal equilibrium at $T \leq 10^{-17}$ GeV, is realized for the value of $g_t \sim 70 g_0$ and $m_t < 60 \text{eV}$ or $1 \text{MeV} < m_t < 10^{15}$ GeV.

Three difficult problems are left unsolved. The first is to know what happen in the $N$-region. This is concerned with the unified theory including gravity and is a problem of future. The second is to solve the Boltzmann equation for the case in which all particles started from a nonequilibrium state such as (5·21). The third is to investigate whether the era corresponding to $T_d \approx T \approx T_G$ in history (II) can be effectively in thermal equilibrium or not. The second and the third need the nonequilibrium statistical mechanics.

Appendix

— The Flat Spacetime Background Approximation for the Reaction Rates —

In this appendix, we will discuss the validity of the flat spacetime background approximation. This picture will be valid when $R(t)$ is much larger than a average wavelength of particles $\langle \lambda \rangle \sim T^{-1}$,

$$ R(t) \gg \langle \lambda \rangle \sim T^{-1}(t), \text{ i.e., } R(t)T(t) \gg 1. \quad (A\cdot 1) $$

Let us estimate $A = RT$ (dim $A = E^0$) after and before PT of GUT. When the universe expands adiabatically, the total entropy

$$ S = \frac{2\pi^2}{45} NT^3 R^3 = \frac{2\pi^3}{45} NA^3 \quad (A\cdot 2) $$

is conserved. There are two chances of entropy production. The one is at $T \approx 10^{18}$ GeV when the universe is expected to be anisotropic and the particle production will be significant.19) (Below $T \approx 10^{18}$ GeV, the particle production is negligible. Further we assume that the universe is isotropic for $T \leq 10^{18}$ GeV.) The other is PT of GUT.

i) The case after PT

At present, $T$ is $3K$ and $R$ is estimated from the average matter density of the universe\(^6\)

$$ R(T=3K) \approx 10^{32} \quad (\text{eV}^{-1}), \quad (A\cdot 3) $$

thus we get
\[ A_{\text{after}} = R(T=3K) \cdot 3K \geq 10^{28} \text{ (eV)} \quad (A \cdot 4) \]

where "after" means the value after PT of GUT. Therefore after PT this approximation is applicable.

ii) The case before PT

Before PT of GUT, along the scenarios of the inflationary and the new inflationary universe,\(^6\),\(^5\)

\[ S_{\text{before}} = \frac{1}{Z^3} S_{\text{after}} , \]

\[ Z \geq 10^{27.7} . \quad (A \cdot 5) \]

Although we cannot obtain any restriction from (A \cdot 4) and (A \cdot 5), \( A_{\text{before}} \) can be estimated in the following way (hereafter \( A_{\text{before}} \) is denoted as \( A_b \)). Let us consider the Einstein equation for the homogeneous and isotropic universe for \( 10^{18} \text{GeV} \geq T \geq 10^{15} \text{GeV} \)

\[ \frac{\dot{R}}{R} = 3 \pi G \rho - \frac{k}{R^2} , \]

\[ \rho = \rho_r + \rho_v = \frac{\pi^2}{30} N T^4 + \rho_v , \quad (A \cdot 6) \]

where \( \rho_v \) is the vacuum energy density \( \sim [10^{15} \text{GeV}]^4 \) and is negligible for this case (\( \dot{R} \) denotes \( dR/dt \), \( k \) is 1, -1 and 0 for close, open and flat case respectively).

ii-a) The case of the closed universe

The solution of (A \cdot 6) is

\[ R(\eta) = R_1 \sin \eta , \quad (A \cdot 7a) \]

\[ t(\eta) = R_1 (1 - \cos \eta) , \quad (A \cdot 7b) \]

\[ R_1 = \left[ \frac{4 \pi^2 N}{45} \right]^{1/2} \cdot \frac{A_{b,2}}{m_p} \approx 10^{17.7} \cdot \frac{A_{b,2}}{m_p} \text{ (eV)} . \quad (A \cdot 7c) \]

This universe has a lifetime \( \approx 2R_1 \sim 10^{9} A_{b,2} / m_p \). If we take \( A_b = 1 \), this lifetime is \( \sim 10^{-41.6} \text{sec} \). It is too short! The universe must survive at least \( T \sim 10^{15} \text{GeV} \). Then \( \rho_v \) term of (A \cdot 6) drives the universe to the de Sitter type. From (A \cdot 7a) and \( R = T / A_b \), we get

\[ \sin \eta \approx 10^{28.7} (TA_b)^{-1} \leq 1 . \quad (A \cdot 8) \]

We set \( T = 10^{15} \text{GeV} \) at (A \cdot 8) and get

\[ A_b \geq 10^{27} . \quad (\text{for } k = 1) \quad (A \cdot 9) \]

In this case \( (k=1, A_b \geq 10^{27}) \), the universe is RDU for \( 10^{18} \text{GeV} \geq T \geq 10^{16} \text{GeV} \) and the de Sitter universe for \( 10^{15} \text{GeV} \geq T \geq 10^{8} \text{GeV} \). The case \( (k=1, A_b \leq 10^{27}) \) is impossible as mentioned above. Therefore in the closed universe the flat spacetime approximation is applicable.
Table II. Various cases in which the flat spacetime approximation is justified (○) or not (×).

<table>
<thead>
<tr>
<th>$A_\phi$</th>
<th>$k$</th>
<th>1(closed)</th>
<th>$-1$ (open)</th>
<th>0(flat)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 &lt; A_\phi \leq 10^{27}$</td>
<td>this universe has too short lifetime</td>
<td>$0 &lt; A_\phi \leq 1$ × C.D.U.</td>
<td>$1 \leq A_\phi \leq 10^{27}$ ○ R.D.U. then C.D.U.</td>
<td>R.D.U. ○</td>
</tr>
<tr>
<td>$A_\phi \geq 10^{27}$</td>
<td>R.D.U. ○</td>
<td>R.D.U. ○</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

ii-b) The cases of the open universe

In the case of the open universe of $(k = -1, A_\phi \geq 10^{27})$ the universe is RDU. However, in the case $(k = -1, 1 \leq A_\phi \leq 10^{27})$, the universe is at first RDU and then become a curvature dominated one (CDU). In this case, we can repeat the same analyses in the text (we use an equation $\dot{R}/R = T/A_\phi$ instead of (4·2), then we get similar consequences to §6. Only in the case $(k = -1, 0 < A_\phi \leq 1)$ the flat spacetime approximation is not justified, thus we omit this case in this paper.

ii-c) The case of the flat universe

In the case of flat universe of $(k = 0)$, $A_\phi$ cannot be determined but the flat spacetime approximation is applicable, of course.

These various cases are summarized in Table II.

The condition for the validity of the weak field approximation is discussed in the framework of PGT. 20)

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References

19) V. N. Lukash and A. A. Starobinsky, Zh. ETF 66 (1974), 1515 [JETP 32 (1975), 742].
20) Private communication with Dr. Fukui.