On the Resonance-Like Behavior in the $pp$ Scattering Amplitudes.

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The series of "dibaryons" is investigated in detail by means of a coupled-channel model of the $NN$, $N\Delta$
and $\Delta\Delta$ channels. The $\pi$ and $\rho$ meson exchange potential is employed with cutoff form factors and inner
hard core. It is shown that the model reproduces the series of the "dibaryons" of the natural-parity states;
that is, the resonance-like behavior (RLB) in the $^1D_2$, $^3F_3$, $^1G_4$, ($^3H_5$) and $^1I_6$ states. The masses and widths
of these "resonances" agree well with those given by Auer et al. A detailed study shows that the RLB is
mainly due to the coupling with the $N\Delta$ aligned states (NDAS) ($L=J-2$ and $S=2$), and that the NDAS
give rise to a spectrum of rotational-like structure. It is suggested that aligned states (SAS) of $L=J$ and
$S=2$ bring about resonance-like behavior for the even-parity states in the higher energy region.

§ 1. Introduction

The recent discovery of characteristic structures in the $pp$ total cross sections has
stimulated many theoretical and experimental investigations on dibaryon resonances.\textsuperscript{11}
The structure of the $\Delta\sigma_{L}^T(=\sigma_{L}^{T+}-\sigma_{L}^{T-})$ can be explained by taking account of the
resonances in the $^1D_2$ and $^3F_3$ states.\textsuperscript{2} Such a resonance has been also suggested in the
$^1G_4$ state from the analyses of $d\sigma/d\Omega$, $P(\theta)$, $\Delta\sigma_{L}^T$, $\Delta\sigma_{T}^T(=\sigma_{T+}^{T+}-\sigma_{T+}^{T-})$ and the spin-spin
correlation parameter $C_{LL}$.\textsuperscript{3,4} The analyses of the $pp \rightarrow \pi^+ \pi^0$ reaction data have yielded a
similar result for these three states.\textsuperscript{5} Furthermore, phase shift analyses of the $pp$
scattering data have been carried out in detail by several groups, and gave the results that the scattering amplitudes in the $^1D_2$ and $^3F_3$ states exhibit clearly resonance-like behavior
(RLB) of counter clockwise loops in the Argand diagrams.\textsuperscript{6}

Many attempts have been done to reveal the origin of the dibaryon resonances. In
the course of a development of the quark models, it has been tried to interpret the
dibaryons as exotic states composed of 6-quarks in a bag. Although these models are
successful in predicting the possible existence of the $^1D_2$ and $^3F_3$ dibaryons, they predict
too many other two-baryon states which still remain unobserved. On the other hand, most of the works are based on the extended OBE models such as $NN\pi$ three-body
models\textsuperscript{7,8} and coupled-channel models of the $N$ and $\Delta$ isobar.\textsuperscript{9,10} Theoretical
calculations using these models reproduce the RLB, particularly for the $^1D_2$ and $^3F_3$ states,
in qualitative agreement with phase shift analyses.

Auer et al.\textsuperscript{4} measured in their recent investigations the $C_{LL}$ of the $pp$ scattering
around $\theta_{CM}=90^\circ$ up to $P_{ab}=5\text{GeV}/c$ and performed an amplitude analysis for the $I=1$
states including all the other available data. From their analysis the candidates of the
natural-parity states, i.e., the singlet or the uncoupled triplet ones, seem to make a series
containing the $^1D_2$, $^3F_3$, $^1G_4$, ($^3H_5$) and $^1I_6$ states. Recently, Niskanen has given an
interpretation that the appearance of the dibaryon series obtained by Auer et al. is a
threshold effect due to the opening of the $N\Delta$ channels, which may give a rotational-like
series owing to the centrifugal barrier.\textsuperscript{12}
In a previous paper\textsuperscript{13}) we have carried out a coupled-channel calculation of the $NN$ and $N\Delta$ channels for the $I=1$ states with the static $\pi$ and $\rho$ meson exchange potential, and shown that a series of the RLB appears for the $NN$ scattering amplitudes up to the $^1I_0^+$ state. In the calculation, the RLB originates from the coupling with the $N\Delta$ aligned states (NDAS), the states of the total angular momentum $J=L+S$ with the spin $S=2$. The purpose of this paper is to give the details of the calculation, and to clarify the mechanism to give rise to the series of the “dibaryon resonances” in a framework of the coupled-channel model. Furthermore, we examine the effect of the $\Delta\Delta$ channels.

In §2 the coupled-channel equations, the potential parameters and the coupling constants are given. The $NN$ and $N\Delta$ states treated in this paper are also given. In §3 the calculated results of the coupled-channel model are shown. The roles of the NDAS and of the transition potential between the $NN$ and $N\Delta$ channels are discussed in detail. The contribution of the $\Delta\Delta$ channels is also examined. Finally, the resonance parameters are estimated by subtracting appropriate background terms. In §4 the conclusions and remarks are given. In Appendix the detailed form of the potential is summarized.

§ 2. $NN$ and $N\Delta$ channel-coupling model

In this section, we describe the $NN$ and $N\Delta$ coupled-channel model. It is straightforward to include the $\Delta\Delta$ channels, whose contributions are described in §3.5.

2.1. Coupled-channel equations

We describe the $NN$ scattering by means of the following coupled-channel equations with the $N\Delta$ channels:\textsuperscript{14}

\begin{equation}
(E_{CM} - E_{c} - T_{c} - V_{c}) \phi_{c} = \Sigma_{c'} V_{cc'} \phi_{c'},
\end{equation}

where $E_{CM}$ is the c.m. kinetic energy of the $NN$ system, $c (c')$ denotes the $NN$ and $N\Delta$ channels, and $E_{NN}=0$, $E_{N\Delta}=M_{\Delta}-M_{N}-(i/2)\Gamma_{\Delta}$. We introduce the imaginary part of the mass of the $\Delta$ isobar $\Gamma_{\Delta}$ to take into account the unstable property of the $\Delta$ and the effects of inelastic channels other than the $N\Delta$. We use an energy independent $\Gamma_{\Delta}(=120\text{MeV})$.\textsuperscript{15}

In Eq. (2.1), $T_{c}$ denotes the relative kinetic energy operator, $V_{cc'}$ is the potential between the channels $c$ and $c'$, and $\phi_{c}$ is the relative wave function in the channel $c$.

We give the expression of the $NN$ scattering $T$ matrix as follows:

\begin{equation}
T^{1s1} = T_{NN}^{1s1} + \Sigma_{LS} T_{N\Delta}^{1s1,LS1}
\end{equation}

and

\begin{equation}
T_{N\Delta}^{1s1,LS1} = \langle \chi_{NN}^{1s1(-)} | V_{NN,N\Delta}^{1s1,LS1} | u_{N\Delta}^{LS1(+)} \rangle,
\end{equation}

\begin{equation}
= \int_{r_{c}}^{\infty} d\gamma F_{NN,N\Delta}^{1s1,LS1}(\gamma).
\end{equation}

In Eq. (2.2), $T_{NN}^{1s1}$ denotes the $T$ matrix of the $NN$ component for the angular momentum $l$ and spin $s$, and $T_{N\Delta}^{1s1,LS1}$ that of the coupling with the $N\Delta$ channel of the angular momentum $L$ and total spin $S$. In Eq. (2.3), $\chi_{NN}^{1s1(-)}$ is the incoming wave of the single-channel $NN$ scattering, and $u_{N\Delta}^{LS1(+)}$ the outgoing wave solution of the full coupled-channel equations in Eq. (2.1).
2.2. Potential and coupling constants

For the interaction potential, we adopt the usual static one constructed from \( \pi \) and \( \rho \) meson exchanges\(^{16}\). In obtaining the potential, we include the effect of the mass transfer and of the \( \Delta \) decay width for the \( \mathcal{N}\Delta \leftrightarrow \mathcal{N}\Delta \) exchange terms but neglect the imaginary part of the potential.\(^{17}\) For the \( \mathcal{NN} \leftrightarrow \mathcal{N}\Delta \) term we omit the energy transfer and the effect of \( \Gamma_s \), because it is impossible to include these within the static approximation. Furthermore it should be mentioned that since the exchange part of the \( V_{\mathcal{N}\mathcal{N},\mathcal{N}\Delta} \) potential is considerably smaller than the direct one, the results are almost independent of whether the mass transfer and the \( \Delta \) decay width are included in the \( \mathcal{N}\Delta \leftrightarrow \mathcal{N}\Delta \) exchange terms. For the short range part of the potentials we introduce a hard core of a radius 0.4fm, and also make use of a cutoff function at each vertex. The form of the potential is presented in the Appendix.

The values of the coupling constants \( f_{\mathcal{NN}}, f_{\mathcal{NL}} \) and \( g_{\mathcal{NN}} \) are derived from experiments,\(^{17}\) and the value of \( f_{\mathcal{NN}} \) is then determined so as to reproduce the \( pp \) scattering phase shifts as accurately as possible at energies lower than \( E_{CM} = 50 \text{MeV} \). For the other coupling constants \( f_{\mathcal{NN}}, f_{\mathcal{NL}}, g_{\mathcal{NN}} \) and \( f_{\mathcal{NN}} \) we use the following relations based on the \( SU(3) \) quark model: \( f_{\mathcal{NN}} = \sqrt{3/2} f_{\mathcal{NL}}, f_{\mathcal{NL}} = f_{\mathcal{NN}} \cdot f_{\mathcal{NL}}/f_{\mathcal{NN}}, g_{\mathcal{NN}} = g_{\mathcal{NL}} \) and \( f_{\mathcal{NN}} = f_{\mathcal{NL}} \cdot f_{\mathcal{NN}}/f_{\mathcal{NL}} \). Since the values of cutoff parameters are not definitely known at present, we take here the widely accepted ones,\(^{18}\) that is, an order of 1 GeV except for \( \Lambda_{\mathcal{NN}} \). We determine the value of \( \Lambda_{\mathcal{NN}} \) to reproduce the energy dependence of the absorption coefficient of the \( ^1D_2 \) state given by the phase shift analysis.\(^{19}\) It is noted that the value of \( \Lambda_{\mathcal{NN}} \) is considerably small compared with the other cutoff parameters.\(^{19}\) The coupling constants and the cutoff parameters are listed in Table I.

2.3. Transition potential and \( \mathcal{N}\Delta \) potential

In this paper we investigate the isospin \( I = 1 \) states with the total angular momentum and parity of \( J^P = 2^+, 3^-, 4^+, 5^- \) and 6+. In Table II we list the states of the \( \mathcal{N}\Delta \) channels which couple with the \( pp \) states having given spins and orbital angular momenta. Among the states, the \( L = J - 2 \) and \( S = 2 \) states have the least centrifugal barriers. In Table II these states (\( L = J - 2 \) and \( S = 2 \)) are underlined and called the \( \mathcal{N}\Delta \) aligned states (NDAS) hereafter, since its orbital angular momentum and spin are maximally aligned.

Table I. The coupling constants of the \( \pi \) and \( \rho \) exchanges, the cutoff parameters and the decay width of the \( \Delta \) isobar.

<table>
<thead>
<tr>
<th>( f_{\mathcal{NN}}/4\pi )</th>
<th>0.08</th>
<th>Exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g_{\mathcal{NN}}/4\pi )</td>
<td>0.52</td>
<td>Exp.</td>
</tr>
<tr>
<td>( f_{\mathcal{NL}}/4\pi )</td>
<td>2.70</td>
<td>Fitted to ( NN ) scattering</td>
</tr>
<tr>
<td>( f_{\mathcal{NL}}/4\pi )</td>
<td>0.35</td>
<td>Width of ( \Delta )-isobar</td>
</tr>
<tr>
<td>( f_{\mathcal{NN}}/4\pi )</td>
<td>11.91</td>
<td>Quark model</td>
</tr>
<tr>
<td>( f_{\mathcal{NL}}/4\pi )</td>
<td>0.005</td>
<td>Quark model</td>
</tr>
<tr>
<td>( f_{\mathcal{NL}}/4\pi )</td>
<td>0.164</td>
<td>Quark model</td>
</tr>
<tr>
<td>( g_{\mathcal{NL}}/4\pi )</td>
<td>0.52</td>
<td>Quark model</td>
</tr>
<tr>
<td>( \Lambda_{\mathcal{NN}} ) and ( \Lambda_{\mathcal{NN}} )</td>
<td>1.20 GeV</td>
<td></td>
</tr>
<tr>
<td>( \Lambda_{\mathcal{NN}} ) and ( \Lambda_{\mathcal{NN}} )</td>
<td>0.56 GeV</td>
<td></td>
</tr>
<tr>
<td>( \Lambda_{\mathcal{NN}}, \Lambda_{\mathcal{NL}} ) and ( \Lambda_{\mathcal{NN}} )</td>
<td>2.00 GeV</td>
<td></td>
</tr>
<tr>
<td>( \Gamma_{\Delta} )</td>
<td>120 MeV</td>
<td></td>
</tr>
</tbody>
</table>

The third column shows the methods to determine the coupling constants. For the details see the text.
other states are generally called the Non-NDAS, and the $L=J$ and $S=2$ states are sometimes referred to as the second aligned states (SAS).

We mention here the characteristics of the potential. The transition potential $V_{NN, N\Delta}$ for the states with $S=1$ in the $N\Delta$ channels is the scalar type and very weak. For the $S=2$ state it is essentially the tensor type. The radial form of the tensor-potential is the same for all the states with $S=2$ and shown in Fig. 1(a). In Table III we give the strengths of the transition potential to the $N\Delta$ states with $S=2$. The Table shows that the transitions to the NDAS are strongest in the odd-parity states. For the even-parity case all states have comparable strength with each other. Therefore, the NDAS are

<table>
<thead>
<tr>
<th>states treated in this work</th>
<th>other $pp$ states</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^1D_1$</td>
<td>$^5S_2$</td>
</tr>
<tr>
<td>$^3F_3$</td>
<td>$^5P_3$</td>
</tr>
<tr>
<td>$^1G_4$</td>
<td>$^5D_4$</td>
</tr>
<tr>
<td>$^3H_5$</td>
<td>$^5F_5$</td>
</tr>
<tr>
<td>$^1I_6$</td>
<td>$^5G_6$</td>
</tr>
<tr>
<td>$^3P_7$</td>
<td>$^5P_1,2$</td>
</tr>
<tr>
<td>$N\Delta$</td>
<td>$^5P_2$</td>
</tr>
<tr>
<td>$^5H_5$</td>
<td>$^5I_6$</td>
</tr>
<tr>
<td>$^5J_5$</td>
<td>$^5P_{1,2}$</td>
</tr>
<tr>
<td>$^5K_6$</td>
<td>$^5F_4$</td>
</tr>
<tr>
<td>$^5D_9$</td>
<td>$^5H_4$</td>
</tr>
<tr>
<td>$^5J_6$</td>
<td>$^5J_6$</td>
</tr>
<tr>
<td>$^3D_2$</td>
<td>$^3F_3$</td>
</tr>
<tr>
<td>$^3I_4$</td>
<td>$^3G_4$</td>
</tr>
<tr>
<td>$^3H_5$</td>
<td>$^3P_7$</td>
</tr>
<tr>
<td>$^3I_6$</td>
<td>$^3H_4$</td>
</tr>
</tbody>
</table>

The underlines denote the $N\Delta$ aligned states (NDAS) of the $J=L+2$ and $S=2$.

Fig. 1. (a) The radial dependence of the transition potential $V_{NN, N\Delta}(r)$ for the $^1D_2(pp)$ state to the $^5S_2(N\Delta)$ state. Only the tensor part is depicted. The transition potential to the other states is obtained by multiplying the respective strength factors listed in Table III.

(b), (c), (d) The radial dependence the $N\Delta$ potential $V_{N\Delta, N\Delta}(r)$ for the NDAS ($L=J-2$ and $S=2$), the SAS ($L=J$ and $S=2$) and the anti-parallel states ($L=J+2$ and $S=2$), respectively.
Table III. The ratio of the strength of the transition potential, $V_{NN,ND}$, to the $ND$ states of the total spin $S=2$.

<table>
<thead>
<tr>
<th>$pp$ states</th>
<th>$N\Delta$ states with $S=2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$J=L+2$</td>
</tr>
<tr>
<td>$^1D_2$</td>
<td>1</td>
</tr>
<tr>
<td>$^2F_3$</td>
<td>$-\sqrt{6}$</td>
</tr>
<tr>
<td>$^1G_4$</td>
<td>$\sqrt{\frac{9}{350}}$</td>
</tr>
<tr>
<td>$^2H_5$</td>
<td>$-\sqrt{20}$</td>
</tr>
<tr>
<td>$^1I_6$</td>
<td>$\sqrt{\frac{117}{220}}$</td>
</tr>
</tbody>
</table>

The strength is normalized to that of the $^2S_0(N\Delta)$ state from the $^1D_2(pp)$ state.

considered to couple most strongly with the $NN$ channels compared with the other states, because the NDAS have the least centrifugal barrier.

Next we show the radial dependence of the $N\Delta$ potential $V_{NN,ND}$ in Figs. 1(b), (c) and (d). As seen in Fig. 1(b) the potential for the NDAS has an attractive nature in the $^6S_2$ and $^6P_2$ states. That for the SAS in Fig. 1(c) is attractive with a long-range part. In Fig. 1(d), we see that the potential for the states with $L=J+2$ and $S=2$ has a repulsive nature.

§ 3. Calculated results

In this section we show the results of the $NN$ and $N\Delta$ coupled-channel calculations. The contribution of the $\Delta\Delta$ channels is presented in §3.5.

3.1. Calculated amplitudes

We show in Fig. 2 the Argand diagrams of the calculated $pp$ scattering amplitudes for the partial waves listed in Table II. The calculated results qualitatively agree with those of the phase shift analyses, although the calculated real phase shifts are somewhat large for low partial waves. In spite of a very simplified model, the characteristic looping behavior is well reproduced.\(^1\)

As shown in later discussions, the "resonance" masses estimated from...
the calculated amplitudes agree well with those given by Auer et al. These features are rather insensitive to the hard core radius because we do not observe any significant effect when changing it from 0.4fm to 0.5fm. Further the dependence on the cutoff parameters except for $\Lambda_{ND}$ is examined by changing their values to the extent of 30%, but these features are modified only slightly. Thus the RLB of the $pp$ scattering amplitudes can be well explained by the channel coupling with the $ND$ channels.

3.2. Dominance of $ND$ aligned states ($NDAS$)

As listed in Table II, there exist several $ND$ channels which couple with a given $NN$ channel. The NDAS have the least centrifugal barriers, and so are expected to play an important role in the $pp$ scattering at lower energies. To investigate the role of the NDAS in the channel coupling, we carry out calculations for the following two cases: One includes only the NDAS in the $ND$ channels, and the other includes only the Non-NDAS. The former is referred to as Case II, and the latter as Case III. The full calculation shown in §3.1 is referred to as Case I.

Calculated results for Cases II and III are given in Figs. 3(a) and (b), respectively. For the $^3F_3$ and $^3H_5$ states, the looping behavior in the Argand plots is clearly seen in both Cases I and II, but not in Case III. This fact means that the looping behavior for the odd-parity states originates from the coupling with the NDAS, and that the Non-NDAS play a small role. For the even-parity states the amplitudes show very clear looping for Case II. This fact means that the NDAS play a very crucial role also for the RLB. For Case III, however, the amplitudes show a behavior like looping in the $^1D_2$ and $^1G_4$ states. This fact means the Non-NDAS have also a considerable contribution in the even-parity states.

To see each contribution of the states in the $ND$ channels, it is convenient to write the $T$ matrix for individual $ND$ states defined in Eqs. (2·2) and (2·3). In Fig.4 we show $|T_{ND}^{LJ,LS}|^2$ as functions of $E_{cm}$ for the $ND$ channels with $S=2$. The contribution of the $S=1$ channels is very small and not shown in Fig.4. From the figures we note that $T$ matrix for the NDAS has a distinct peak (the solid line) for every partial wave in the energy region relevant to the
Fig. 4. The absolute square of the $T$ matrix defined in Eq. (2-3), $|T_{N_{if}^{(L,J,L_{F},S_{F})}}|$, as a function of the c. m. kinetic energy $E_{CM}$ of the $pp$ system. The solid lines denote the contribution from the NDAS, the dashed lines that from the SAS ($L=J$ and $S=2$), and the dotted lines that from the anti-parallel states ($L=J+2$ and $S=2$). The ordinates are in arbitrary scales.

Fig. 5. The radial dependence of the absolute value $|F_{N_{if}^{(L,J,L_{F},S_{F})}}|$ of the integrand of the $pp$ scattering $T$ matrix defined in Eq. (2-3). The states of the $N\Delta$ channels are taken to be the NDAS. The numbers denote the c. m. kinetic energy $E_{CM}$ of the $pp$ system.

dibaryon candidates, and that for the other states gives only a small contribution around the peak. Each energy of the peak agrees quite well with that of the minimum of the absorption coefficient. These results indicate that the coupling with the NDAS gives rise to the RLB corresponding to the dibaryon candidates.

As mentioned for the even-parity states, the contribution of the Non-NDAS cannot be neglected; that of the SAS gives rise to large and broad bumps at higher energy region as seen in Fig.4. Thus, it is considered that the looping behavior appearing in Case III is due to the coupling with the SAS. The effect of the interference between the NDAS and the SAS is rather large, as evidenced by the hairpin shape of the looping in Case I. It should be noted that the transition strengths to the SAS are comparable to those to the NDAS in case of the even-parity states. The
bumps from the SAS suggest the existence of resonance-like behavior at higher energies.

In Fig.5, we show the radial dependence of \( |F_{NN,N\Delta}^{16J,LS}(r)| \) defined in Eq. (2.3) for the NDAS calculated by using the solution of Case I. We observe that the height of the peak grows with energy and reaches its maximum at energy where the \( T \) matrix takes a maximum as seen in Fig.4 (solid lines), and that the radial position at the peak is almost independent of the partial waves and is about 1.2 fm, which coincides with approximately the range of the transition potential \( V_{NN,N\Delta} \). This suggests that the RLB originates from a dynamics at the surface region and may be related to a rotational band-like structure in the \( N\Delta \) configurations for the NDAS.

3.3. Roles of \( N\Delta \) potential and \( \Delta \) decay width

The looping behavior for the calculated scattering amplitudes might be ascribed to the existence of bound states or resonances in the \( N\Delta \) configurations, especially in the \( ^1D_2 \) and \( ^3F_5 \) states, since the \( N\Delta \) potential for the \( ^1S_2 \) and \( ^3P_2 \) states is considerably attractive as seen in Fig. 1(b). We perform a calculation for the case where the \( N\Delta \) potential outside the hard core is put to be zero. Although the calculated amplitude for the \( ^1D_2 \) state suffers a slight change, the \( N\Delta \) potential \( V_{N\Delta,N\Delta} \) is found to play only a minor role for the RLB.

So far to take into account the decaying property of the \( \Delta \) isobar we have used a constant width parameter \( (\Gamma_\Delta = 120 \text{MeV}) \). To see the effect of \( \Gamma_\Delta \) we carry out a calculation in which \( \Gamma_\Delta \) is taken to be zero in the full coupled-channel case. For all the partial waves the calculated amplitudes show looping behavior,\(^*) \) but they move rapidly with energy in the Argand diagram. This indicates the necessity to use the large value for \( \Gamma_\Delta \) to get an agreement with experiment. We also note that the real phase shift of the \( ^1D_2 \) state shows a cusp in this case, manifesting a threshold effect. The cusp in the \( ^1D_2 \) state is actually smeared out by the effect of the large width \( \Gamma_\Delta \).\(^10\)

3.4. Role of transition potential

As shown in §3.2, the NDAS play a crucial role in showing the looping behavior in the Argand plots. However, as noted in §3.3, the \( N\Delta \) potential \( V_{N\Delta,N\Delta} \) gives little effect on the RLB. It may be considered that an effect of the strong coupling with the NDAS plays an important role in the RLB.

In order to see the coupling effect with

\(^*) \) The statement in Ref. 13) that the amplitudes cease to show the looping behavior for the case \( \Gamma_\Delta = 0 \) in the higher partial waves is incorrect.
the NDAS in detail, we introduce a factor \( g \) which is multiplied to all the transition potentials between the NDAS and the others. Then we can write the \( S \) matrix as a power series of \( g^2 \). Here, a term of the \( n \)-th power in \( g^2 \) involves the interaction through the NDAS \( n \) times. It should be noted that every order term of \( g^2 \) includes all order terms about the Non-NDAS.

To see the dependence of the amplitudes on \( g^2 \), it is useful to subtract the zeroth-order terms of \( g^2 \) from the amplitudes. To do this, we define the following subtracted \( S \) matrix \( \tilde{S} = S - S_{bg} \), where \( S \) is the \( S \) matrix of the full calculation and \( S_{bg} \) denotes that of a background term which is taken here to be that in Case III. In Fig. 6 we show the calculated results of \( \tilde{S} \) for the \( \,^1D_2, \,^3F_3 \) and \( \,^1G_4 \) states as functions of \( g^2 \) and energy \( E_{cm} \). Comparing the amplitudes for \( g^2 = 1 \) in Fig. 6 with those of Fig. 2, we can see that the resulting amplitudes exhibit very clear looping behavior and that the resonance parts are extracted well by subtracting the background terms. However, that of the \( \,^1D_2 \) state is somewhat obscure. The reason is probably an interference between the NDAS and the SAS, since the transition strengths to these states from the \( NN \) channel are comparable and the coupling between the NDAS and the SAS is also strong. To see the amplitude without the contribution of the Non-NDAS, we calculate another subtracted \( S \) matrix \( \tilde{S}' = S' - S(NN) \) for the \( \,^1D_2 \) state. Here, \( S' \) is the one calculated for Case II with the \( g \) factor, and \( S(NN) \) the single-channel \( S \) matrix of the \( pp \) scattering. The calculated \( \tilde{S}' \) is shown in Fig. 6(d). The resulting amplitude has a clear looping behavior, as expected. The amplitude is still distorted, which is probably due to a threshold effect appearing as the cusp for the case \( \Gamma_2 = 0 \).

Now, let us see the \( g^2 \) dependence of the amplitudes in Fig. 6. We note that the amplitudes depend almost linearly on \( g^2 \) at fixed energies. This means that the coupled-channel effect of the NDAS can be well described by the Born terms. At the resonance region, however, the amplitudes deviate rather appreciably from the linear dependence on \( g^2 \); that is the multiple terms about the NDAS contribute to a certain extent in the resonance region. This may suggest that the RLB are something more than the threshold effect of the opening of the \( NA \) channels.

3.5. Contribution of \( \Delta \Delta \) channel

So far, we have neglected the contributions of the \( \Delta \) channels. Their threshold is about the mass of the \( \,^1G_4 \) dibaryon candidate. Then, the contributions may influence importantly on the RLB of the higher partial waves. In view of this, we carry out the coupled-channel calculation including both the \( NA \) and \( \Delta \) channels. It is noted that the inclusion of the \( \Delta \) channels is straightforward (\( E_{\Delta} = 2E_{NA} \) in Eq. (2.1)) and does not need any new parameters other than those in \$2.2 \). The results are plotted by dashed lines in Fig. 2. Compared with the solid lines calculated previously, we note that the contributions of the \( \Delta \) channels are rather small, and that the RLB is little affected by the inclusion even for the higher partial waves we considered. Thus, the results obtained in the previous subsections are not altered.

3.6. Calculation of resonance parameters

In this subsection we extract the "resonance" parameters from the calculated amplitudes. For this purpose, we need to separate the "resonance" amplitudes from the background terms. It is, however, difficult to perform the separation rigorously, since
Table IV. The resonance parameters calculated by the full coupled-channel model.

<table>
<thead>
<tr>
<th>$p\bar{p}$ states</th>
<th>$E_{\text{exp}}$(GeV)</th>
<th>$E_{\text{calc}}$(GeV)</th>
<th>$\Gamma_{\text{el}}/\Gamma_{\text{tot}}$</th>
<th>$\Gamma_{\text{el}}/\Gamma_{\text{tot}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^3D_2$</td>
<td>2.14 – 2.17</td>
<td>2.14</td>
<td>0.08</td>
<td>0.10</td>
</tr>
<tr>
<td>$^3F_3$</td>
<td>2.20 – 2.25</td>
<td>2.25</td>
<td>0.16</td>
<td>0.18</td>
</tr>
<tr>
<td>$^1G_4$</td>
<td>2.43 – 2.50</td>
<td>2.41</td>
<td>0.09</td>
<td></td>
</tr>
<tr>
<td>$^3H_8$</td>
<td>2.70 ± 0.10</td>
<td>2.59</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>$^1I_6$</td>
<td>2.90 ± 0.10</td>
<td>2.91</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$E_{\text{exp}}$ denote the masses of the resonance candidates reported by Auer et al. F. $E_{\text{calc}}$ and $\Gamma_{\text{el}}/\Gamma_{\text{tot}}$ are the calculated masses and inelasticities using the one-level Breit-Wigner formula, respectively. The fifth column is the inelasticities obtained by the pole search for the result of the phase shift analysis.

there are some ambiguities to determine the background terms. In this paper, based on the studies so far, we define the background terms to be the scattering amplitudes of Case III, which are calculated without the NDAS. Then, the background subtracted S matrix is just given by $\hat{S}$ in §3.4 with $g=1$. By assuming a one-level formula for the “resonance” part, the “resonance masses and widths” (or inelasticities) are determined so as to fit the amplitudes calculated from $\hat{S}$. The “resonance parameters” obtained are shown in Table IV. The “resonance masses” agree well with the values of Auer et al., and the inelasticities for the $^1D_2$ and $^3F_3$ states are also in good agreement with the predictions by Kanada et al.²⁰

The extracted “resonance masses” are plotted versus $J(J+1)$ in Fig. 7, and compared with the masses of the dibaryon candidates given by Auer et al. Here, the $R_{H}$ candidate at 2.7 GeV is assigned as the $^3H_8$ state. We notice here that the “resonance masses” show a rotation-like spectrum. The solid and dashed lines in Fig. 7 display the spectra calculated with the formula $\sqrt{s}=M_{d}+M_{N}+L(L+1)/2\mathcal{J}$, where $L$ is the orbital angular momentum between the $N$ and $\Delta$, and the inertia parameter is taken to be $\mathcal{J}=r_{0}^{2}M_{N}\cdot M_{d}/(M_{N}+M_{d})$. The calculated masses are well in accordance with the curve of the lowest band, which is constructed from the NDAS ($L=J-2$), when we take $r_{0}$ to be about 1.0 fm. The formula is based on the rotational-band model of the $ND$ system.²¹ Then, the $^1D_2$
state is the band head, and is just the lowest dibaryon candidate. In §3.2, we have shown that the $F_{\pi N N, S_L S_L}(r)$, the integrand of the $T$ matrix for the coupling to the NDAS, is spatially localized around 1.2 fm. This may support the above consideration. It is interesting to recall here that the SAS give rise to broad bumps at higher energies for the even-parity states (see §3.2). We indicate the energies at the bumps with the triangles in Fig. 7. They are placed roughly on the curve of the next band of the natural-parity states ($L=J$ and $S=2$), although the inertia parameter seems large.

Recently, Kanada et al.\textsuperscript{20} carried out a phase shift analysis using new data and suggested the existence of RLB in the $^3P_2$ and $^3P_0$ states from a pole search. The masses of the $^3P_2$ and $^3P_0$ are also shown in Fig. 7. The $^3P_2$ and $^3P_0$ candidates may be a candidate for the lowest and next rotational bands of the unnatural-parity states ($L=J-1$ and $L=J+1$ with $S=2$), respectively.

§ 4. Concluding remarks

We have examined the resonance-like behavior (RLB) in the elastic $pp$ scattering in terms of the $NN$, $N\Delta$ and $\Delta\Delta$ coupled-channel model with the $\pi$ and $\rho$ meson exchange static potential.

The results of the present analysis are summarized as follows:

1) The coupled-channel model reproduces well the series of the "dibaryons" of the natural parity from the $^1D_2$ state up to $^1I_0$.

2) The $N\Delta$ channels give essential contributions to the RLB. The contribution of the $\Delta\Delta$ channels is considerably small in the energy region under consideration.

3) Among the intermediate $N\Delta$ states, the $N\Delta$ aligned states (NDAS) play a crucial role to bring about the RLB.

4) The series of the "dibaryons" show a systematics analogous to a rotational band. Then, it is reasonable to consider that the $^1D_2$ candidate is the band head of the series. The $R_{II}$ candidate at 2.7 GeV is assigned to the $^3H_2$ member of this band.

5) The $N\Delta$ potential $V_{N\Delta N\Delta}$ has little influence on the RLB. Therefore, it is considered that the RLB is not due to the bound states or resonances by $V_{N\Delta N\Delta}$.

6) The characteristic features of the RLB can be well described using the transition potential $V_{NN, N\Delta}$ of the $\pi$ and $\rho$. The repeated effect of the coupling between the $NN$ and $N\Delta$ channels gives a certain contribution to the RLB.

7) To obtain a quantitative agreement with the RLB, it is necessary to take about 120 MeV for $\Gamma_{\Delta}$.

8) There appears an even-odd-parity dependence in the scattering amplitudes, which is due to the property of the transition potential $V_{NN, N\Delta}$. For the even-parity states, the coupling to the $N\Delta$ second aligned states (SAS) is rather strong, and resonance-like behavior may appear as a series in the higher energy region.

Now, we mention several points to be examined further in our analysis. We have taken into account the effects of all the inelastic channels by introducing the complex mass for $\Delta$ isobar. It is desirable to evaluate the effect of every inelastic channel explicitly, especially $NN\pi$ channel. Moreover, in the present calculations the $NN$ phase shifts of low partial waves at low energies are not reproduced satisfactorily, because the potential includes only by the $\pi$ and $\rho$ exchanges. To obtain a quantitative agreement it is necessary to use more realistic potential for the $NN$ interaction. However, we expect
that the RLB is not qualitatively altered by more realistic potential in the $NN$ channel, since the transitions to the $N\Delta$ channels are essentially described by the $\pi$ and $\rho$ exchanges.

In this paper we do not study such $pp$ states as the $^3P_l$ states. There has been pointed out a possibility of existence of the RLB in the scattering amplitudes for the $^3P_2$ and $^3P_0$ states (see §3.6). It is then interesting to do a similar calculation to these states. On the other hand, the situation for the existence of the “dibaryon states” in the isospin $I=0$ states is not clear in experiments.\textsuperscript{22) Therefore, it is also interesting to study the $I=0$ states in the coupled-channel model, where only the coupling with the $\Delta\Delta$ channels influences the $NN$ scattering.

Appendix

In this appendix we give the explicit formulas of the $NN$ and $N\Delta$ potentials of the one $\pi$ and one $\rho$ exchanges. The direct and transition potentials are expressed in an operator form as

$$V = \mathcal{D} \cdot \mathcal{T} (V_{\text{s}} + S \cdot S \cdot V_{\text{s}} + S_{12} \cdot V_{\text{T}}), \quad (A \cdot 1)$$

where $S$, $\mathcal{T}$ and $S_{12}$ are the generalized spin, isospin and tensor operators, respectively. The explicit form of $S$ is given by the following matrix:

$$S = \begin{bmatrix} \sigma & \Sigma^\dagger \\ \Sigma & S \end{bmatrix}, \quad (A \cdot 2)$$

where $\sigma$ is the Pauli matrix, $\Sigma$ the transition spin operator between the $N$ and $\Delta$, and $S$ the spin operator of the $\Delta$. The operator $\mathcal{T}$ is given as similar to the $S$. In Eq. (A\cdot1), $V_{\text{s}}$ ($V_{\text{s'}}$) and $V_{\text{T}}$ are the radial parts of the scalar and tensor potentials, respectively. Including the $\pi$ and $\rho$ exchange contributions we obtain

$$(S \cdot S \cdot V_{\text{s}})_{ij,kl} = f_{ikn}f_{jln}F_{\sigma}(r, m_{\pi}, e_{\pi}, \Lambda_{\pi}^1, \Lambda_{\pi}^2) + 2f_{ikn}f_{jlp}F_{\sigma}(r, m_{\rho}, e_{\rho}, \Lambda_{\rho}^1, \Lambda_{\rho}^2), \quad (A \cdot 3)$$

$$(S_{12} \cdot V_{\text{s}})_{ij,kl} = f_{ikn}f_{jln}F_{T}(r, m_{\pi}, e_{\pi}, \Lambda_{\pi}^1, \Lambda_{\pi}^2) - f_{ikn}f_{jlp}F_{T}(r, m_{\rho}, e_{\rho}, \Lambda_{\rho}^1, \Lambda_{\rho}^2), \quad (A \cdot 4)$$

$$(V_{\text{s'}})_{ij,kl} = \delta_{ij} \delta_{jk} \delta_{kl} g_{i}^{\text{exchange}}(i,g_{\text{exchange}}) F_{\sigma'}(r, m_{\rho}, e_{\rho}, \Lambda_{\rho}^1, \Lambda_{\rho}^2), \quad (A \cdot 5)$$

where $f_{ikn}$, $f_{ikp}$ and $g_{i}$ denote the coupling constants, and $i$, $j$, $k$ and $l$ are $N$ or $\Delta$. $F_{\sigma}$, $F_{T}$ and $F_{\sigma'}$ are given as

$$F_{\sigma}(r, m_{\pi}, e_{\pi}, \Lambda_{1}, \Lambda_{2}) = \frac{1}{3m^{2}} \frac{1}{(2\pi)^{3}} \int \frac{m^{2}(S \cdot S)}{e^{2}-q^{2}-m^{2}} \frac{\Lambda_{1}^{2}-m^{2}}{\Lambda_{1}^{2}+q^{2}} \frac{\Lambda_{2}^{2}-m^{2}}{\Lambda_{2}^{2}+q^{2}} \exp(iq \cdot r) d^{3}q, \quad (A \cdot 6)$$

$$F_{T}(r, m_{\pi}, e_{\pi}, \Lambda_{1}, \Lambda_{2}) = \frac{1}{m^{2}} \frac{1}{(2\pi)^{3}} \int \frac{(q \cdot S)(q \cdot S)}{e^{2}-q^{2}-m^{2}} \frac{m^{2}(S \cdot S) / 3}{\Lambda_{1}^{2}-m^{2}} \frac{\Lambda_{2}^{2}-m^{2}}{\Lambda_{2}^{2}+q^{2}} \exp(iq \cdot r) d^{3}q, \quad (A \cdot 7)$$

$$F_{\sigma'}(r, m_{\rho}, e_{\rho}, \Lambda_{1}, \Lambda_{2}) = \frac{1}{(2\pi)^{3}} \int \frac{1}{e^{2}-q^{2}-m^{2}} \frac{\Lambda_{1}^{2}-m^{2}}{\Lambda_{1}^{2}+q^{2}} \frac{\Lambda_{2}^{2}-m^{2}}{\Lambda_{2}^{2}+q^{2}} \exp(iq \cdot r) d^{3}q, \quad (A \cdot 8)$$

where $\Lambda_{1}$ and $\Lambda_{2}$ denote the cutoff parameters and $e$ the mass transfer.
On the Resonance-Like Behavior in the $pp$ Scattering Amplitudes

Now, we write the spin and angular momentum parts of the $NN$ and $N\Delta$ wave functions, respectively:

$$q_{NN}^{LS} = \frac{1}{2} \left\{ \left| l\left(\frac{1}{2} \frac{1}{2}\right) s J;\left(\frac{1}{2} \frac{1}{2}\right) I \right| - \left(\frac{1}{2} \frac{1}{2}\right) s J;\left(\frac{1}{2} \frac{1}{2}\right) I \right\},$$  \hspace{1cm} (A\cdot9)$$

$$q_{N\Delta}^{LS} = \frac{1}{\sqrt{2}} \left\{ \left| L\left(\frac{1}{2} \frac{3}{2}\right) s J;\left(\frac{1}{2} \frac{3}{2}\right) I \right| - \left(\frac{3}{2} \frac{1}{2}\right) s J;\left(\frac{3}{2} \frac{1}{2}\right) I \right\}. \hspace{1cm} (A\cdot10)$$

Then, the matrix elements of the $S \cdot S$ and $S_{12}$ are given by

$$\langle \lambda(s_1 s_2) s_3 | S \cdot S | \lambda'(s_1' s_2') s_3' \rangle$$

$$= \delta_{s_3 s_3'} \delta_{\lambda \lambda'} (-)^{s_3 + s_3' + s_1} s_1 \left\{ \begin{array}{c} s_2 \ s_3 \\ s_2' s_1' \end{array} \right\} <s_1\|S\|s_1'> <s_2\|S\|s_2'> \hspace{1cm} (A\cdot11)$$

and

$$\langle \lambda(s_1 s_2) s_3 | S_{12} | \lambda'(s_1' s_2') s_3' J \rangle$$

$$= (-)^{s_3 + s_3'} \sqrt{30} s_3 s_3' \lambda \lambda' \left\{ \begin{array}{c} \lambda' \ 2 \ \lambda \\ 0 \ 0 \ 0 \end{array} \right\} s_1 \left\{ \begin{array}{c} \lambda \ 2 \ -\lambda' \\ s_1' \ s_2' \ s_3' \end{array} \right\} <s_1\|S\|s_1'> <s_2\|S\|s_2'> \hspace{1cm} (A\cdot12)$$

where $A(\lambda')$ and $s_3(s_3')$ denote the orbital angular momentum and the total spin of the $NN$ or $N\Delta$ system, respectively, and $s_i(s_i')$ for $i=1,2$ the spins of the $N$ or $\Delta$. $\lambda$ represents $\sqrt{2\lambda+1}$. The reduced matrix element is given by

$$\langle s_1\|S\|s_2\rangle = \begin{cases} \sqrt{6} & s_1 = s_2 = \frac{1}{2}, \\ 2\sqrt{15} & s_1 = s_2 = \frac{3}{2}, \\ 2 & \text{otherwise}. \end{cases} \hspace{1cm} (A\cdot13)$$

For the $T \cdot T$ we have an expression of the matrix element similar to that of the $S \cdot S$.

References

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