Anti-Baryon Inclusive Spectra
in Hadron Fragmentation and the Recombination Mechanism

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We extend our quark cascade recombination model to include the successive type baryon and anti-baryon emission such as quark \( \rightarrow \) anti-diquark \((\bar{q}q)\)+baryon. We compare our predicted single particle spectra, i.e., \( p \rightarrow \bar{p}X, \bar{p}X, \Lambda X, \Lambda X ', \) meson \(+X, \pi^{+}\rightarrow \pi^{+}X, K^{+}X, K^{+}X, \) \( pX, \Lambda X, \Delta^{+}X, \Delta^{-}X, \rho^{+}X, \rho^{0}X, \) and particle ratios \( p \rightarrow \Lambda/\Lambda, \pi^{+}/\pi^{-}, K^{+}/K^{-}, \Sigma^{+}/\Sigma^{0} \) with the experimental data. We discuss the type of recombination mechanism in the final stage of quark diquark cascade.

§ 1. Introduction

The inclusive spectra of hadrons in small \( p_{T} \) reactions such as \( p \rightarrow pX, K^{+}X, \pi^{+}\pi^{+}X, \) \( \Delta^{+}X, \pi^{+}\rightarrow \pi^{+}X, K^{+}X, \pi^{+}\pi^{+}X, \) and \( e^{+}e^{-} \rightarrow \pi^{+}X, K^{+}X \) have been analyzed successfully and in a unified way using quark cascade recombination model which takes into account the recombination mechanism proposed by Fukuda and one of the authors (C. I.).\(^{1-8} \)

Most of the predictions from our model fit nicely to the experimental data, including the leading particle spectra such as \( \pi^{+}p \rightarrow \pi^{+}X \) and \( pp \rightarrow pX, \) and two particle correlations.\(^{4} \)

This model is too simple to predict the anti-baryon inclusive spectra in baryon fragmentation and the baryon and anti-baryon spectra in meson fragmentation. Several authors\(^{9,10} \) took into account diquark states to predict these spectra. The quark-diquark cascade model\(^{10} \) by Kinoshita, Noda, Mizouchi and one of the authors (T. T.) well describes these spectra except for the leading particle spectra at large \( x \) region.

In order to analyze rare hadron spectra like

\[
\begin{align*}
\text{meson} & \rightarrow \text{baryon} + X, \\
& \rightarrow \text{anti-baryon} + X, \\
\text{baryon} & \rightarrow \text{anti-baryon} + X,
\end{align*}
\]

(1.1)

it is necessary to extend the previous quark cascade recombination model\(^{1-8} \) by introducing diquark states. Then hadrons are emitted through the following successive processes:

\[
\begin{align*}
(\bar{q}q) & \rightarrow \bar{B} + q, \\
(\bar{q}q) & \rightarrow m + (\bar{q}q), \\
q & \rightarrow B + (\bar{q}q),
\end{align*}
\]

(1.2)

besides the usual successive meson emission from quarks.

\(^{4} \) Our prediction is the one for the low \( p_{T} \) reaction (low \( Q^{2} \) ) at the extremely high energy and approximate \( x \)-scaling for the initial energy is assumed. This model can be applied to fragmentation region only and not to central plateau in which region more complex mechanism like pionization becomes important.
\[ q \rightarrow m + q. \] (1.3)

By this modification of our previous model we are obligated to introduce the following recombination mechanism into hadrons at the final stage of the quark diquark cascade,

\[ (\bar{q} q)(q q) \rightarrow m + m, \quad (\bar{q} q)(\bar{q} \bar{q}) \rightarrow B + m, \]

\[ (\bar{q} q)(q q) \rightarrow B + B \quad \text{for baryon fragmentation}, \quad (1.4a) \]

\[ (\bar{q} q) \rightarrow B, \quad q(q q) \rightarrow B, \]

\[ (\bar{q} q)(q q) \rightarrow m + m \quad \text{for meson fragmentation}, \quad (1.4b) \]

besides the usual type of recombination, \( qqq \rightarrow \text{baryon} \). In our model the recombined hadron has relatively large fraction of momentum compared with that of the successively emitted hadrons. Therefore, if the recombinations like (1.4) occur with the same probability as the usual type recombination, we may have meson and anti-baryon spectra with relatively large momentum fraction. The probabilities of recombinations are arbitrary parameters and in this note we take the extreme two cases and check which kind of recombination we should assume:

Model I) The probabilities of all type of recombinations like (1.4) are the same as the usual one (\( qqq \rightarrow \text{baryon in baryon fragmentation and } \bar{q}q \rightarrow \text{meson in meson fragmentation} \)).

Model II) Only the probability of the usual type of recombination is large and those of recombination like (1.4) can be ignored effectively. As we see later these two types of recombination mechanism predict very similar spectra for reactions such as \( p \rightarrow p X, \Delta^{+}X, \pi^{\pm}X, \) but there are differences in the reactions \( p \rightarrow \bar{p} X, \pi^{\pm}X, \) and \( \pi^{+} \rightarrow p X, p X \) due to the differences of recombination mechanism and we may choose what kind of recombination mechanism we should assume. In \( \S 2 \) we explain our modified quark cascade recombination model and describe the whole processes of proton and meson jets. We also explain our two extreme models. In \( \S 3 \) the single particle spectra \( p \rightarrow \bar{p} X, \bar{p} X, p X, \Lambda X \) and particle ratios \( p \rightarrow \bar{A}/A, \pi^{+}/\pi^{-}, K^{+}/K^{-}, \Sigma^{+}/\Sigma^{-} \) by our two models are compared with the experimental data to fix the parameters and to check which kind of recombination mechanism is desirable. In \( \S 4 \) the single particle spectra \( \pi^{+} \rightarrow \text{meson} + X, \pi^{+} \rightarrow \text{Baryon} + X \) predicted by our model are compared with the experimental data. Finally conclusions and discussion are given in \( \S 5 \).

\section*{\S 2. Models}

We modify our quark cascade recombination model in Refs. 1)\textemdash}5), and the hadron jet in hadron's fragmentation region proceeds in the following way (Fig. 1);

(i) The \( \pi^{+} \)-meson beam separates into one \( u \)-quark and one anti-\( d \)-quark, where we assign Feynman scaling variable \( x = 2p_{u}/\sqrt{s} \) of these quarks to be \( x_{0}' \) and \( x_{0}'' \), respectively. We assume equipartition \( x \) distribution;

\[ dx_{0}' dx_{0}'' \delta(x_{0}' + x_{0}' - 1). \] (2.1)

(i') For the proton beam which separates into two \( u \)-quarks and one \( d \)-quark, in Ref. 1) we assumed the equipartition \( x \) distribution;
This simple model with this initial quark distribution predicts that the $\pi^+ / \pi^-$ ratio is less than 2 and does not agree with the experimental data, thus in Ref. 5) we parametrized $x$ distribution of the initial quarks as follows:

$$\frac{6}{3-p} \delta(x_0' + x_0'' + x_0''' - 1) \times \delta(x_0 + x_0'' + x_0''' - 1), \quad (0 \leq p \leq 1)$$

where $x_0'''$ is the initial Feynman scaling variable of $d$ quark, and $x_0'$ and $x_0''$ are those of $u$ quarks.

We choose $p = 1$ in this paper as in Ref. 5). This choice of the initial quark distribution corresponds to the QCD spin effects as was expected by Lepage and Brodsky.\(^{11}\)

(ii) Each quark with $x_0$ emits a meson or a baryon and becomes a quark or an anti-diquark with $x$. The anti-diquark emits a meson or an anti-baryon and becomes an anti-diquark or a quark. These successive meson, baryon or anti-baryon emissions are formulated as in Ref. 1) in the form of diffusion equation (or in the form of integral equation). We write the quark and anti-diquark spectrum functions as

$$Q(x, t) = \begin{bmatrix}
  u(x, t) \\
  d(x, t) \\
  s(x, t) \\
  (\bar{u}\bar{u})(x, t) \\
  (\bar{u}\bar{d})(x, t) \\
  (\bar{d}\bar{d})(x, t) \\
  (\bar{u}\bar{s})(x, t) \\
  (\bar{d}\bar{s})(x, t) \\
  (\bar{s}\bar{s})(x, t)
\end{bmatrix}, \quad (2.4)$$

where $x$ and $t$ are Feynman scaling variable and time, respectively. The diffusion equation for the quark anti-quark cascade process starting from initial Feynman parameter $x_0$ is
\[ \frac{dQ(x, t)}{dt} = -\lambda' Q(x, t) \]
\[
\begin{bmatrix}
1 & 1 & c \\
& D & \\
-\lambda & D & \\
& D & \\
D((1-F)+F(1+c)/2) & D((1-F)+F(1+c)/2) & D((1-F)+F\cdot c)
\end{bmatrix} \cdot Q(x, t)
\]
\[ + \lambda \int_x^{x_0} dx' K(x, x') Q(x', t), \]  
\]
\[ K(x, x') = \begin{bmatrix}
(1-E) \cdot T_{q,q}(x, x') & (1-F) \cdot DT_{q,q}(x, x') \\
E \cdot DT_{q,q}(x, x') & F \cdot DT_{q,q}(x, x')
\end{bmatrix} \]  
\]
\[ \begin{bmatrix}
1-a-b & a & b' \\
& a & \\
b & b & c-2b'
\end{bmatrix}, \]  
\]
\[ T_{q,q} = \begin{bmatrix}
A_q^2 & A_q^2 & A_q^2 \cdot c \\
2A_q^2 & 2A_q^2 & 2A_q^2 \cdot c \\
A_q^2 & A_q^2 & A_q^2 \cdot c \\
2A_q \cdot B_q & 2A_q \cdot B_q & 2A_q \cdot B_q \cdot c \\
2A_q \cdot B_q & 2A_q \cdot B_q & 2A_q \cdot B_q \cdot c \\
B_q^2 & B_q^2 & B_q^2 \cdot c
\end{bmatrix}, \]  
\]
\[ \begin{bmatrix}
1-a-b & a/2 & 0 & b'/2 & 0 & 0 \\
a & 1-a-b & a & b'/2 & b'/2 & 0 \\
0 & a/2 & 1-a-b & 0 & b'/2 & 0 \\
b & b/2 & 0 & (1-a-b+c-2b')/2 & a/2 & b' \\
0 & b/2 & b & a/2 & (1-a-b+c-2b')/2 & b' \\
0 & 0 & 0 & b/2 & b/2 & c-2b'
\end{bmatrix} \]  
\]
\[ T_{q,q} = \begin{bmatrix}
A_q & A_q & A_q & A_q & A_q & A_q \\
A_q & A_q & A_q & A_q & A_q & A_q \\
B_q & B_q & B_q & B_q & B_q & B_q
\end{bmatrix}, \]  
\]

where \( \lambda \) and \( \lambda' \) are “decaying factor” for hadron production and “recombination factor”, respectively. \( f_{q,q}(x, x') \) is the probability that \( Q' \) (a quark or an anti-diquark) with momentum fraction \( x' \) emits a meson, a baryon or an anti-baryon with \( x'-x \) and becomes \( Q \) with \( x \) and normalized by
Anti-Baryon Inclusive Spectra in Hadron Fragmentation

\[ \int_0^{z_0} f_{\varphi, \varphi}(x, x_0) dx = 1. \quad (2.11) \]

We assume the following emission function:

\[ f_{\varphi, \varphi}(x, x_0) = \frac{1 + a_{\varphi, \varphi}}{x} \left( \frac{x}{x_0} \right)^{a_{\varphi, \varphi}}. \quad (2.12) \]

Judging from our former results,\(^{1\text{-}8}\) we take

\[ a_{\varphi, \varphi} = 1.5, \]

and for simplicity we assume

\[ a_{\bar{s}q, q} = a_{q, \bar{s}q} = a_{q, q} = a_{q, \varphi}. \quad (2.13) \]

The constants \( a, b, b', \) and \( c \) in matrix \( T_{\varphi, \varphi} \) is the parameters defined in Ref. 1) and represent the probabilities that quarks emit mesons in a flavour dependent way. The constants \( A_f \) and \( B_f \) in the matrices \( T_{q, q} \) and \( T_{q, q} \) are parameters a la Field and Feynman,\(^{12}) \) that is, \( A_f \) and \( B_f \) represent the parameters for \( uu(d\bar{d}) \) and \( ss \bar{s} \) pair production, respectively, and satisfy the relation \( 2A_f + B_f = 1. \) The probability of a quark (an anti-diquark) to emit a meson and the one to emit a baryon (an anti-baryon) are \( 1 - E(F) \) and \( E(1 - F) \), respectively. In our previous papers\(^{1\text{-}8}\) we set our parameters as

\[ b/a = 0.15, \quad b'/b = 3, \quad c = 0.6, \]

\[ \lambda'/\lambda = 0.5 \quad (2.14) \]

and required the relation \( 1 - a - b = a/2. \) We also set the other parameters as

\[ E = \begin{cases} 0.05 & \text{for baryon fragmentation}, \\ 0.1 & \text{for meson fragmentation}, \end{cases} \]

\[ F = 0.33, \quad A_f = 0.45, \]

\[ D = 1 \text{ or } 50 \text{ (depends on the model described in later)}. \quad (2.15) \]

We consider the direct production of vector mesons as well as \( ps \) mesons from quarks or di-quarks, and we simply assume that the vector and pseudo-scalar meson branching ratios are equal,

\[ a_{\psi} / a_{ps} = b_{\psi} / b_{ps} = b'_{\psi} / b'_{ps} = 1. \quad (2.16) \]

The time of the hadron emission from \( u \) and \( d \) quarks is characterized by a parameter \( \lambda \), while the time of hadron emission from \( s \) quark (\( uu \) diquark, \( u\bar{s} \) diquark, \( \cdots \)) is characterized by \( \phi \lambda \left( D\lambda, D((1 - F) + F(1 + c)/2)\lambda, \ldots \right) \).

(iii) After such incoherent successive type hadron emissions those quarks or (anti) diquarks recombine into hadrons. The time when the recombination occurs is determined by a parameter \( \lambda' \). The ratio \( \lambda' / \lambda \) is taken to be \( \sim 0.5 \). In this note we assume the following extreme cases:

Model 1) The probabilities of all types of recombination have the same value; that is, in baryon fragmentation quark and anti-quark systems recombine into hadrons as

\[ qqq \rightarrow \text{one baryon} \quad (\text{Fig. 1(a)} \sim (c)), \quad (2.17a) \]
\((\bar{q} \bar{q}) qq \rightarrow \text{two mesons} \quad \text{(Fig. 1(d))},\)
\((\bar{q} \bar{q})(\bar{q} \bar{q}) q \rightarrow \text{one anti-baryon and one meson} \quad \text{(Fig. 1(e))},\)
\((\bar{q} \bar{q})(\bar{q} \bar{q})(\bar{q} \bar{q}) \rightarrow \text{two anti-baryon} \quad \text{(Fig. 1(f))}.\)  \hspace{1cm} (2
d On the other hand, the following recombinations are expected for meson beam,
\[ q \bar{q} \rightarrow \text{one meson}, \]  \hspace{1cm} (2.18a)
\[ (\bar{q} \bar{q}) \bar{q} \rightarrow \text{one anti-baryon}, \]  \hspace{1cm} (2.18a)
\[ q(qq) \rightarrow \text{one baryon}, \]  \hspace{1cm} (2.18a)
\[ (\bar{q} \bar{q})(qq) \rightarrow \text{two mesons}. \]  \hspace{1cm} (2.18a)

This recombination mechanism is characterized by putting the parameter \(D\) to be 1. Model II) We assume the probability that the same kind of hadron with the initial one is produced at the final stage of quark cascade to be large; that is, the following recombinations
\[ \text{baryon} \rightarrow qqq \rightarrow \text{one baryon} \]  \hspace{1cm} (2.17a)
and
\[ \text{meson} \rightarrow q\bar{q} \rightarrow \text{one meson} \]  \hspace{1cm} (2.18a)
dominate in baryon and meson fragmentation, respectively. This kind of recombination is achieved by taking the value of \(D\) to be large. This implies that di-quarks last very short time as compared with quarks in the cascade process. Then the chance of di-quark to recombine into meson or baryon becomes smaller compared with that of quark to recombine into meson or baryon.¹ We take \(D=50\) and call this case Model IIa. In order to clarify the difference of the recombination mechanism straightforward, we can also make a model in which the successive hadron emissions occur with \(D=1\) as Model I but at the end of the cascade we force the di-quark (anti-diquark) to emit a baryon (an anti-baryon) and there is no diquark (anti-diquark) just before the recombination. Then only recombination (2.17a) or (2.18a) occurs. We call this latter model Model IIb. These two models are essentially the same one, because the large value of \(D\) in Model IIa forces Model IIa to take the same recombination mechanism with that in Model IIb.

(iv) For simplicity we consider vector meson and decouplet baryon decay as
\[ \rho^+ \rightarrow \pi^+ + \pi^-, \quad \rho^0 \rightarrow \pi^+ + \pi^-, \]  \hspace{1cm} (2.19)
\[ \omega^0 \rightarrow \pi^+ + \pi^- + \pi^0, \quad \phi \rightarrow K + \bar{K}, \]
\[ \Delta^{++} \rightarrow p + \pi^+, \quad \Delta^- \rightarrow n + \pi^-. \]  \hspace{1cm} (2.19)

In this note we use the modified Monte Carlo simulation program from that in Ref. 6) by including the diquark cascade with recombination, and the method of generation of cascade with recombination is explained in the appendix of Ref. 6).

¹ In our formulation (2.5) we take simply the recombination probability of all kinds of quarks or anti-diquarks to be equal, then the probability conserves at each step of cascade. Otherwise the model becomes very complicated in order to conserve the probability. Therefore we adjust parameter \(D\) to change the recombination mechanism effectively. If we take the uncorrelated jet model like Van Hove,⁷ we can adjust \(\lambda'\) easily.
§ 3. Hadron spectra in proton fragmentation region

The most outstanding feature of our model is that the recombination mechanism is considered there. As the recombined hadron spectra have broad $x$-dependence in large $x$ region, the spectra near $x \sim 1$ depend much on the model of recombination. By the modification in this note we can calculate anti-baryon spectra.

In Fig. 2 the experimental data\textsuperscript{(13),14,*) on $p \to \bar{p}X$ spectra are compared with the

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{The anti-proton invariant spectra in proton's fragmentation region. Model IIA with $E=0.05$ and $D=50$ (solid curve ---), Model III with $E=0.05$ and $D=1$ (dash-dotted curve -----), Model I with $E=0.05$ and $D=1$ (dotted curve ------), and Model IIa with $E=0.1$ and $D=50$ (dashed curve ---), are given for comparison with the experimental data (Refs. 13) and 14)).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3.png}
\caption{The $\bar{A}/A$ ratio of inclusive spectra in proton's fragmentation region. The experimental data are taken from Refs. 13) -- 15).}
\end{figure}

\*) The original experimental data are the invariant cross section at $p_T=0.5$ GeV/c. We assume the $x$-independent Gaussian $p_T$ distribution as

\[ E_{p_T} \frac{d^2\sigma}{dp_T^2} = A(x)e^{-8p_T^2}. \]

Then

\[ \frac{1}{\sigma} \frac{2x_0}{\pi} \frac{d\sigma}{dx} = \left. E_{p_T} \frac{d^3\sigma}{dp_T^3} \right|_{p_T=0} e^{8p_T^2}. \]

We take $B=3.04$ and plot the experimental data\textsuperscript{14} in Fig. 2. By changing $B$ from 2.66 to 3.4 the experimental plot changes only several percent in this case.
predictions from models I and II, using the same values of parameters (2.14) as in Ref. 6) and adjusting the new parameters (2.15). The magnitude of $E$ is sensitive to the absolute value of the spectra, and the type of the model (of recombination) is sensitive to the shape of the spectra. The models IIa and IIb with parameter $E=0.05$ nicely reproduce the experimental inclusive spectra $p \rightarrow \bar{p}X$. On the other hand model I gives broader spectra which comes from additional type of recombination mechanism, such as $(\bar{q}q)(\bar{q}q)q \rightarrow \bar{B}+m$, and fails to reproduce the experimental data. In Fig. 3 the ratio of the experimental\textsuperscript{138}--\textsuperscript{160} inclusive spectra $(p \rightarrow \bar{X})/(p \rightarrow X)$ vs $x$ is compared with the results calculated by model I and models IIa and IIb with parameter $E=0.05$. From these comparisons of the experimental anti-baryon production spectra with our calculated one in baryon fragmentation, it is seen that the models IIa and IIb with parameter $E=0.05$ give nice fit with the experimental data. In Fig. 4 the predicted inclusive anti-baryon spectra in proton fragmentation are given using model IIa. By making the parameter $E$ twice the diquark production from quark becomes twice. Then baryon and anti-baryon spectra also becomes twice, as we see in Figs. 2 and 4. Therefore we can fix parameter $E$ using the experimental anti-baryon spectra to be 0.05 in proton's fragmentation region.

In Fig. 5 the experimental data on the inclusive $\pi^\pm$ meson spectra in proton fragmentation are compared with the predicted spectra by models I, IIa and IIb. By
model I the recombination mechanisms \((\bar{q}q)qq \to m+m\) etc. enlarge the meson spectra in large \(x\) region. (The spectrum coming from recombination mechanism by model I is also shown in Fig. 5 and the predicted spectra by model I disagree with the experimental one. However, the predicted one by models IIa and IIb give nice fit with the experimental data.) The meson spectra in proton fragmentation predicted by models IIa and IIb give almost the same spectra as those by our previous model without considering diquark. In Figs. 6 and 7 the \(\pi^+/\pi^-\) and \(K^+/K^-\) ratios in proton fragmentation are plotted with the experimental data. These meson ratios also support models IIa and IIb and do not support model I.

In Fig. 8 the experimental inclusive proton and \(\Lambda^{++}\) spectra are compared with our predicted ones by model I and models IIa and IIb. The predicted spectra by both models give almost the same spectra by our previous simple model and all these models nicely fit with the experimental data. Our model nicely predicts most of the experimental data on hadron inclusive spectra, but there still remain some discrepancies. In Fig. 9 the data on inclusive \(\Lambda\) and \(\bar{\Lambda}\) spectra are compared with our predicted ones. The data on \(\Lambda\) decrease with increasing \(x\) faster than the predicted spectra. This may show that our parameterization on the strangeness flow is still not suitable or we should consider baryon resonance more realistically. In Fig. 10 the inclusive spectra ratio \(\Sigma^+/\Sigma^{++}\) vs \(x\) in proton fragmentation is compared with our predictions. As we have seen in Ref. 7), in order to get correct \(\Sigma^-\) multiplicity we have to choose larger \(\lambda\) than Eq. (2.14).
The other charge ratio such as $\pi^+/\pi^-$ can be fitted with data by modifying the equipartition initial $x$ distribution of quarks to non-equipartition one (2·3), but the ratio $\Sigma^-/\Sigma^+$ comes from the recombination in our model and it is difficult to adjust by changing the initial quark distribution.

§ 4. Hadron spectra in pion fragmentation

Now our interest is whether it is possible to explain the hadron spectra in pion fragmentation with the same parameter as the ones for proton fragmentation. For meson fragmentation we take the equipartition $x$ distribution (2·1) and parameters (2·14) and (2·15).

The inclusive $\pi^\pm$ and $K_s^0$ meson spectra in pion fragmentation are given in Fig. 11. Both models I, IIA and IIB predict almost the same spectra as those in Refs. 1)~3) and are in good agreement with the data\textsuperscript{22,23}. This is due to the small value of $E$ giving small effects of diquark on $\pi^+$ spectra. Contributions from recombination mechanism for $\pi^-$ spectra are also represented in Fig. 11. (We assume the same model as in Refs. 1) and 2). All the recombined meson with $I=1$ and $I_S=-1$ are recombined by the second kind
Anti-Baryon Inclusive Spectra in Hadron Fragmentation

Fig. 10. \( \Sigma^+(1385)/\Sigma^*(1385) \) ratio in proton's fragmentation region. The solid line is the predicted ratio by Model IIa with \( D=50 \) or by Model IIb with \( D=1 \), and the dotted line is the one by Model I with \( D=1 \), where the parameter \( E \) is taken to be 0.05. The experimental data from Refs. 14, 15, 20 and 21) are given for comparison with the predicted ratios.

Fig. 11. The inclusive \( \pi^\pm \) and \( K^\pm \) meson spectra in \( \pi^\pm \) meson fragmentation. The experimental data are taken from Refs. 22) and 23).

The invariant inclusive spectra for the reactions \( \pi^+ \rightarrow \pi^+ X, \pi^- X, K^+ X, K^- X, \rho X \) and \( \bar{\rho} X \) are given in Fig. 12(a) and (b) by the model I, IIa and IIb by taking parameter \( E=0.05 \) and 0.1. For comparison we plot the data at \( p_T=0.3 \) GeV/c. We take the ratio \( (1/\sigma)(2\pi/\pi_0)(d\sigma/dx)/\rho_3(d^3\sigma/d^3p) \) to be 1/37, 1/25 and 1/50 GeV²/mb in order to fit the experimental data for \( \pi^+ \), \( K^+ \) and \( \rho \) and \( \bar{\rho} \) spectra with the calculated one, respectively. (For \( \sigma_{\text{tot}}=23.5 \) mb and \( p_T \) dependence parameter \( B=3 \) (GeV/c)^{-2} this ratio is 1/27 GeV²/mb.) The meson spectra depend very weakly on the parameter \( E \), but \( \rho \) and \( \bar{\rho} \) spectra do depend on parameter \( E \). The \( \rho \) and \( \bar{\rho} \) spectra can be explained by our model II by taking a little bit larger value of \( E \) than that in proton fragmentation; that is, \( E=0.1 \) or a little bit larger one. Models I, IIa and IIb predict similar \( \pi^\pm \) and \( K^\pm \) meson spectra in pion fragmentation, but predict quite different one for \( \rho \) and \( \bar{\rho} \) spectra. Comparing with the experimental data, it seems plausible to adopt the models IIa and IIb also in pion fragmentation.

In Fig. 13 the invariant \( \Lambda \) particle spectra in \( \pi^+ \) meson fragmentation are compared with the predicted ones. In Fig. 14 the inclusive spectra \( \pi^+ \rightarrow \Delta^+(528) X \) and \( \pi^- \rightarrow \Delta^{--}(1236) X \) are shown for comparison.
Fig. 12. The invariant cross sections for the reactions (a) $\pi^+ \to \pi^+X$ and $K^+X$, and (b) $\pi^+ \to \rho X$ and $\bar{\rho}X$ in $\pi^+$ meson fragmentation. The experimental data are taken from Ref. 24 and we take the ratio $\langle 1/\sigma \rangle (2x_0/\pi)(d\sigma/dx)/E_\pi(d^2\sigma/dE^2)_{p_T=0.3 GeV/c}$ to be $1/37, 1/25, 1/50$ and $1/50$ GeV/c mb.

Fig. 13. The invariant $\Lambda$ production cross section in $\pi^+$ meson fragmentation. The experimental data are taken from Refs. 23 and 25.

Fig. 14. The inclusive $\Delta^{++}$ spectra in $\pi^+$ meson fragmentation. The experimental data are taken from Ref. 26.
Anti-Baryon Inclusive Spectra in Hadron Fragmentation

Fig. 15. The predicted inclusive baryon and anti-baryon spectra in $\pi^+$ meson fragmentation from Model IIa with parameters $E=0.1$ and $D=50$.

$\Delta^{++}X^{26)$ are given. From these comparisons of the data and the predicted baryon spectra in $\pi$ meson fragmentation ($\pi^+\rightarrow pX$, $\bar{p}X$, $\Lambda X$, $\Delta^{++}X$, $\Delta^{-}X$), the larger value of parameter $E=0.1$ is suitable than the smaller one $E=0.05$ which is taken in proton fragmentation. The agreement of the predicted spectra with the measured data is satisfiable. In Figs. 15(a) and (b) we show the predicted inclusive baryon and anti-baryon spectra in $\pi^+$ fragmentation from model IIa with $E=0.05$. Although we have taken a large value of $D$, we can see contributions from the recombination mechanism at $x\approx1$. The symmetric distribution between $u$- and $\bar{d}$-quark in $\pi^+$ beam gives similar $x$-dependence for, e.g., $n$ and $\bar{p}$.

The separation of $ps$ meson, and vector and tensor meson resonance depends on the additional parameter or on the assumption about the model. It is interesting to know whether we can explain the vector meson spectrum in a consistent...
way or without an additional assumption. In Fig. 16 the $\rho^\pm$ and $\rho^0$ meson spectra in $\pi^+$ meson fragmentation are compared with the measured values.\textsuperscript{27,28,29} The $\rho^+$ and $\rho^0$ meson spectra in $\pi^+$ meson fragmentation fit quite nicely with the predicted ones by our model. However, the predicted $\rho^-$ meson spectra do not decrease with increasing $x$ in contrast to the experimental data, and the predicted $\rho^-$ meson spectra without recombination decrease with increasing $x$ much faster than the data. In order to explain the spectra of mesons in which both quark and anti-quark have different flavours with that in the incident meson, we need to construct more skillful model.

§ 5. Conclusions and discussion

In this note we extend the quark cascade recombination model to consider the anti-baryon production in baryon fragmentation region and baryon and anti-baryon production in meson fragmentation region. For this purpose other models, such as dual topological unitarization model\textsuperscript{30} and recombination model\textsuperscript{31} which were applied to the small $p_T$ reactions need also some extension.

As we have seen in the previous sections, (anti-) baryon productions in meson fragmentation and anti-baryon productions in baryon fragmentation are well described by Model II (a and b) where hadrons are emitted through processes (1·2) and (1·3), and recombinations (2·17a) and (2·18a) dominate over recombinations (2·17b) and (2·18b). For simplicity, we take the particle independent form (2·12) of emission function with the same values of parameters $a_{q,q,a}=a_{a,q,q}=a_{q,q,a}=a_{q,a}=1.5$. This choice of the emission function corresponds to the Bremsstrahlung like emission of hadron, and a quark or a diquark keeps a large momentum fraction after successive particle emission. Consequently the recombined hadron tends to have a large momentum fraction.

The small $p_T$ hadron reaction should be analyzed using the non perturbed QCD in future, and the recombination mechanism of quarks or diquarks into hadron is one of the most interesting and difficult subject from the QCD theoretical view point. It is very much interesting to analyze whether all the allowed recombinations occur at the final stage of quark diquark cascade or only special kind of recombination mechanism dominates at the final stage. The phenomenological analysis of the following single particle inclusive reaction is useful for this purpose:

\[ \pi^+ \rightarrow B + X, \quad \pi^+ \rightarrow \bar{B} + X, \]

\[ B \rightarrow m + X, \quad B \rightarrow \bar{B} + X. \tag{5·1} \]

As we see in the previous sections, to get the correct inclusive spectra the probabilities of recombination mechanisms such as those in Eqs. (1·4a) and (1·4b) must be suppressed, and the following recombination process should dominate,

\[ q\bar{q} \rightarrow \text{one meson for meson fragmentation}, \]

\[ qqq \rightarrow \text{one baryon for baryon fragmentation}. \tag{5·2} \]

This is achieved by Model IIa or Model IIb.

The most outstanding success of this model with the recombination mechanism is that we can explain naturally both data on single particle spectra with sharp $x$-dependence like
Anti-Baryon Inclusive Spectra in Hadron Fragmentation

\[ p \rightarrow \pi^\pm X \] and \[ e^+ e^- \rightarrow \pi^\pm X \], and ones with broad \( x \)-dependence like \[ p \rightarrow p X, \Delta^{++} X \] and \( \pi^\pm \rightarrow \pi^\pm X \) in a unified way. In our quark cascade recombination model the conservation of probability and that of quantum number are considered exactly. However, this model is based on the classical cascade model and the effects of, e.g., the phase and spin are ignored in our model. Without solving the problem quantum mechanically some of the discrepancies between the simple-minded model and experimental data were overcome phenomenologically by modifying the model:

1. We have modified\(^9\) the initial quark distribution in proton beam as Eq. (2-3). By this modification \( \pi^+ / \pi^- \) ratios such as those in proton fragmentation (with and without trigger) are explained by our model.

2. We take the asymmetric parametrization of matrix \( T \) as \( b'/b \neq 1 \) but \( b'/b = 3 \).\(^{1-8} \) Otherwise, \( u \) or \( d \) quark gradually changes to \( s \)-quark after successive type meson emission and the probability that the recombinet hadron has strangeness becomes too large.

3. We cannot assume the democratic quark recombination but must specify the type of recombination:

3a. We have assumed\(^{12-2} \) that in \( ps \) meson fragmentation the recombinet meson by the second kind mechanism is only vector meson and not \( ps \) meson to fit the predicted spectra with data.

3b. As we saw in this paper, in the final stage of quark diquark cascade we assume that the recombination process (5-2) dominate.

Though the success of our quark diquark cascade model with recombination mechanism is remarkable, there still remain some discrepancies between our model and the experimental data or some lack of unified parametrization even after the modification of the simple model:

(i) We must choose the parameter \( E \) for the diquark anti-diquark pair creation to be different for the proton beam data and pion beam data; \( E = 0.05 \) for proton beam and \( E = 0.1 \) for pion beam.

(ii) The predicted \( \Lambda \) spectra in proton fragmentation is flat in \( x \), but the experimental data decrease with increasing \( x \). (Fig. 9)

(iii) The experimental \( \Sigma^{**}/\Sigma^{**} \) ratio in proton fragmentation is much less than the predicted one. (Ref. 7) and Fig. 10)

(iv) The measured \( \rho^- \) spectra decrease with increasing \( x \) faster than the predicted ones. (Fig. 16)

These discrepancies may come from the fact that we took too simple model for resonances. Also there is a possibility that for the recombinet hadron we may have a possibility to change the ratios of vector meson to \( ps \) meson and decuplet baryon to octet baryon as we did in Ref. 2). The discrepancy at small \( x \) region (0 < \( x < 0.1 \)) where the predicted spectra are larger than the data may come from the ignorance of diquark mass (and \( \Lambda \) mass).

Anyway more detailed consideration on the recombination mechanism, if possible using QCD, is useful. We should solve why it is necessary to modify the simple model as (1) \( \sim (3) \), and how we can modify our model to explain the discrepancies (i) \( \sim (iv) \). Though there are some discrepancies, our model explains the most of the spectra including diffractive components in a simple and unified way.

Finally we make a comment on the relation with the other model\(^{19} \) which was
proposed by Kinoshita, Noda, Mizouchi and one of the authors (T. T.). These models are similar ones but there are some differences between them. In this paper we have assumed that the proton beam breaks up into three independent quarks and used the Bremsstrahlung like emission of hadrons (Eq. (2·13)). This choice of the cascade mechanism ensures the beam like particle to have a large momentum fraction by the recombination. This mechanism gives also the beam unlike particles such as $\rho^-, \rho^0, K^\pm$ and $\phi$ mesons from $\pi^+$ beam, and $n, \Delta^+, \Lambda, \Sigma^+, \Xi$ and $\Sigma^*$ particles from proton beam a large momentum fraction. Some of the spectra in which the changes of quantum numbers are large like $\pi^+\rightarrow \rho^-X, p\rightarrow \Sigma^-X$ and $p\rightarrow \Lambda X$ contradict with the data, and the remaining ones fit well with the data. On the other hand, the quark-diquark cascade model in Ref. 10) has different types of breaking up of beam hadrons into constituents and different $x$-dependences of emission functions $f_{\alpha, \beta}(x, x_0)$ from the present model. For example the proton beam breaks up into a quark with a small momentum fraction and a diquark with a large one. Instead of the Bremsstrahlung like emission, equipartition distribution for $q\rightarrow q + M(a_{\alpha, \beta} = 0)$ and an energetic baryon and a wee meson emission from a diquark were assumed. After one or more steps of the cascade, only a small momentum fraction is left in the quark to recombine into hadrons. In this way the different $x$-dependences between beam-like ($\Delta^+, p$) and beam unlike ($\Sigma^-, \Xi$) baryons in proton fragmentation are explained. The beam like particle spectra are not large enough near $x \sim 1$, and we may need to include the recombination even in this non-Bremsstrahlung like cascade model.

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We refer to the references in this paper for the related experiment.
Anti-Baryon Inclusive Spectra in Hadron Fragmentation


The simple-minded extension of this model is difficult because of the conservation of probability (see Ref. 7).