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Using the Rational Method for Design in Complex Urban Basins

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Because of its simplicity the rational method is still frequently used in urban planning and design. In this paper using the time-area method and statistically derived design storms, the design peak flow is computed for urban basins of different complexity and compared with the design flow determined from the rational method. It is shown that the design flow is underestimated using the rational method unless a reduced time of concentration is used. The relation between travel times in different parts of a drainage system is used to correct the design flow computed by the traditional rational method.

Introduction

Until the early 1980s the rational method served as the leading method for design of urban storm sewers, *e.g.* McPherson (1977). Still, although advanced urban rainfall-runoff models are commonly used, the rational method is used for design of small systems and when estimating peak flows in the early stages of planning large urban drainage systems, Lyngfelt (1991a). The basic assumption behind the rational method, originally emanating from Kuichling (1889) is that the mean maximum rain intensity over a period equal to the concentration time of the urban basin, when the entire basin contributes to runoff, corresponds to the specific runoff from the impervious surfaces within the basin. Lyngfelt showed that, when the time of concentration can be accurately determined, the rational method is a useful tool for drainage system planning.

The rational method is used with block rain, *i.e.* constant rain intensity during the period of rain duration, and therefore the distribution of floating times to the basin outlet and the complexity of the urban basin do not influence the peak runoff. The time of concentration of a basin is related to the basin configuration and the basin size, but it is one single measure of the basin hydrology and the basin hydraulics. Recently Hromadka and Whitley (1996) derived the rational method from the balanced design storm hydrograph approach, US Army Corps of Engineers (1982). Still, the approach requires that the floating time to the basin outlet is evenly distributed between time zero and the basin concentration time. However, a rainfall with a peaked intensity distribution causes a higher runoff peak, if the travel time distribution is uneven, for example a surface area being narrow at the top, wide half-way downwards to the outlet and then narrow again, as compared to when the floating time is evenly distributed between time zero and the basin concentration time. In a complex urban basin, where the drainage water moves in different directions before it reaches the basin outlet, the travel time distribution is usually uneven. When applying the rational formula in a straight-forward way not accounting for the complexity of the basin and not for the complexity of the rainfall, the peak flow is not correctly estimated.

Prior to when mathematical urban rainfall-runoff models where used, the design flow from urban basins having uneven travel time-contributing area distribution was often determined from a time-area approach. When the travel time is evenly distributed, *i.e.* each time increment representing equal area, the time-area approach gives the same peak flow for a design storm as the rational method does. Lyngfelt (1985) has shown that the time-area approach gives runoff hydrographs corresponding to rainfall-runoff model computed hydrographs and also corresponding to observed hydrographs, provided the travel time-contributing area distribution is determined correctly. Therefore, the exercise to show the agreement between time-area computed and kinematically computed hydrographs is not done here. To determine the travel time distribution is not straightforward. It might be easier to apply mathematical runoff models, unless the drainage system can be divided into components in a systematic way.

The intention of this paper is to show that using the rational method for design in the conventional way with correctly determined concentration time, the peak flow is underestimated, and also to show how the basin complexity can be accounted for by dividing the drainage system into sub-components and determining correction factors by which the peak flows determined from the traditional rational method can be increased to get a better estimate of expected design flows. It is also shown how an effective concentration time, which give correct peak flows when inserted in the rational formula, can be estimated in a simple way.

Design Storm

When using the rational method, apart from the impermeable area and maybe a loss coefficient, the basin concentration time and the design storm must be known. The concentration time is discussed in the next chapter. Here the focus is on the design storm and its intensity distribution.

Design storms are derived by statistically analyzing historical storms and are given as rainfall depth or maximum mean intensities for rainfalls of different duration and return periods. These intensity-duration-frequency curves show decreasing intensities with increasing duration. The maximum mean intensity-duration relationship has been expressed in different analytical forms. A common relation used in Sweden and Denmark, Dahlström (1979), Danish Engineering Society (1974), is

$$p(t_d) = a\, t_d^{-b} \tag{1}$$

where a, b are parameters which vary with location and return period. For most cities in Sweden and Denmark b is about 0.7.

A synthetic design storm with intensity distribution matching the maximum mean intensity of Eq. (1) can be derived. The simplest one is the block rain with constant rain intensity during the duration of the storm. The peak runoff at the outlet from a basin of non-uniform travel time distribution may, however, be quite different, if the rain intensity distribution is peaked as compared to the runoff produced by a block rain. The maximum mean intensity-duration relationship should be valid for all durations relevant for urban runoff, *i.e.* a few minutes up to almost an hour. This criteria requires a peaked design storm with varying rain intensity. Such an intensity distribution is found after derivation of Eq. (1), Bengtsson (1991) and Hromadka and Whitley (1996)

$$i(t) = \alpha |t|^{-b} \tag{2}$$

where $i(t)$ is rain intensity, which has an infinity high peak value at $t = 0$. When t is less than 0, the absolute value of t must be used. The coefficient α is

$$\alpha = 0.5^{b}(1-b)a \tag{3}$$

There are many synthetic intensity distributions presented in the literature of which the most well-known probably is the Chicago-storm, Keifer and Chu (1957), but these intensity distributions are less suitable for analytical mathematical treatment than Eq. (2). Neither the intensity distribution Eq. (2) nor the Chicago-storm are good representations of real storm distributions, and should therefore be used only to determine design flows.

Time of Concentration

Apart from a given design storm, the basin concentration time, t_c, must be known when applying the rational method. The basis of the rational method is that a storm, block rain, has constant rain intensity, p, during a period which is considered to be its time of duration, t_d, and that a block rain of duration corresponding to the basin concentration time is chosen for determining the design flow. This leads to the rational method on the form

$$q = p; \quad p = \mathrm{func}(t_d); \quad t_d = t_c \tag{4}$$

where q is specific runoff from impermeable surfaces within the basin and where the maximum mean rainfall intensity, p, is a function of rainfall duration. This rain intensity-duration function is determined from observations and given as graphs for different recurrence intervals or in the form of Eq. (1) with different parameters for different return periods. The coefficient a is increasing with increasing return period while the coefficient b is almost constant. The specific runoff reaches its highest value for a storm of duration equal to the urban basin concentration time.

The time of concentration for an urban basin is not constant but depends on the flow, usually the higher the flow, the higher the wave celerity and the shorter the travel time. For constant width overland flow along a slope, the kinematic wave theory, *e.g.* the text book by Eagleson (1970), gives the concentration time as

$$t_c \equiv C^{1/d} \, p^{1/d-1} \tag{5}$$

where $C \equiv L/c$ with L being the length of the slope and c, d coefficients in the friction formula

$$v = c\, h^{d-1} \tag{6}$$

v being flow velocity, h flow depth and c a friction coefficient. When the Manning formula is used as friction formula $d = 5/3$ and $c = S^{0.5}/n$ with S as slope and n as the Manning number.

Eqs. (1), (4) and (5) can be combined to explicitly find the design runoff as

$$q(t_c) \equiv C^{\frac{-b}{b+d-bd}} \, a^{\frac{d}{b+d-bd}} \tag{7}$$

When the Manning formula and, as suggested from the studies carried out in the Nordic countries, $b = 0.7$, the two exponents of Eq. (7) are -0.58 and 1.39. This means that if the rain intensity is doubled, *i.e.* a is doubled, the runoff peak for overland flow increases 2.6 times.

Analytical expressions for the time of concentration can also be found for gutter flow and pipe flow, if it is assumed that the lateral inflow along the gutter or pipe is constant in time, for example Akan (1984). Introducing the Manning friction formula into the gutter or pipe continuity equation shows that the wet area, A, along a characteristic $dx/dt = 4/3\ C\, A^{1/3}$ increases with time, t, as $A = q_l\, t$, where q_l is lateral inflow. Integrating dx/dt along the gutter/pipe of length L gives the concentration time

$$t_c \equiv C^{3/4}\, L^{3/4}\, q_l^{-1/4} \tag{8}$$

where the coefficient C is for gutter and pipe flow, respectively,

$$C_p = \frac{S^{1/2}}{n} \sqrt[3]{\frac{1}{4\Pi}} \tag{9a}$$

$$C_g = \frac{S^{1/2}}{n} \sqrt[3]{\frac{\sin\theta}{8}} \tag{9b}$$

where S and n are slope and Manning number, and θ is the full bottom angle of a triangular shaped gutter.

The assumption of constant lateral inflow limits the applicability of the derived concentration time. However, if the overland flow concentration time is considerably smaller than the time of concentration for the gutter/pipe flow, the lateral inflow produced by a block rain is almost constant. Then, a doubling of the rain intensity should result in a doubling of the lateral inflow, which should reduce the time of concentration to 0.81 of the time of the lower rain intensity. From the concept of the rational formula, $t_c = t_d$, and the intensity-duration relationship, Eq. (1), with $b = 0.7$, the reduced concentration time and the doubled rain intensity result in an increase of the peak flow by a factor 2.3. The influence of rain intensity on the time of concentration is thus less pronounced for gutter/pipe flow than for overland flow.

In the previous example when the concentration time of pipe flow was computed, it was assumed that the pipe ran full. However, even during a storm corresponding to the design storm a pipe runs full only for an extremely short period. The flow velocity and the wave celerity depend on the flow depth. The dependence of the relative depth (depth/diameter) is minor, if the relative depth is not very small. Theoretically using the Manning formula, the flow velocity does not deviate more than 15% from the full pipe flow velocity when the water depth is in the range 0.35-1.0 of the pipe diameter. Except when the depth is very close to the top of the pipe, when the discharge theoretically does not increase with increasing wet area, the wave celerity for different depths is also within 15% of the full pipe wave celerity. Experiments have shown that the dependence of depth actually is even less than what is theoretically found from the Manning equation, Höganäs (1975).

Since, in a pipe, the kinematic wave celerity is rather independent of the flow depth and thus of the discharge, the concentration time for a pipe system is rather independent of the rain intensity. When the Manning formula is used for a pipe running full, the wave celerity is 4/3 of the flow velocity, and the time of concentration for a pipe is, with D as pipe diameter

$$t_c = 0.75 \frac{L\, n}{S^{1/2}} \left(\frac{D}{4}\right)^{2/3} \tag{10}$$

The flow paths in an urban drainage system are first overland flow and gutter flow, which is followed by flow in conduits. As just have been discussed, the overland flow-travel time is highly dependent on the rain intensity while the conduit flow travel time is not, unless the flow depth in the conduit is small. Usually the overland flow travel time constitutes only a small fraction of the total concentration time for an urban drainage system. Thus, it is sufficient to use a rough estimate of the contribution of the overland flow to the total concentration time. For example, assuming first 10 m overland flow (Manning *n* 0.02, slope 2%), then 10 m gutter flow (90° degree triangular shaped, Manning *n* 0.02, slope 1%) before the storm water reaches a 500 m long pipe (*n* 0.015, slope 0.2%), then the total travel time is 9 min for 1 mm/min rain intensity and 10 min for 4 mm/min intensity. Because the travel time is dominated by the conduit flow-travel time, the total travel time is rather independent of the rain intensity. When the pipe is running full only for a short time, the storage capacity of the pipe may have to be considered. A wave traveling through an empty conduit moves with the flow velocity. For that situation the concentration time as given by Eq. (10) should not be multiplied by 0.75.

Influence of Travel Time Distribution on Peak Flow

In the rational method it is assumed that peak flow occurs when the entire impermeable part of a basin contributes to runoff, and then the specific runoff is the mean rainfall intensity of a storm of duration corresponding to the basin concentration time. This is true, when the travel time is evenly distributed between time zero and the basin concentration time, *i.e.* each time fraction contributes with equal runoff contributing area fraction, regardless of the rainfall intensity distribution. However, as is obvious but still shown here, the maximum specific runoff does not correspond to the mean maximum rainfall intensity over a time corresponding to the concentration time, if both the travel time and the rainfall intensity are unevenly distributed.

Consider overland flow on a sloping surface, where the width increases linearly to reach a maximum width halfway down to the outlet, and then narrows again as sketched in Fig. 1. The frequency distribution of floating times is triangular shaped as is also shown. The floating time interval 0.4-0.6 relative the full travel time represents 36% of the travel time of all water particles falling on the area. A rain with most of its volume concentrated within a shorter period than the time of concentration gives the design peak runoff.

It is convenient to use block rain in the rational method, but the peak flow from an area shaped as the one in Fig. 1 is much lower for a block rain of duration equal to the basin concentration time than for a storm of high intensity over a shorter period. The previously derived design storm with variable intensity fulfilling the statistical

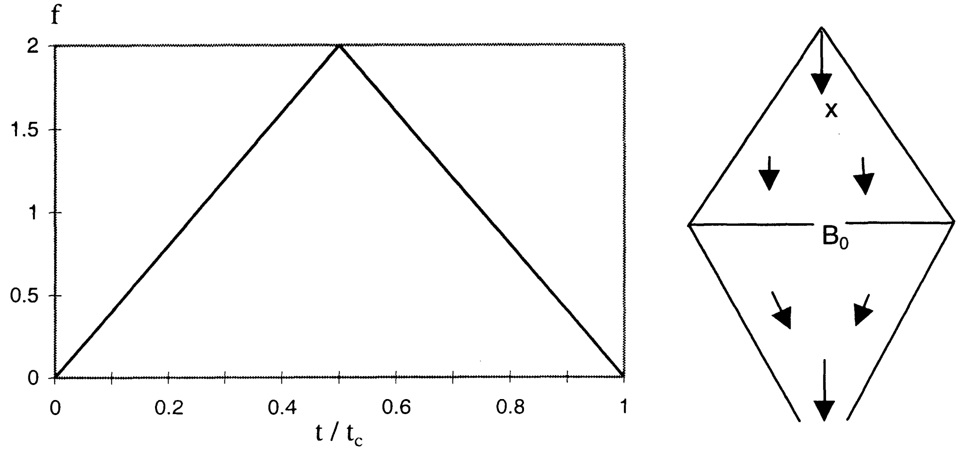

Fig. 1. Relative contributing runoff area $f = 1/A\ \partial A/\partial t$ as function of relative time, t_c, for the basin shown to the right.

requirement of rainfall depth for all durations is used to compute the peak flow and the age distribution of water particles reaching the outlet. The outflow as function of time, $Q(t)$, from the area shown in Fig. 1 is computed as

$$Q(t) = \int_0^L i(t-\frac{x}{v})\, B(x)\, dx \tag{11}$$

where i is rain intensity as function of time, v is flow velocity assumed to be constant equal to L/t_c with t_c as the travel time or concentration time for the whole basin, L being the length from the top of the basin to the outlet. The width $B(x)$ is linearly distributed with maximum width B_0 at $x = L/2$, with the origin of x at the very upstream point. Substituting $\tau \equiv t - x/v$ and dividing by the basin area gives the specific runoff, q, as

$$q(t) \equiv \frac{4}{t_c^2} \left\{ \int_{t-t_c/2}^{t} i(\tau)(t-\tau)\, d\tau + \int_{t-t_c}^{t-t_c/2} i(\tau)(t_c-t+\tau)\, d\tau \right\} \tag{12}$$

When the intensity distribution of Eq. (2) is inserted, the integrals above can be solved The peak runoff occurs when $t = t_c/2$. The ratio peak runoff to maximum mean rainfall intensity during the time of concentration for the basin is 8.3, when the rain intensity exponent is 0.7. Clearly, the rational method can not be used in a straightforward way to compute the peak flow from a surface area of non-uniform width. The distribution of the time for the water to reach the basin outlet must be accounted for.

Time-Area Calculations

Accounting for different floating times representing differently large area fractions of a basin, as was done in the previous sections, is basically a time-area approach. The general form of time-area calculations of runoff is

$$q(t) = \int_0^{t_c} i(t-\tau)\frac{\partial A}{\partial \tau}\, d\tau \tag{13}$$

where t_c is basin concentration time and $\partial A/\partial\tau$ is the travel time distribution, *i.e.* the relative area contribution for different floating times.

Now the situation of evenly distributed lateral flow along a draining pipe or canal is considered, as sketched in Fig. 2. The lateral flow may be overland flow or flow in many small pipes or gutters. Even in this simplified situation of lateral flow followed by longitudinal flow, different travel time and travel distances represent runoff contributing areas of different size. The cumulative area distribution is found by calculating contributing area as a function of floating time. When x is the travel distance in the L_x long large pipe for which the travel time is t_x, and y is the travel distance for the L_y long lateral slope for which the travel time is t_y, the relative contributing area is (assuming $x/L_x = t/t_x$ and $y/L_y = t/t_y$)

$$A_{rel} = \begin{cases} \dfrac{t^2}{2\, t_x\, t_y} & t \le t_y \\[2ex] \dfrac{t}{t_x} - \dfrac{t_y}{2\, t_x} & t_y \le t \le t_x \\[2ex] 1 - \dfrac{(t_x+t_y-t)^2}{2\, t_x\, t_y} & t_x \le t \end{cases} \tag{14}$$

where for simplicity only the situation when t_x is larger than t_y is shown.

Derivation of Eq. (14) gives the travel time distribution

$$\frac{\partial A_{rel}}{\partial t} = \begin{cases} \dfrac{t}{t_x\, t_y} & 0 \le t \le t_y \\[2ex] \dfrac{1}{t_x} & t_y \le t \le t_x \\[2ex] \dfrac{t_x+t_y-t}{t_x\, t_y} & t_x \le t \le t_x + t_y \end{cases} \tag{15}$$

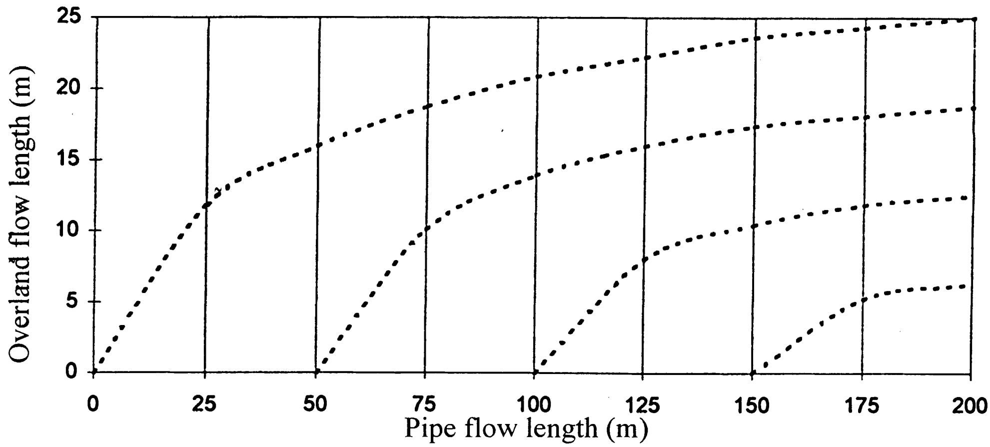

Fig. 2. Layout of a small urban basin with 25 m overland flow or flow in many small parallel pipes draining into a pipe or gutter, 200 m long. Isolines of travel times 2, 4, 6 and 8 min are shown with dashed lines.

The travel time distribution is shown as isolines in Fig. 2. The relative contributing area is dominated by travel times around half the total concentration time, *i.e.* $t = 0.5\,(t_x + t_y)$.

The specific runoff is found when Eq. (15) is inserted into Eq. (13), which gives after coordinate transformation, new $\tau = t -$ old τ,

$$q(t) = \frac{1}{t_x t_y}\left\{\int_{t-t_x-t_y}^{t-t_x} i(\tau)\,(t_x+t_y-t+\tau)\,d\tau + \int_{t-t_x}^{t-t_y} i(\tau)\,t_y\,d\tau + \int_{t-t_y}^{t} i(\tau)\,(t-\tau)\,d\tau\right\} \tag{16}$$

The solution of Eq. (16) for the intensity distribution of Eq. (2) is shown as Eq. (20) in the next section. For a block rain, when the rainfall duration equals the basin concentration time, the ratio peak flow to mean rain intensity is unity, and is less than unity for shorter durations.

The above exercise is simply the rational method applied to a limited part of a drainage basin. Even when a block rain is used as design storm, the rational method applied to a complex basin may give erroneous design flows.

Runoff Computed from Lateral Inflow Hydrographs

The way shown in the previous section of first calculating the contributing area distribution as a function of travel time and then multiplying by a rainfall intensity distribution is rather complicated. It is more straightforward to successively compute the hydrograph from each component of a basin. In the example of lateral inflow along a pipe, first the lateral inflow hydrograph can be computed for a given rainfall distribution, and then the outflow from the pipe for the then already computed inflow hydrograph. In this section, analytical solutions for the specific runoff resulting from the peaked design storm of Eq. (2) is derived.

Following the same procedure as for the overland flow of Fig. 1, the overland or small-pipe flow, which constitutes the lateral inflow to the pipe in Fig. 2, is

$$q_0(t) = \frac{1}{t_0} \int_{t-t_0}^{t} i(\tau)\,d\tau \tag{17}$$

with q_o as specific runoff and t_o as the overland flow concentration time. When the intensity distribution of Eq. (2) $i \equiv \alpha/t/^{-b}$ is inserted, the lateral flow is

$$q_0(t) = \beta\{t|t|^{-b} - (t-t_0)|t-t_0|^{-b}\} \tag{18a}$$

with

$$\beta \equiv \frac{\alpha}{t_0(1-b)} \equiv 2^{-b}\frac{a}{t_0} \tag{18b}$$

Thereafter, the flow in the large pipe is computed. Above, previously used index y is replaced by o for overland-flow. Below, the previously used index x is replaced by p for pipe flow. Consequently the concentration times are t_o and t_p. The downstream outflow from the pipe as specific runoff, q_p, is

$$q_p(t) = \frac{1}{t_p} \int_{t-t_p}^{t} q_0(\tau)\,d\tau \tag{19}$$

which is

$$q_p(t) = \gamma\{|t|^{2-b} - |t-t_0|^{2-b} - |t-t_p|^{2-b} + |t-t_0-t_p|^{2-b}\} \tag{20a}$$

with

$$\gamma = \frac{\beta}{t_p(2-b)} \tag{20b}$$

Eq. (20) can, as has previously been discussed, also be derived by inserting the rain intensity distribution into Eq. (16).

The peak of overland flow is obtained when $t = t_o/2$ and is $a\ t_o^{-b}$, which is the mean maximum rainfall intensity during the concentration time. The travel time distribution for overland flow is uniform. Since this is not so for the flow reaching the pipe outlet, *c.f.* Fig. 2, the peak runoff from the whole area does not correspond to the mean maximum rainfall intensity over the concentration time for the whole area, $t_o + t_p$: The peak runoff occurs when $t = 0.5\ (t_o + t_p)$. The ratio q_{peak} to maximum mean rainfall intensity, $p = a\ (t_o + t_p)^{-b}$, is

$$r = \frac{q_{\text{peak}}}{p} = \frac{0.5(t_0+t_p)^b}{(2-b)\,t_0\,t_p}\ \{\,|t_p+t_0|^{2-b} - |t_p-t_0|^{2-b}\} \tag{21}$$

When t_p is much larger than t_o, the ratio approaches unity. The maximum possible value of the ratio is for a basin where the travel times t_o and t_p are equal. Then $r = 2/(2\text{-b})$, which for $b = 0.7$ is 1.5.

Adding Drainage Basin Components

When the structure of a drainage system is known in detail, detailed mathematical modeling is also the best method to determine design flows. However, in the planning process the structure of the drainage system is known only roughly. This knowledge should allow for a crude division of a drainage basin into sub-components with respect to flow paths. Water moves as overland flow over sloping surfaces and in gutters, a process which in urban basins only extends over a few minutes. The overland flow enters the pipe system through inlets. In the upper parts of a drainage system, the pipes are of diameter 300-400 mm and laid in a regular pattern following the block street system. A block may be considered as a drainage unit. The travel time as overland flow, gutter flow and flow in very small pipes is t_o. The travel time as pipe flow within the small drainage unit to a larger pipe connecting different blocks is t_p: Such a crude discretization level, when overland flow and small-pipe flow are represented by one unit has using a kinematic approach been suggested by Marsalek (1983) and Lyngfelt (1991b).

A schematic picture of a complex symmetric urban basin is shown in Fig. 3. An urban sub-basin conduit (major street pipe), represented by the travel time t_c; collects water from the street block units (travel time $t_o + t_p$) within the sub-basin and distributes the storm water to a large conduit. The travel time within the sub-basin is thus $t_o + t_p + t_c$.

The urban basin of the previous section with travel times t_o and t_p is made more complex simply by adding a very large number of such small drainage units to a large pipe. The pipe flow as computed in the previous section is assumed to be lateral

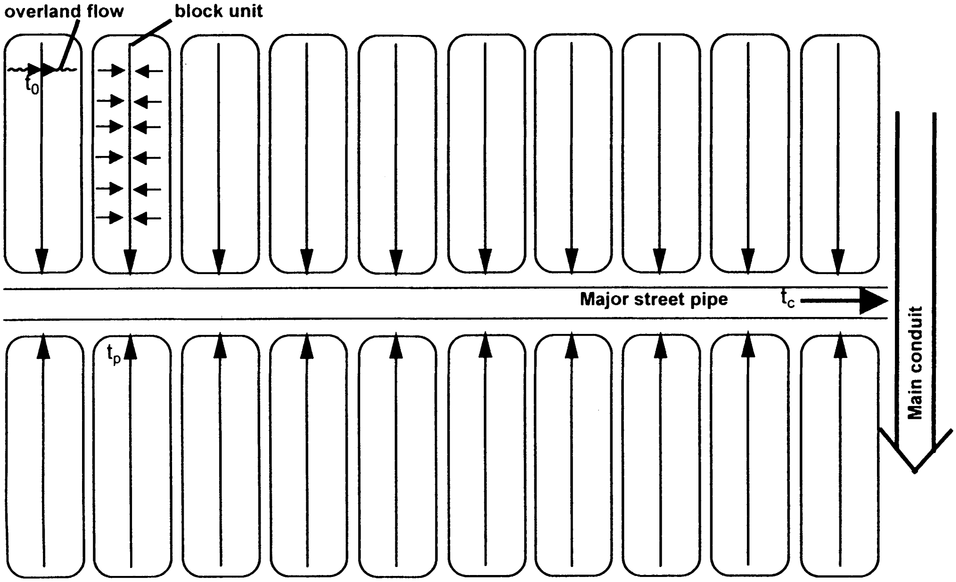

Fig. 3. Complex urban basin with drainage from a sub-basin into a main sewer. The sub-basin, for which the kinematic travel time of the major street pipe is t_c, consists of 20 identical street block drainage units with overland flow of concentration time t_o and pipe flow of concentration time t_p.

inflow evenly distributed along this sub-basin conduit. The downstream sub-basin conduit flow, q_c as specific runoff, is

$$q_c(t) \equiv \frac{1}{t_c} \int_{t-t_c}^{t} q_p(\tau)\, d\tau \tag{22}$$

which has the solution

$$q_c(t) = K \left\{ \begin{array}{ll} t|t|^{2-b} - (t-t_0)|t-t_0|^{2-b} - (t-t_p)|t-t_p|^{2-b} & \\ + (t-t_0-t_p)|t-t_0-t_p|^{2-b} & - (t-t_c)|t-t_c|^{2-b} \\ + (t-t_0-t_c)|t-t_0-t_c|^{2-b} & - (t-t_c-t_p)|t-t_c-t_p|^{2-b} \\ - (t-t_0-t_p-t_c)|t-t_0-t_p-t_c|^{2-b} & \end{array} \right\} \tag{23a}$$

with

$$K = \frac{\gamma}{t_c(3-b)} \tag{23b}$$

Peak runoff occurs when $t \equiv 0.5\,(t_o + t_p + t_c)$ which gives the ratio between peak basin runoff and mean maximum precipitation intensity

$$r = \frac{q_{peak}}{p} = \frac{(t_0+t_p+t_c)^b}{4(2-b)(3-b)t_0 t_p t_c}\,\Psi$$

$$\Psi = \begin{Bmatrix} (t_c+t_p+t_0)^{3-b} - |t_c+t_p-t_0|^{2-b}(t_c+t_b-t_0) \\ -|t_c-t_p+t_0|^{2-b}(t_c-t_p+t_0) + |t_c-t_p-t_0|^{2-b}(t_c-t_p-t_0) \end{Bmatrix} \qquad (24)$$

When t_c is much greater than t_p and t_o, the ratio approaches unity. When only t_o is small, the ratio is as given by Eq. (21) when t_p; t_o are replaced by t_c and t_p. The highest possible ratio 1.7 (the intensity exponent $b = 0.7$), is when $t_c = t_p = t_o$.

An urban basin can be extended even further by adding sub-basins and connecting them to a main sewer, as is sketched in Fig. 3. However, it is from an engineering point of view not feasible to include more than three individual travel times.

Example of Computation Procedure for Determining Design Flows

Prior to applying the rational method, the drainage basin concentration time must be determined also when using the rational method in the conventional way. This concentration time must be computed by adding contributions from different parts of the drainage system. When knowing these concentration times of different parts of a complex drainage system, the ratio between peak specific runoff and mean maximum precipitation intensity can be determined using the analytical solutions given above. Thus, if the concentration time is well known, it does not require much effort to account for the basin complexity.

It is practical to divide an urban basin into three drainage components, overland flow including gutter flow, pipe flow within a street block, and major street pipe flow. Consider a symmetric complex urban basin as the one in Fig. 3 with 20 equal street block units, 10 on each side of the major street pipe in the basin. The total area of the basin is 20 ha, 500 m long and 400 m wide, and the impermeable part is 25%. Each street block unit, consisting of overland travel time t_o and street block pipe flow time t_p', is 50 m times 200 m. The overland flow slope is 2% and the overland travel distance is 10 m; the block street pipe and the major street pipes all slope at 1%.

A 2-year storm in Stockholm, Höganäs (1975), corresponds to $a = 4$ in Eq. (1) when b is put to 0.7, t is in minutes, and the intensity is in mm/min. Following the rational method approach, the overland design flow can be determined directly from Eq. (7), from which thereafter the overland flow time of concentration is found from

Eq. (5). For this intense storm t_o is very close to 1 minute and thus the peak flow as specific runoff at the pipe inlet 4 mm/min. To determine the flow from the block unit some initial estimate of the pipe dimension is required. The flow velocity in a 300 mm pipe sloping at 1% and having a Manning n of 0.015 is 1.2 m/s. The particle travel time in a 200 m long pipe is almost 3 minutes, but the kinematic travel time is only slightly more than 2 minutes. The mean rain intensity of 3 minutes duration (overland flow + pipe flow concentration time) is from Eq. (1) 1.85 mm/min. Because of the lower intensity of a storm of 3 min duration compared to a storm of 1 min duration, the overland flow concentration time increases to somewhat more than 1 min, but this is not accounted for here, since the overland flow time is so short. The complexity of the basin, lateral flow followed by flow in a pipe, is accounted for using Eq. (21). The ratio peak flow to maximum mean rainfall intensity is, when t_o/t_p is 0.5, 1.3. The design flow is then, provided the pipe diameter 300 mm is a proper choice, 1.85 times 1.3 mm/min times the impermeable area of the block unit, which gives a flow of 0.100 m^3/s. The chosen pipe can, however, only carry 0.080 m^3/s, when running full. When instead a pipe of diameter 400 mm is chosen for the downstream 100 m of the street, the kinematic wave travel time in the pipe is only decreased by about 10 sec, so the concentration time for the street block unit can still be taken as 3 min and the design flow as 0.100 m^3/s. A pipe of diameter 400 mm, which can carry 0.180 m^3/s, should be chosen for the downstream part of the street.

To start the computation of the dimension of the major street pipe, a diameter of 400 mm is assumed for the first $^1/_4$ of the street, then a diameter of 500 mm, then 600 mm, and for the downstream $^1/_4$ of the street a diameter of 800 mm. This will give a kinematic wave transport time in the major street pipe of $3^1/_2$ minutes. For the total concentration time $6^1/_2$ min (3 min within the block unit and $3^1/_2$ in the major street pipe) the mean maximum rain intensity is 1.08 mm/min. The peak runoff is increased relative the rain intensity by a factor 1.4, determined from Eq. (24) with t_0 as 1 min, t_p as 2min and t_c as $3^1/_2$ min. The design flow at the outlet from major street is 1.4 times the rain intensity 1.08 mm/min times the impermeable area of the major street unit (25% of 200 m times 500 m) and is 0.630 m^3/s (1.51 mm/min). A 700 mm pipe, although seldom available, can carry 0.800 m^3/s, while a 600 mm one can carry only 0.530 m^3/s, so the initial assumptions are appropriate.

To show the influence of the distribution of the floating time, the calculations are repeated but for an elongated basin. The 20 street blocks are now assumed to be situated on only one side of the major street pipe, which is doubled from 500 to 1,000 m. The kinematic wave traveling time in the main conduit should be close to double that of the almost squared basin. For a pipe increasing from 400 mm to 800 mm along the 1,000 m long pipe, the kinematic travel time is 7 min. The concentration time for the whole basin is then 10 min, which of course is longer than for the squared basin, since the travel route is longer. The mean maximum rain intensity of 10 minutes duration is 0.80 mm/min. From Eq. (24) the ratio peak specific runoff

Table 1 – Maximum specific runoff from the different drainage units of Fig. 3 as computed from the rational method and the intensity-duration function Eq. (1) using correction factors accounting for travel times within the individual drainage units.

	over-land unit	400 m × 800 m basin block unit	major street	200 m × 1000 m basin major street
individual concentration time min	1	2	$3^1/_2$	7
total concentration time min	1	3	$6^1/_2$	10
mean rain intensity mm/min	4.00	1.85	1.08	0.80
correction factor	1.00	1.30	1.40	1.28
max spec runoff mm/min	4.00	2.40	1.51	1.02

to rain intensity is computed as 1.28. This ratio is lower for an elongated basin than for an approximately squared basin. The design flow for the elongated basin as specific runoff is 1.02 mm/min, which is much lower than the design specific runoff from the squared basin (1.51 mm/min). The results of all the above calculations are summarized in Table 1.

Areal Rainfall

Rain intensity does not only, as in the analysis above, vary in time but also in space. In the preceding analysis the design storm was derived from point rainfall statistics. When a storm moves over a basin, the storm center moves so that the intensity at a point varies much more than the areal rain intensity. Niemczynowicz (1985) gives statistically derived mean maximum rainfall intensities as function of duration, frequency and area. His data, although given in graphs and tables, can be put on the same form as Eq. (1), *i.e.* $p = a\,t^{-b}$ with t as duration. Both the parameters a and b decrease with increasing area. For the area 5 km^2 a is reduced to 0.78 of the point value and b is reduced from 0.70 to 0.65. For 10 km^2 a is 0.65 of the point value and b is 0.61. The smaller b is, the smaller is the ratio peak flow to mean maximum rain intensity as determined from a time-area approach. When the urban basin is less than one km^2, the areal rainfall is almost the same as the point rainfall, and it is not necessary to account for the spatial distribution.

In large basins, say 4 km^2 and a concentration time of 30 minutes, the rain intensity coefficient a is about 0.8 of the point value and the exponent b is about 0.66. The mean maximum areal rain intensity of 30 min duration is then only 8% lower than the point value. Thus, the influence of areal rainfall distribution on design peak flow is much less than the effect of the rainfall intensity distribution in time, even if the urban basin is large.

Accounting only for the Main Conduit Travel Time

The rational method is a rough tool for estimating design flows. To determine concentration times is on the other hand a very advanced procedure, if done with accuracy. Therefore, to match the simple procedure of the rational method, the concentration time is usually estimated in a crude way. In spite of what is stated that the design flow is underestimated using the rational method, the method has been used quite successfully in urban storm drainage design. The reason for not underestimating might be that the concentration time is not determined correctly but is estimated to a too short time. If only the travel time of the main drainage component is accounted for and is considered to be the basin concentration time, the storm duration used in the rational method is shorter and the mean maximum rain intensity is higher than if the concentration time for the whole basin is used as storm duration. It has already been shown that if the travel time of one of the drainage components dominates the travel time of a drainage system, the peak runoff corresponds to the mean maximum rainfall intensity for a storm of a duration corresponding to the basin concentration time, *i.e.* the rational method gives a correct peak flow. The objective of this section is to find how the time-area computed peak flows are affected, if only the main travel time component in a drainage system is accounted for.

To simplify, a drainage system with only two components, individual concentration times t_0 and t_p; is considered. The peak flow can be determined from Eq. (21) when the mean maximum precipitation over the time $t_o + t_p$ is used. Eq. (21) can be developed further and be put on the form ($r_o = t_o / t_p$)

$$q_{\text{peak}} = a\, t_p^{-b} \left\{ \frac{0.5}{(2-b)\, r_0} \left[(1+r_0)^{2-b} - (1-r_0)^{2-b} \right] \right\} \tag{25}$$

The peak flow computed this way is to be compared with the mean maximum precipitation intensity over the time corresponding to the longest individual drainage component travel time $a\, t_p^{-b}$. Looking at Eq. (25) it is seen that the expression within the major brackets is a ratio between peak flows when full account is taken of the basin complexity and when only the major drainage component is accounted for. In an elongated basin where r_o approaches zero, the ratio approaches unity. The minimum value of the expression within brackets is 0.95 when the traveling times t_o and t_p are equal. Thus, if only the concentration time of the drainage unit with the longest concentration time is accounted for, the estimated design flow is always within 5% of the design flow obtained when the complexity of the basin is considered.

Including a third drainage unit, Eq. (23) can be developed to an expression like Eq. (25) above. By inserting different ratios t_o/t_c and t_p/t_c, it can be calculated that as long as t_c exceeds the sum of t_o and t_p, the design flow found from a simple rational method approach using only t_c, the concentration time of the drainage unit with the longest concentration time, never deviates more than 5% from the flow computed using a full approach accounting for all the individual travel times. However, if all the three traveling times are about equal, the very simple approach of accounting only for one of the traveling times will result in an overestimated design flow by about 20%.

It is reasonable to believe that the successful use of the rational method in the past is due to extreme oversimplification. By not considering all flow paths, the fact that the rainfall intensity distribution is peaked is compensated for.

Conclusions

In the planning stage of an urban drainage project, the rational method is still used to estimate design flows. However, if the concentration time of a complex drainage basin is correctly estimated, the conventional use of the rational method results in underestimation of design flows. By dividing a drainage basin into drainage components for which the individual travel times are determined, correction factors can be determined and introduced into the rational method to give better estimates of the design flows. Design flows derived from the conventional rational method are thus multiplied by travel time ratio dependent correcting factors accounting for the structure of the drainage system. The correction factor may exceed 1.5 in a basin with a squared shape, but approaches unity in elongated basins.

If such a crude method as the rational method is used to estimate design flows, it is reasonable also to use some rough method to estimate the basin concentration time or the design precipitation duration to be used. It is shown that considering the travel time of a drainage component, for which the travel time is at least 50% of the concentration time of the whole basin (usually the main conduit of the basin under consideration), and, theoretically falsely, putting this travel time equal to the basin concentration time and to the duration of a rainfall of maximum mean constant intensity, will lead to a proper estimate of the peak flow.

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