Pergamon

Wat. Sci. Tech. Vol. 37, No. 11, pp. 73–79, 1998.
IAWQ

0273–1223/98 $19·00 + 0·00

PII: S0273-1223(98)00318-7

TEMPORAL RAINFALL DISAGGREGATION BASED ON SCALING PROPERTIES

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ABSTRACT

The present study concerns disaggregation of daily rainfall time series into higher resolution. For this purpose, the scaling-based cascade model proposed by Olsson (1998) is employed. This model operates by dividing each rainy time period into halves of equal length and distributing the rainfall volume between the halves. For this distribution three possible cases are defined, and the occurrence probability of each case is empirically estimated. Olsson (1998) showed that the model was applicable between the time scales 1 hour and 1 week for rainfall in southern Sweden. In the present study, a daily seasonal (April-June; 3 years) rainfall time series from the same region was disaggregated by the model to 45-min resolution. The disaggregated data was shown to very well reproduce many fundamental characteristics of the observed 45-min data, e.g., the division between rainy and dry periods, the event structure, and the scaling behavior. The results demonstrate the potential of scaling-based approaches in hydrological applications involving rainfall.

KEYWORDS

Rainfall; daily time series; disaggregation; scaling; cascade.

INTRODUCTION

The present study focuses on disaggregation of rainfall data into sets of higher resolution. The development of accurate methods for rainfall disaggregation is presently of prime importance in applied hydrology since the solution of hydrological problems often requires rainfall data of a higher resolution than the available measurements. For temporal rainfall, available data is generally of daily resolution whereas hydrological models often require hourly or even finer values, particularly in urban applications.

For their calibration, temporal disaggregation models developed to date generally require access to high-resolution measurements, i.e., measurements of the same resolution as the model is intended to disaggregate the available data to, from a station nearby the location of interest (e.g., Hershenhorn and Woolhiser, 1987). Firstly, such data may not be available. Secondly, due to microclimatological differences it is difficult to judge the representativity of such data for the location of interest (e.g., Econopouly *et al.*, 1990). To overcome these difficulties, the disaggregation could be based on scaling properties of the rainfall data. Then the model could, in principle, be calibrated on the very same data which are to be disaggregated and the uncertainty associated with using data from other locations would thus be eliminated.

During the present decade, so-called scaling properties have successively come to be regarded as a fundamental feature of the rainfall process. Scaling in general refers to a statistical symmetry across scales manifesting itself in relationships valid over a range of scales. Pioneering work in the field was performed by Lovejoy (1982) and later a distinction was made between simple scaling, where the scaling is defined by a single exponent, and multiscaling, where a function is required (e.g., Schertzer and Lovejoy, 1987; Gupta and Waymire, 1990). Scaling and multiscaling properties of rainfall time series have been found in a number of empirical investigations (e.g., Hubert *et al.*, 1993; Olsson *et al.*, 1993; Olsson, 1995; Burlando and Rosso, 1996; Cârsteanu and Foufoula-Georgiou, 1996; Harris *et al.*, 1996; Svensson *et al.*, 1996, Tessier *et al.*, 1996; Menabde *et al.*, 1997). So-called cascade processes have been proposed as a possible mechanism to account for the scaling properties as well as the hierarchical and clustered structure found in rainfall observations (e.g., Schertzer and Lovejoy, 1987; Gupta and Waymire, 1990, 1993; Tessier et al., 1993; Over and Gupta, 1994). Cascade processes originate from turbulence theory and describe how some quantity is transferred and concentrated from larger to smaller scales in the process (e.g., Yaglom, 1966; Mandelbrot, 1974). The theoretical basis of using cascade processes to model rainfall is yet unclear, but their use is supported mainly by empirical evidence.

The relevance of scaling properties for rainfall disaggregation has recently been recognized in some studies. Bo *et al.* (1994) used the Bartlett-Lewis rectangular pulses to disaggregate daily rainfall into hourly values, and argued that the successful result was due to a scaling (power-law) behavior of the power spectrum. For spatial rainfall, Perica and Foufoula-Georgiou (1996) developed a disaggregation model partly based on scaling of probability distributions of rainfall fluctuations.

Olsson (1998) proposed a temporal rainfall disaggregation model based on a conceptually simple cascade scheme. The model showed to be applicable between the approximate time scale limits 1 hour and 1 week for rainfall in southern Sweden. However, in Olsson (1998) daily values were not disaggregated due to the temporal resolution of the data used. Furthermore, seasonal nonstationarities in the data were not explicitly taken into account, only their magnitude were estimated. Therefore the aim of this paper is to further test the model of Olsson (1998) by performing disaggregation of daily values from a physically homogeneous season.

METHODS

Model

The model used is thoroughly described in Olsson (1998), and for detailed information the reader is referred to that paper. Here only a summary of the model is given.

Figure 1 illustrates the basic idea of how a conceptually simple cascade scheme was used in Olsson (1998) to represent the temporal structure of rainfall. In the cascade process, a certain time period T associated with a certain rainfall volume V was divided into two halves, T_1 and T_2, each receiving a part of V, $V_1= W_1 \cdot V$ and $V_2=W_2 \cdot V$, where W_1 and W_2 are multiplicative weights ($0 \le W \le 1$). Three types of division were considered, (1) $W_1=0$ and $W_2=1$, (2) $W_1=1$ and $W_2=0$, and (3) $W_1=x$ and $W_2=1-x$, $0<x<1$. The probability of having a certain type of division was denoted $P(0/1)$, $P(1/0)$, and $P(x/x)$, respectively, with $P(0/1)+P(1/0)+P(x/x)=1$. Within a certain range of time scales, these probabilities were assumed to be constant and the variable x was assumed to be associated with a theoretical probability distribution. These assumptions thus define the scaling property of the present model, i.e., the scale-independence of the probability values and x-distribution. Furthermore, the probability values were assumed to depend on two characteristics of the time period T to be divided: (1) position in the rainfall sequence and (2) associated rainfall volume V.

An advantage of the present model, as compared to most other cascade models developed for rainfall (e.g., Tessier *et al.*, 1993; Over and Gupta, 1994), is that the weights may be extracted from a time series and the applicability of the model thus evaluated in a direct fashion. Extraction of the weights is done by aggregating the time series values two by two, i.e., "running the cascade backwards", that way obtaining empirical estimates of $P(0/1)$, $P(1/0)$, and $P(x/x)$. Olsson (1998) evaluated the model using 8 min rainfall data from southern Sweden and found it applicable for time scales between approximately 1 hour and 1 week with a uniform distribution of x values. At smaller scales, below the mean event duration, the probability values and the x-distribution significantly changed. At larger scales, the absence of zero-values constituted a conflict

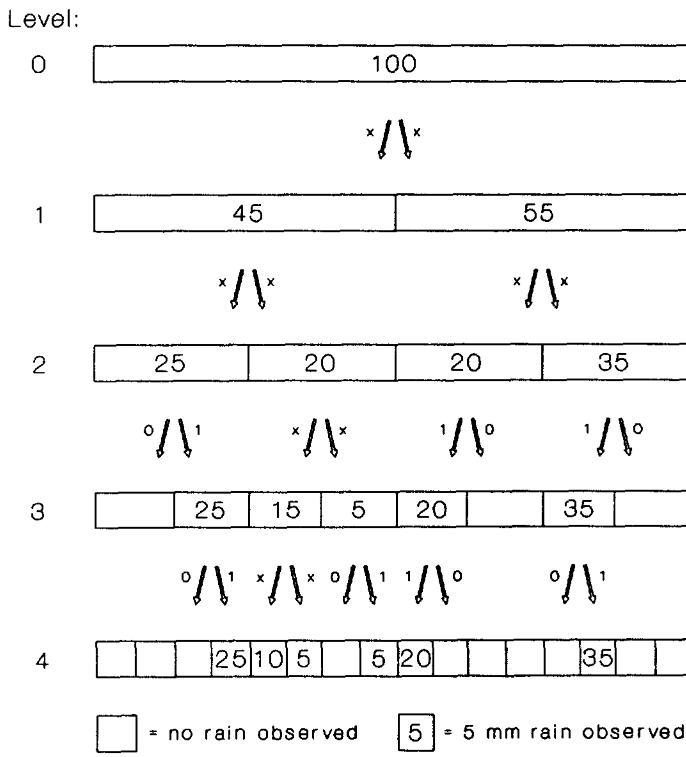

Figure 1. Principal structure of the cascade scheme used in the present study to represent the temporal rainfall process.

with the model formulation ($P(x/x)$=1). Time periods T characterized by different positions and volumes were, as assumed, associated with different sets of P(0/1)-, P(1/0)-, and $P(x/x)$-values .

Rainfall data

The data employed originate from a detailed observation program of short-term rainfall properties carried out in the city of Lund, southern Sweden, 1979-1981 (e.g., Niemczynowicz, 1984). The rainfall intensity was measured with a time resolution of 1 min by small tipping-bucket gauges with an intensity resolution of 0.033 mm/min. The aim of the present study is to disaggregate daily values into successively halved time periods, which means that the resolution of the data must be 24 hours divided by an integer power of 2. Since Olsson (1998) found the present model valid down to at least 1 hour for similar data, the 1-min values are here aggregated into 45-min values (24 hours divided by 2^5 equals 45 min). In Olsson (1998) it was also revealed that the model parameters, i.e., probability values, showed a slight seasonal nonstationarity. Therefore only spring data (April-June) are used in the present study and the total number of values in the time series is 8192.

Disaggregation

As previously mentioned, in the present study the model is used to disaggregate a time series from 1-day to 45-min resolution. To achieve this, the probability values P(0/1), P(1/0), and $P(x/x)$ must first be estimated. For this purpose, the weights W were extracted by aggregating the series from 45-min to 1-day resolution. In the disaggregation, the mean of the empirical probabilities in the range 45 min to 1 day was used as probability values. See Olsson (1998) for more detailed information about this parameter estimation procedure.

The model may then be applied for temporal rainfall disaggregation in a straightforward way, as demonstrated by Olsson (1998). Starting from the time series at 1-day resolution, the position and volume of each non-zero value is firstly used to determine the set of probability values to be used. Then, for each non-zero value, a random number is drawn to determine the type of division. In case of type 3, a x value is drawn from its corresponding theoretical probability distribution to specify the amount of rainfall in each half. When all non-zero values have been disaggregated accordingly, a series of 12-hour resolution has thus been produced. The procedure was then repeated four times to finally arrive at a series of 45-min resolution. The entire disaggregation from 1-day resolution was repeated 10 times, i.e., 10 realizations of the 45-min series was produced.

To evaluate the accuracy of the data generated by the model, these were compared with the observed data in terms of mean and standard deviation of the following five variables: (1) percentage of zero-values, (2) rainfall volume of individual values, (3) rainfall volume of events, (4) duration of events, and (5) length of dry periods between events. At all scales, an event was defined as a sequence of consecutive non-zero values. Furthermore, for all variables but the first quantile-quantile plots were made to evaluate the agreement of the entire cumulative distribution function. Finally the ability of the model to reproduce the scaling behavior of the observed data was studied by calculating statistical moments of various orders at different scales. Scaling of moments is manifested in a power law relationship between moment and scale (see, e.g., Svensson *et al.*, 1996).

RESULTS AND DISCUSSION

Table 1 shows the overall results from the disaggregation. The model results are obtained as averages over the 10 realizations produced by the model. From Table 1 it is evident that the model performs very well in reproducing all the five variables considered. The only pronounced discrepancy is an underestimation of the

Table 1. Comparison between observed data (Obs) and data generated by the model (Model), in terms of five variables at all scales to which disaggregation was performed (mean±std).

Scale	Data	Zero values (%)	Individual volume (mm)	Event volume (mm)	Event duration (hrs)	Dry period (hrs)
12 hrs	Obs	0.82	2.9±3.2	5.0±6.7	21.0±14.2	98±94
	Model	0.82	2.8±3.5	5.2±6.0	22.3±14.1	101±92
6 hrs	Obs	0.89	2.2±2.5	3.9±5.2	10.4±7.2	67±89
	Model	0.88	2.1±2.6	3.7±4.4	10.4±5.9	65±86
3 hrs	Obs	0.93	1.7±2.0	3.0±3.6	5.2±3.4	56±85
	Model	0.93	1.7±2.1	2.9±3.7	5.0±2.9	52±81
1.5 hrs	Obs	0.95	1.3±1.6	2.3±2.9	2.6±1.8	52±82
	Model	0.95	1.3±1.6	2.5±2.9	2.7±1.7	55±82
45 min	Obs	0.97	1.0±1.3	1.8±2.3	1.3±0.9	41±75
	Model	0.97	1.0±1.2	2.0±2.6	1.5±1.0	46±77

standard deviation of event volume and duration at the larger scales. Otherwise the agreement between observed and generated data is nearly total. It is interesting to note that the result does not deteriorate with decreasing scale, as could be suspected beforehand, but also the 45-min data appear very well reproduced by the model.

Figure 2 shows typical examples of quantile-quantile plots comparing the entire distribution of four variables in the observed and generated 45-min data. Overall the agreement is good, but some differences exist. Individual values (Fig. 2a) larger than 3-4 mm are generally somewhat overestimated by the model, whereas event volumes (Fig. 2b) are somewhat underestimated. The latter may be due to an excessive tendency of the disaggregation procedure to split up events in two parts at some scale. This also results in a slight underestimation of the event durations (Fig. 2c; note that one point may represent many identical pairs of values). Finally, dry period lengths (Fig. 2d) are well reproduced, which is expected since the main dry

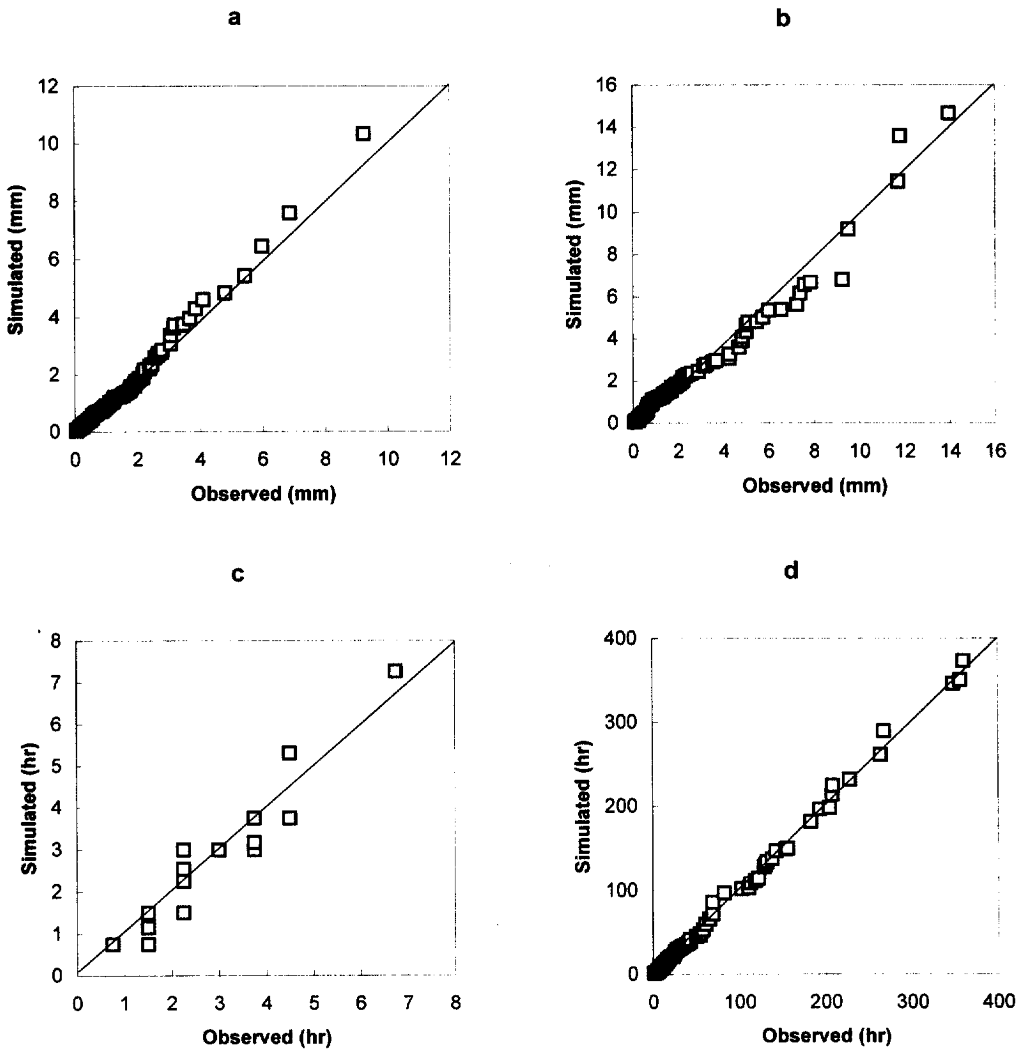

Figure 2. Quantile-quantile plots comparing the distribution of (a) individual volumes, (b) event volumes, (c) event durations, and (d) dry period lengths in observed and simulated 45-min data, respectively.

periods in the 1-day data are preserved by the model.

Figure 3 shows moments of various orders as a function of scale for the observed and generated data. The log-log linear (power-law) curves of the observed data confirm the scaling behavior. The disaggregated data very well match these curves and it is evident that the model preserves the observed scaling behavior.

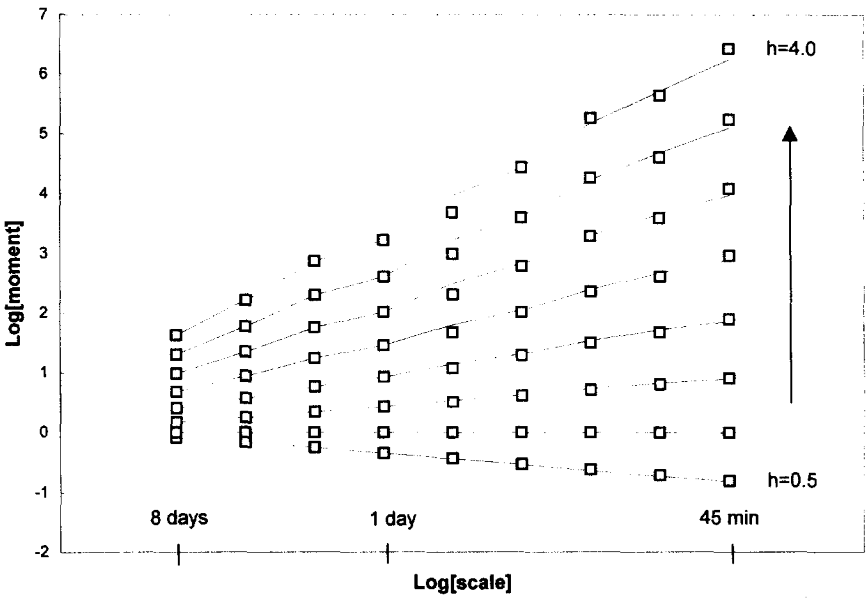

Figure 3. Statistical moments of order h ranging from 0.5 to 4.0 as a function of scale for observed (solid lines) and simulated (point values) 45-min, respectively.

CONCLUSIONS

It is encouraging that the conceptually simple cascade model was able to disaggregate the daily time series into 45-min resolution with many fundamental rainfall characteristics accurately reproduced. This demonstrates the potential of scaling-based approaches in hydrological applications involving rainfall. However, in order for the model to be useful in real-world applications, it should be possible to accurately estimate its parameters using larger-scale data only. This possibility could not be tested in the present study due to the limited amount of data available. The results by Olsson (1998), however, indicated that this can be performed with a high accuracy. In that study, disaggregation was performed using parameter values estimated from time scales larger than approximately 1 day. The accuracy of the model generated data was similar as compared to using parameter values estimated from the entire scale interval, i.e., down to the resolution to which the disaggregation was performed. Further evaluation of the model using large databases from different geographical regions will be the subject of future research.

Generally, more analyses of the scaling properties of rainfall data, not least continuous time series, urgently need to be carried out. In real-world applications of scaling-based hydrological methodologies, such as the present one, an assumption of scaling from the available scale to the desired scale must be made, unless some smaller-scale data are available for the model calibration. Due to the small number of empirical analyses performed to date, such an assumption is at present rather daring, although the few analyses performed nearly unanimously point at the existence of scaling down to small time and space scales. However, more analyses confirming the scaling behavior of rainfall in different geographical regions would substantially increase the confidence in scaling assumptions.

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