Nordic Hydrology 6, 1975, 73–92
Published by Munksgaard, Copenhagen, Denmark

THE DEVELOPMENT OF A SNOW ROUTINE FOR THE HBV-2 MODEL

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The development of a snow routine for the HBV-2 rainfall-runoff model is described. Experiences from field investigation that have contributed to the work are briefly discussed.
In the snow routine, snow accumulation and snowmelt are simulated using daily totals of precipitation and mean daily temperatures. The application of the model to a 2178 km^2 forested basin in central Sweden is shown. A critical study of the interaction and effects of the parameters is carried out.

Snow accumulation and snowmelt are important factors in the hydrology of the Nordic countries. For instance, about 50 % of the annual runoff in the Lappträsket Representative Basin in northern Sweden occurs during the spring flood in May. Therefore a reliable snow routine is needed before any mathematical runoff model can be successfully applied.

At the Swedish Meteorological and Hydrological Institute, a simple rainfall-runoff model has been developed, the HBV-2 model (Bergström 1972, Bergström & Forsman 1973). The structure of this model is shown in Fig. 1.

The objective has been to develop a model with few parameters and limited demand on input data in order to make it easy to handle and evaluate, and to make it applicable even when data coverage is poor. In the development of a snowmelt routine for the HBV-2 model the same principles have been applied.

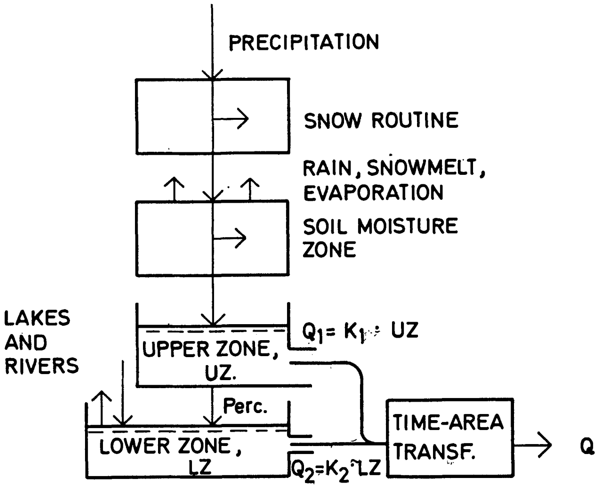


Fig. 1.
The structure of the HBV-2 model.

METHODOLOGY

In the initial phase, automatic optimization was used for the evaluation of parameters and to test the performance of different routines. The limitations of this method soon became obvious. It was difficult to define the best adjustment of the observed hydrograph with one simple criterion of fit. One reason is that the acceptable error varies. The time errors of the rising limbs of spring floods, for example, are of a quite different magnitude than those during a low flow recession. The crudeness of the criterion of fit makes the model relatively insensitive to small changes in some of the parameters, changes that can be easily observed with the eye. In addition to this, interdependence between parameters will create long flat ridges in the error function topography, often giving us parameter values on the physical constraints, while multiple extremes will make it difficult to find the absolute minimum value of the error function. Finally, the size and capacity of the SAAB-D22 computer used in this investigation, and the long period needed for the calibration of the model, made automatic optimization too time-consuming to be practical. These considerations led to the conclusion that a more subjective fitting by visual inspection should be used. As a complement to this, the efficiency of the model has been computed according to:

$$R^2 = \frac{\Sigma(Qo - \bar{Q}o)^2 - \Sigma(Qo - Qc)^2}{\Sigma(Qo - \bar{Q}o)^2} \qquad (1)$$

where

Qo = observed discharge
$\bar{Q}$o = mean of observed discharge
Qc = computed discharge

The performance of the model has been verified with split sample testing. This has been found to be the only way to distinguish modelling from curve-fitting.

An investigation of the error function topography has been carried out in order to estimate the significance and interaction of parameters.

THE SNOW ROUTINE

The snow routine is developed as a subroutine accumulating snow and releasing water which is fed into the soil moisture zone (Fig. 1). Snowmelt water in the soil is treated the same as rainfall. Snowfall on lakes is not processed through the snow routine, as the pressure effect on the ice will give the same effect as rainfall on an ice-free lake.

A physically correct snowmelt model should take into account the whole energy budget of the snowpack. This includes consideration of fluctuations in sensible and latent heat, radiation, exchange with the ground, contributions from precipitation, as well as the thermal quality of the snowpack itself. The damping of the response in the large basin tested, uncertainties in the available data, and the wish to avoid unjustified complexities made us start with a very simple, but less physical, approach. The air temperature was chosen as an index of all factors affecting snowmelt, using a modified degree-day method.

It should be noted that this routine is developed and tested in a forested basin only, and that the shading effect of the trees is important. It seems reasonable that other meteorological factors cannot be neglected in a mountainous or more open area.

All computations in the model are made on a daily basis with daily totals of precipitation, mean daily temperature, and mean daily discharge. Monthly totals of potential evaporation are computed by Penman's formula, but these values are used under snow-free conditions only. The evaporation losses from the snowpack are assumed to be nil.

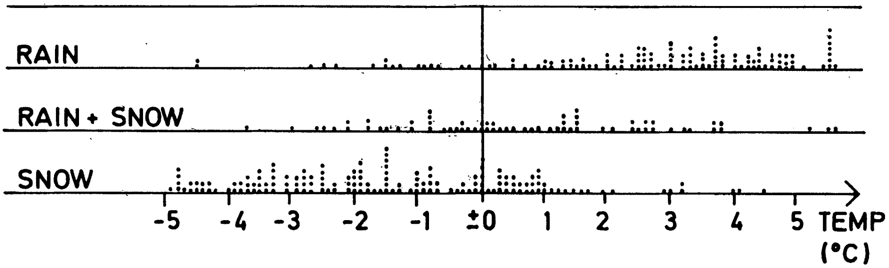


Fig. 2.
The observer's note on the type of precipitation related to mean daily temperatures. Each point represents one event.

Snow accumulation

One important problem is to decide whether precipitation accumulates in the form of snow, or enters the soil moisture zone directly as liquid water.

At the Lilla Tivsjön climate station in central Sweden an attempt has been made to relate the observer's notes on the type of precipitation to the mean daily temperature. The results are shown in Fig. 2.

As can be seen from this figure, temperature is a rather weak indicator of the type of precipitation. It should be noted, however, that the figure represents precipitation before it reaches the ground, thus neglecting the temperature of the ground itself. In spite of these uncertainties, mean daily temperatures have been used in the accumulation procedure of the model. This has been done mainly because it is a simple method and because the amount of precipitation in question is small compared to the total snowpack.

The use of rain gauges for measurement of snowfall means that large errors in the average catch over the basin could be expected. The errors can, for example, be caused by the aerodynamic effect around the gauges and by poor representativeness. Fortunately, the length of the accumulation period helps to reduce the random errors, thus giving one a possibility to correct the systematic error. This is done by multiplying the precipitation totals with a factor, C_{sf}, if it is accumulated as snow in the model. This factor accounts for the errors introduced when neglecting evaporation from the snowpack as well. C_{sf} is a very important parameter in the model, being a means of controlling the total volumes during the spring flood.

Snow accumulates differently in the different parts of a catchment. Investigations in Sweden (Waldenström 1975) and Finland (Seppänen 1961), indicate that the density of a forest is one factor influencing the water equivalent of the snowpack. This effect has not been taken into account so far, as it was felt unwise to introduce so little known a distribution at this stage of work.

Snowmelt at a point

Studies of snowmelt in the Representative Basins in Sweden have shown that if a degree-day approach is used, it must be combined with a parameter which allows the effect of temperature on the snowpack to increase as the snowmelt proceeds. This is clearly shown in Fig. 3, where the accumulated temperature is plotted against the accumulated release from snow plates in different locations in the Velen Representative Basin in southern Sweden.

Bergström (1971) and Jönsson (1975) came to the same conclusions when investigating recordings from snow pillows in the Lappträsket Representative Basin in northern Sweden.

The increasing degree-day factor can be due to the change of the physical properties of the snow during melt. The albedo of the snow cover, for example, decreases with time.

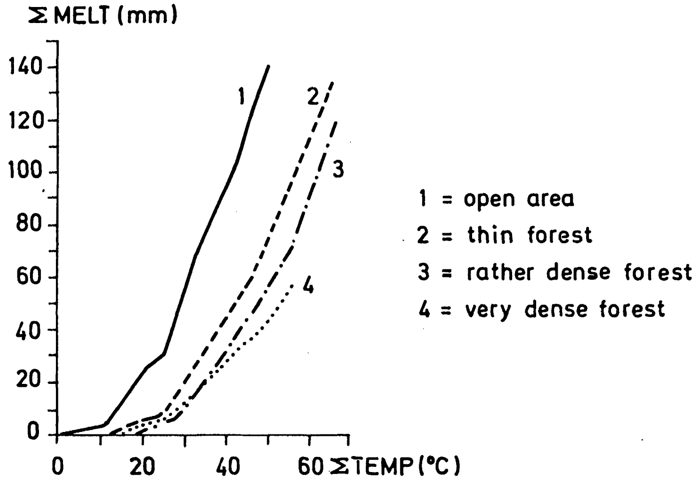


Fig. 3.
The accumulated temperature plotted against accumulated melt during one melt period in the Velen Representative Basin (1970).

In the model we have attempted two ways of representing the increase of the degree-day factor with the accumulated melt:

$$M = C_0(1 + C_{eff} \cdot \frac{\Sigma M}{S_0})(T - T_0) \tag{2}$$

$$M = C_0(1 + C_{eff} \cdot \Sigma M)(T - T_0) \tag{3}$$

where

M = snowmelt in mm
S_0 = initial water equivalent of the snowpack
C_0 = initial degree-day factor
C_{eff} = rate of increase of the degree-day factor
T = mean daily temperature
T_0 = threshold value of the temperature

Eq. (2) is based on a relative measure of accumulated melt while eq. (3) uses the absolute value. Results from 12 years of calibration and testing indicate that eq. (3) is the most appropriate form to use. Thin snowpacks have a tendency to disappear too rapidly if eq. (2) is used. T_0 is a parameter reflecting the representativeness of the temperature station. T_0 was suspected of being strongly dependent on the other parameters in the snowmelt routine. Therefore $T_0 = 0°C$ was used as a starting point in this investigation. We have not found any reason to change this value.

Multiple layers

The introduction of a variable degree-day factor entails difficulties in handling fresh snow on ripening or melting snow. To overcome this, the fresh snow is stored in a separate layer on top of the old snowpack. Snowmelt will occur in this layer until its degree-day factor equals the factor of its underlying layer, then the two layers will be one again. The dynamics of such a routine is shown schematically in Fig. 4.

Thanks to the consolidation routine the maximum number of snow layers was limited to four during the twelve years of investigation.

Liquid water in the snowpack

When snowmelt starts, a certain amount of water is retained in the snow by capillary action, or stored at the bottom of the snowpack as an effect of damming or similar action. Even if these effects are easy to imagine, they are not easy to model. Variability in the snowpack and in the soil amplifies the dif-

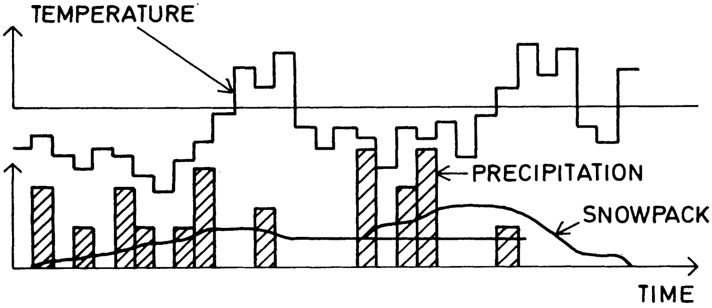


Fig. 4.
The build-up of multiple layers and consolidation in the snow routine.

ficulties. Therefore it is necessary to use great simplifications. Two ways to model liquid water retention have been tested in the snowmelt routine for the HBV-2 model. The first one was to apply a constant water-holding capacity, C_{wh}, as a percentage of the total water equivalent of the snowpack.

The second one was to use this water-holding capacity in combination with a constant storage capacity, W_b, at the bottom of the lowest snow layer. Both methods imply that the accumulated melt must exceed a threshold value before the snowpack will release any water into the soil-moisture zone.

Visual inspections of the results made us choose the last method, even though the improvement was not reflected in the R^2 criterion. This method was shown to give the small spring floods better timing.

Refreezing

If snowmelt is interrupted by cold weather, melting stops and refreezing is simulated by eq. (3). Refreezing is limited to water which is present in liquid form in the snow. It starts at the top and proceeds downwards through the consecutive layers as liquid water is used. The degree-day factor is not reduced more than that which corresponds to the change of accumulated melt when refreezing the free water.

Distribution of the degree-day factor

In Fig. 3 is shown not only the change of the degree-day factor during the melt period but also its variability over the catchment. The variability of melt and the areal extent of the snow cover were assumed to be such important factors that the lumped approach was abandoned in favor of a more distributed one. The degree-day factor was chosen as the parameter to distribute.

The catchment was divided into 10 parts and C_0 in eq. (3) was given a rectangular distribution ranging from $C_{0\,min}$ to $C_{0\,max}$. The effect of this distribution was a less abrupt start of the snowmelt period in the catchment. Problems arose with the evaluation of $C_{0\,min}$ and $C_{0\,max}$, as they could not be optimized unambiguously with the R^2 criterion. Therefore likely values of these parameters were chosen from field investigations.

IMPROVEMENT OF THE TIME-AREA TRANSFORMATION

Problems arose in fitting the timing of the flood peaks. The great variability in the magnitude of the floods made the simplified time-area transformation used in the HBV-2 model unsuitable.

The original time-area transformation is schematically shown in Fig. 5. In order to reflect the effect of flow velocities on the time-area transformation, the base of the triangular function, filtering generated runoff to give discharge, was made dependent on the magnitude of the runoff values. This was expressed as:

$$B = B_{max} - C_{route} \cdot Q_g \tag{4}$$

where

B = base of the triangular function
B_{max} = max base for low flows
C_{route} = lapse rate of B
Q_g = generated runoff

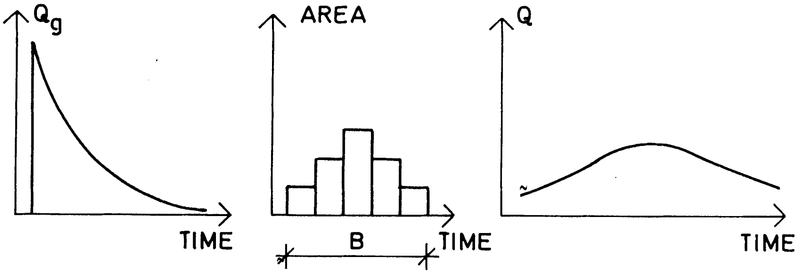


Fig. 5.
The time-area transformation of the HBV-2 model.

The B values ranged from 40 days for low flows to 16 days for the highest peaks during the 12 investigated years. The improvement of the model performance due to the introduction of this routine was substantial. The error function is studied, in a later section.

APPLICATION OF THE MODEL

The snow routine was applied to the modified HBV-2 model and tested on the River Gimån in central Sweden (Fig. 6). The Gimån River is a tributary of the Ljungan River, which flows to the Baltic.

The characteristics of the basin are as follows:

drainage area	2178 km^2
lakes	15 %
soil	morain
predominant vegetation cover	mainly coniferous forest
altitude range	300 m
number of precipitation stations	3
number of temperature stations	1
number of climate stations	1

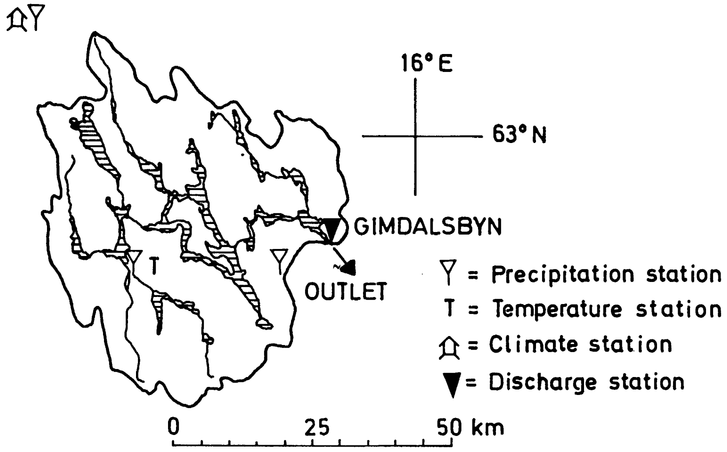
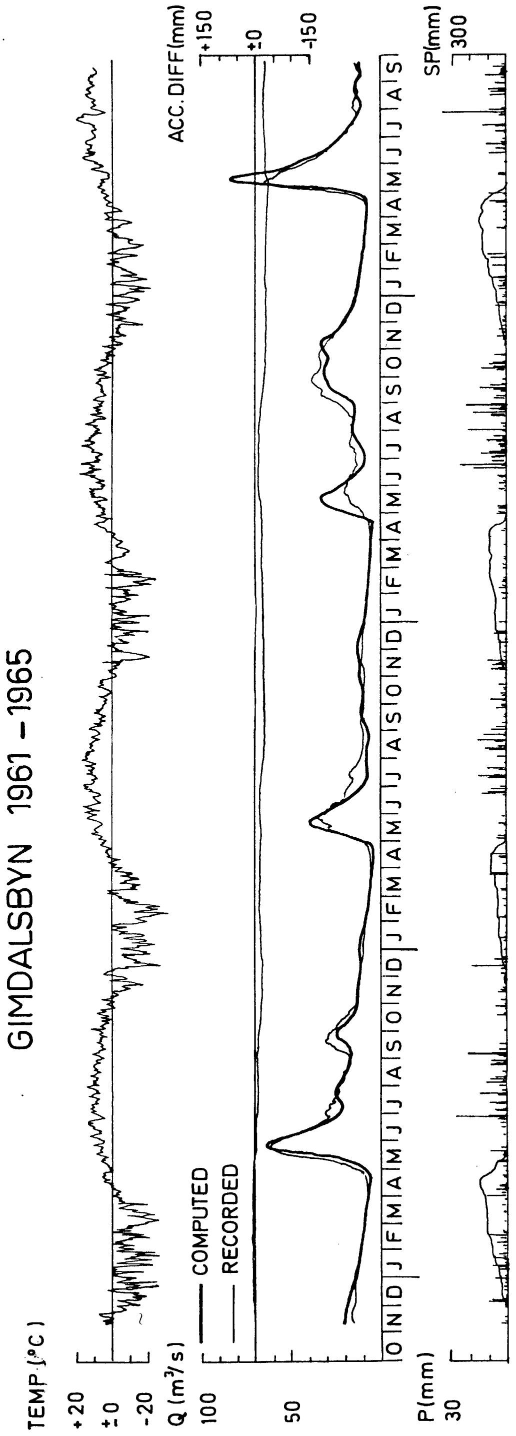


Fig. 6.
The Gimån Basin in central Sweden which was used for testing the model.


GIMDALSBYN 1961 – 1965
TEMP (°C)
+20
±0
-20
Q (m³/s)
100
50
COMPUTED
RECORDED
ACC. DIFF (mm)
+150
±0
-150
O N D J F M A M J J A S O N D J F M A M J J A S O N D J F M A M J J A S O N D J F M A M J J A S
P(mm)
30
SP(mm)
300

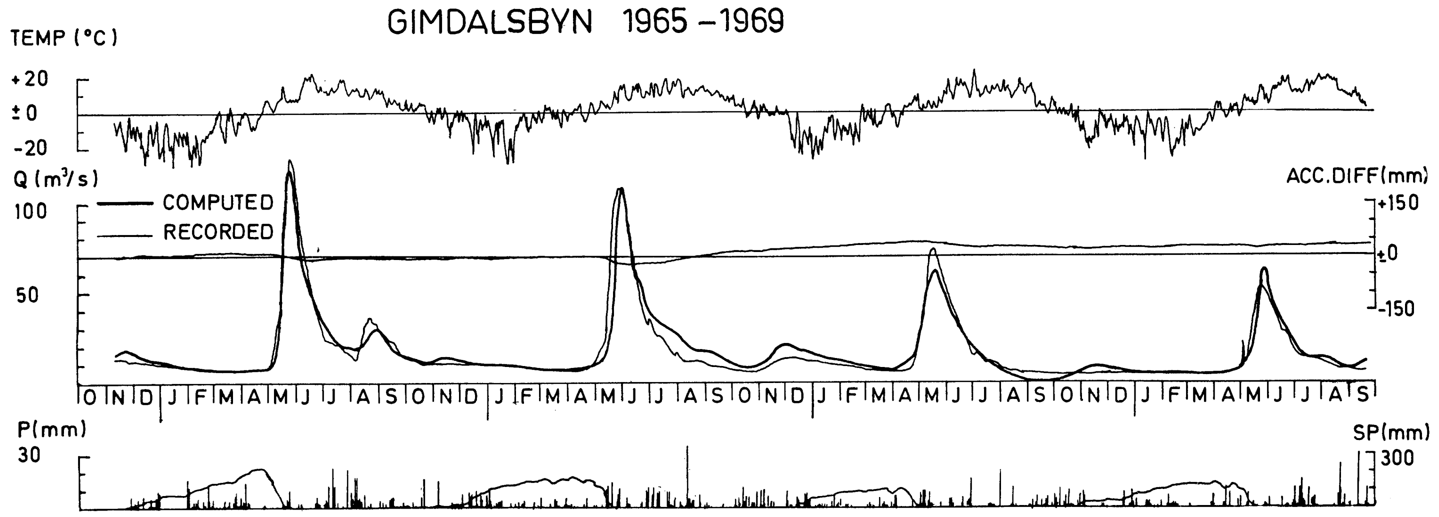


Fig. 7.
The period used for calibration of the model (1961–1969).

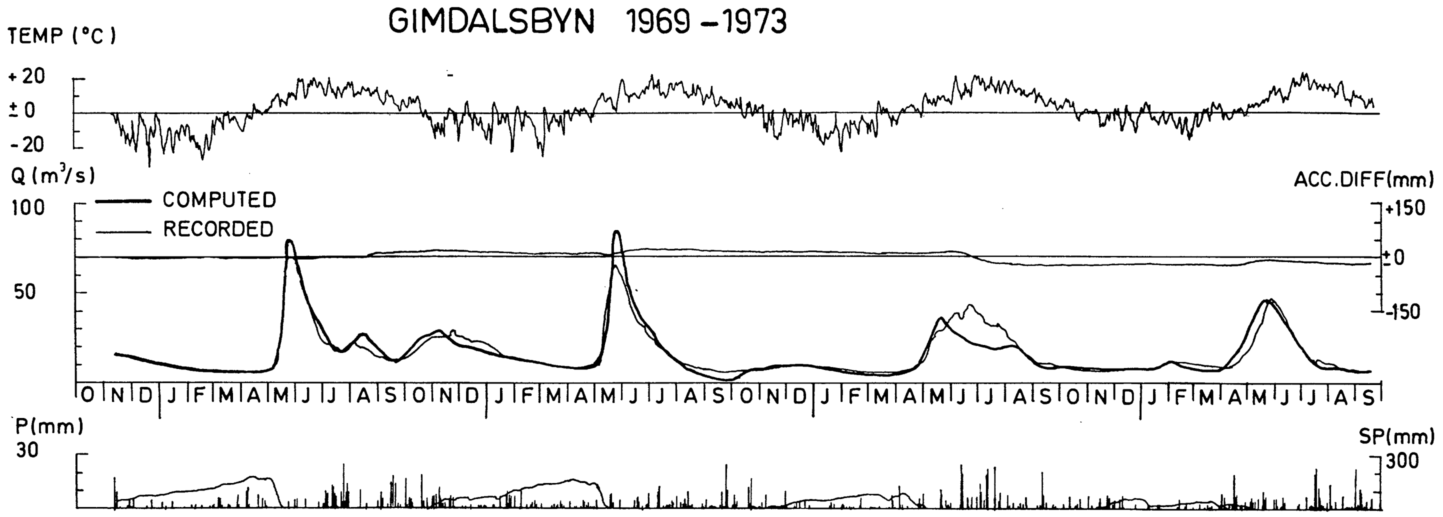


Fig. 8.
Simulation over the independent test period (1969–1973).

It was soon found that, due to the great climatic variability and the possibility of the effect of one single flood dominating the spring flood, quite long records were needed for the calibration of the model. 4 years of calibration proved to be too short. The model was easy to fit but gave poor results over the independent test period. Fitting 8 years proved to be more difficult but, once the model was calibrated, the independent test period gave comparable results. Therefore a period of 8 years was concluded to be adequate for calibration.

Calibration and test

Fig. 7 shows the fitted period used for evaluation of the parameters. As can be seen in the figure, the recession part of the high flows and the timing and shape of the low flows cause problems in certain years. However, the errors were not found to be so large as to justify the introduction of new parameters, taking into consideration the errors in input and limited degrees of freedom for each new parameter estimate.

In Fig. 8, an independent period, 1969–73, is shown with the parameters evaluated for 1961–69. The parameters are shown in Table 1.

The first two years coincide well, while the summer of 1972 is less satisfactory. The variability problem is clearly demonstrated by these twelve years. The very extreme winter conditions of 1973 (see Fig. 8, snowpack) are unique, as nothing of their kind appeared during the preceding years. However, since 1969–73 is the independent period, this period has not affected the structure of the model or its parameters.

The R^2 values defined in eq. (1) are shown in Table 2. As can be seen from the table the R^2 values are of the same magnitude. This indicates that the length of the calibration period is appropriate.

ALTERNATIVES TESTED

A few modifications of the model were tested without significantly better results. The first one was a distribution of the soil moisture zone in accordance with the distribution of the snow cover. The routine was the same in principle, but the output from the ten areally distributed snow covers were processed through ten separate soil moisture storages and then collected in the upper zone. The results obtained from this method were almost identical with those obtained from the original model with collection of snowmelt in one single soil moisture storage.

Table 1.
Parameter values in the Gimån basin.

C_{sf}	= 0.8	
$C_{o\ min}$	= 1.0 mm/°C · day	
$C_{o\ max}$	= 1.5 mm/°C · day	
C_{eff}	= 0.01 mm^{-1}	
T_o	= 0 °C	
C_{wh}	= 5 %	
W_b	= 10 mm	
B_{max}	= 40 days	
C_{route}	= 0.000023 days/((l/s) · 10)	
Fc	= 200 mm	1)
Lp	= 200 mm	1)
Beta	= 1.8	1)
Perc	= 0.6 mm/day	1)
K_1	= 710 l/s · mm	Fig. 1.
K_2	= 350 l/s · mm	Fig. 1.

1) See, for example, Bergström & Forsman (1973).

Table 2.
R^2 values for river Gimån.

1961–1965	fitted	0.86
1965–1969	fitted	0.91
1969–1973	test	0.86

The second modification was the introduction of a time lag to the evaporation from lakes. This was introduced because it was felt that the temperature lag in the lakes would strongly affect the timing of the runoff from the lower zone. A 30 days time lag in the evaporation from lakes was hardly discernible in the output from the model.

ERROR FUNCTION TOPOGRAPHY

Studies of the error function response are often used for detailed investigations of the effect of different components of the model. Plottings of the error function inform about the significance of an introduced parameter and its interaction with others. It is thus a valuable help in keeping down the complexity of the model, and to determine which parameters can be evaluated with automatic methods.

There are, however, two major points that must be considered when analysing the shape of the error function.

1. The error function topography only reflects how the error function responds to changes in the parameters. If the error function does not exactly represent the intentions of the modeller, the interpretation must be made with great care. The R^2 value used in this work has been found to be a useful measure of the overall fit, but the best fit according to visual inspection and highest R^2 value do not always coincide. Detailed studies showed, for example, that the R^2 criterion primarily serves to adjust the high magnitude of the errors during the rapid rise of spring flood in certain years. This can mask the effect of a parameter that was introduced to improve the modelling of another sequence of the hydrograph where the errors are of much smaller magnitude.
2. It is hard to visualize the effect of more than two parameters at the same time. Therefore the parameters are studied in pairs with all the others at constant values. A well-defined minimum can thus develop into a long ridge if one more parameter is let free. On the other hand, a long ridge with an ill-defined minimum in the three-dimensional error function topography is strong evidence of the fact that this pair of parameters cannot be optimized simultaneously with automatic methods.

In the following, a study of the response of the error to a few parameters will be shown.

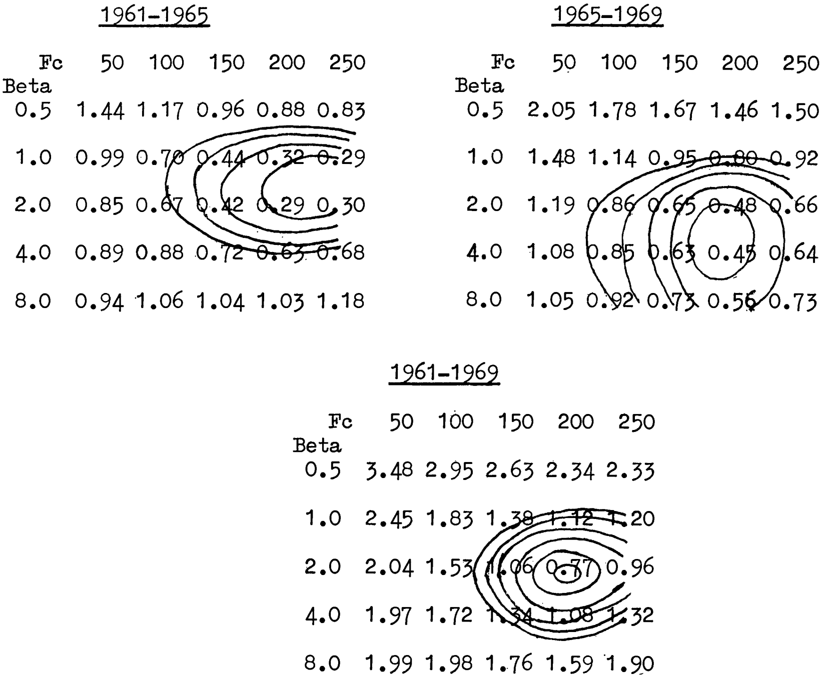
The error is expressed as:

$$\Sigma(Q_o - Q_c)^2 \cdot 10^{-5},$$

with Q_o and Q_c in m^3/s. The investigation is based on the parameters found through visual inspection over the period 1961–69.

Fc and Beta

These are parameters in the soil-moisture zone (Bergström & Forsman 1973). Fitted values were Beta = 1.8 and Fc = 200. In Fig. 9 are shown the results



1961-1965

Beta \ Fc	50	100	150	200	250
0.5	1.44	1.17	0.96	0.88	0.83
1.0	0.99	0.70	0.44	0.32	0.29
2.0	0.85	0.67	0.42	0.29	0.30
4.0	0.89	0.88	0.72	0.63	0.68
8.0	0.94	1.06	1.04	1.03	1.18

1965-1969

Beta \ Fc	50	100	150	200	250
0.5	2.05	1.78	1.67	1.46	1.50
1.0	1.48	1.14	0.95	0.80	0.92
2.0	1.19	0.86	0.65	0.48	0.66
4.0	1.08	0.85	0.63	0.45	0.64
8.0	1.05	0.92	0.73	0.56	0.73

1961-1969

Beta \ Fc	50	100	150	200	250
0.5	3.48	2.95	2.63	2.34	2.33
1.0	2.45	1.83	1.38	1.12	1.20
2.0	2.04	1.53	1.06	0.77	0.96
4.0	1.97	1.72	1.34	1.08	1.32
8.0	1.99	1.98	1.76	1.59	1.90

Fig. 9.

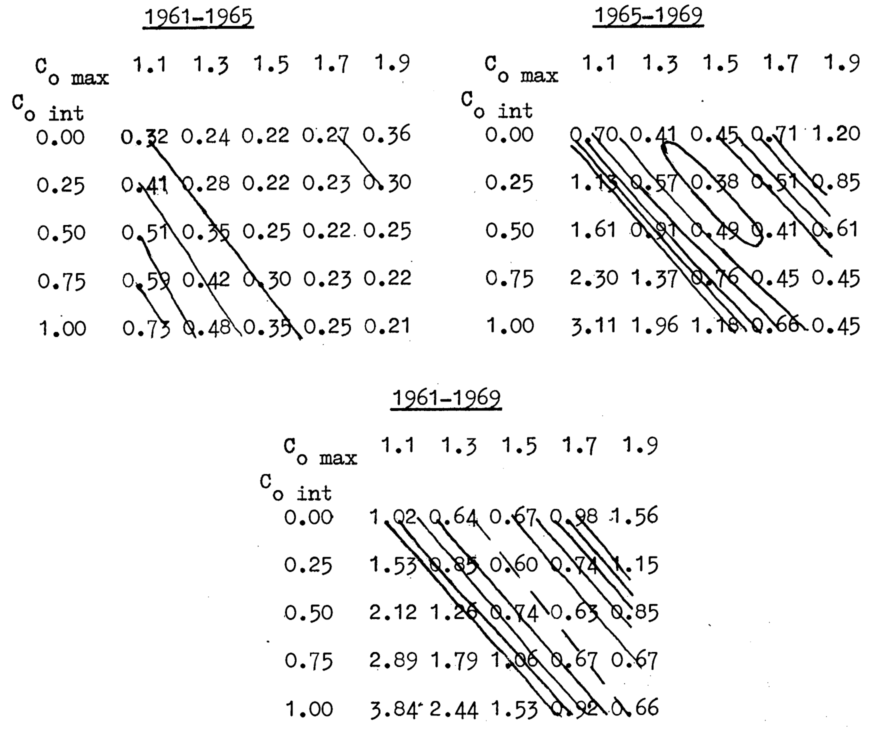
Error function response to Beta and Fc.

if the error function is studied over two four-year periods and over the whole period separately. As can be seen, the error function has a well-defined minimum in the vicinity of the fitted values. The two four-year periods give different optimum values indicating that four years is too short a period for the evaluation of stable parameter values.

$C_{0\ \max}$ and $C_{0\ \min}$

In Fig. 10 is shown the effect of detoriations of the degree-day factors in the distribution function described earlier. In the figure $C_{0\ \mathrm{int}}$ is defined as $C_{0\ \max} - C_{0\ \min}$.

It is evident that the distribution of the degree-day factor has very little effect on the error function as long as the average value is correct. The valley in the error function follows the equation $(C_{0\ \max}+C_{0\ \min})\ /\ 2 \equiv 1.40$ (broken



1961-1965

$C_{o\,max}$ $C_{o\,int}$	1.1	1.3	1.5	1.7	1.9
0.00	0.32	0.24	0.22	0.27	0.36
0.25	0.41	0.28	0.22	0.23	0.30
0.50	0.51	0.35	0.25	0.22	0.25
0.75	0.59	0.42	0.30	0.23	0.22
1.00	0.73	0.48	0.35	0.25	0.21

1965-1969

$C_{o\,max}$ $C_{o\,int}$	1.1	1.3	1.5	1.7	1.9
0.00	0.70	0.41	0.45	0.71	1.20
0.25	1.13	0.57	0.38	0.51	0.85
0.50	1.61	0.91	0.49	0.41	0.61
0.75	2.30	1.37	0.76	0.45	0.45
1.00	3.11	1.96	1.18	0.66	0.45

1961-1969

$C_{o\,max}$ $C_{o\,int}$	1.1	1.3	1.5	1.7	1.9
0.00	1.02	0.64	0.67	0.98	1.56
0.25	1.53	0.85	0.60	0.74	1.15
0.50	2.12	1.26	0.74	0.63	0.85
0.75	2.89	1.79	1.06	0.67	0.67
1.00	3.84	2.44	1.53	0.92	0.66

Fig. 10.

Error Function response to $C_{o\,max}$ and $C_{o\,int}$ where $C_{o\,int} = C_{o\,max} - C_{o\,min}$.
Fitted values: $C_{o\,max} = 1.5$, $C_{o\,int} = 0.5$.

line in Fig. 10) implying a constant mean in the distribution. To fit these parameters with an automatic method without knowledge of the error function topography would yield misleading results. Values found reasonable from field experience and used in this work are $C_{0\,max} = 1.5$ and $C_{0\,int} = 0.5$. As can be seen, they deviate somewhat from the values in the trough.

In order to test the significance of the C_{eff} value used in eq. (3), the study was carried out with C_{eff} put at zero. The result is shown in Fig. 11.

A comparison between Fig. 10 and 11 shows that the effect of C_{eff} can easily be absorbed by a change in the C_0 values. This means that the change in the degree-day factor with accumulated melt, observed at point stations, is not reflected by the R^2 criterion if $C_{0\,max}$ and $C_{0\,min}$ are free parameters.

1961-1969

$C_{o\,max}$ $C_{o\,int}$	1.5	1.7	1.9	2.1	2.3
0.00	1.20	0.81	0.64	0.64	0.75
0.25	1.54	1.01	0.72	0.63	0.67
0.50	1.98	1.33	0.89	0.68	0.64
0.75	2.47	1.70	1.17	0.81	0.66
1.00	3.05	2.16	1.50	1.05	0.77

Fig. 11.
Error function response to $C_{o\ max}$ and $C_{o\ int}$ with $C_{eff} \equiv 0$.

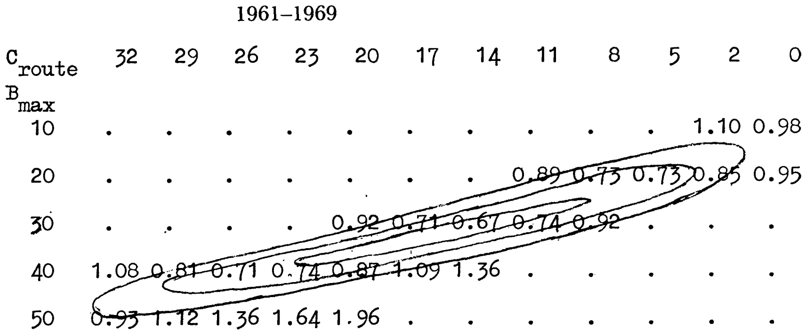
C_{wh} and W_b

The conflict between the R^2 criterion and visual inspection is clearly shown, if C_{wh} and W_b are studied. Earlier two ways of modelling liquid water retention in the snowpack were described. A separation in two components with 5 % water-holding capacity (C_{wh}) and 10 mm bottom storage (W_b) was found to be better than a 15 % water-holding capacity alone. Fig. 12 indicates the

1961-1969

W_b C_{wh}	0	5	10	15	20
0.00	1.26	0.66	0.63	0.73	0.87
0.05	0.67	0.62	0.74	0.89	0.99
0.10	0.60	0.71	0.88	0.99	1.07
0.15	0.66	0.83	0.97	1.04	1.11
0.20	0.76	0.90	1.01	1.08	1.14

Fig. 12.
Error function response to W_{hc} and W_b.



1961–1969

C_{route} B_{max}	32	29	26	23	20	17	14	11	8	5	2	0
10	.	.	.	.	.	.	.	.	.	.	1.10	0.98
20	.	.	.	.	.	.	.	0.89	0.73	0.73	0.85	0.95
30	.	.	.	.	0.92	0.71	0.67	0.74	0.92	.	.	.
40	1.08	0.81	0.71	0.74	0.87	1.09	1.36	.	.	.	.	.
50	0.93	1.12	1.36	1.64	1.96	.	.	.	.	.	.	.

Fig. 13.
Error function response to C_{route} and B_{max}. (C_{route} is multiplied by 10^6).

opposite, even if the difference is small. It can generally be said that Fig. 12 reflects the overall effect of these two parameters on the output, but the improvement found when introducing W_b is not discernible due to the rigidity of the R^2 criterion.

C_{route} and B_{max}

Finally, the effect of the parameters in the variable time-area transformation has been investigated (Fig. 13). Interdependence is strong, but the improvement of the model is clearly shown. The old version corresponds to $C_{route} = 0$.

CONCLUSIONS

The investigations in the Gimån basin show that the simple HBV-2 model can be used in rather large basins. The difficulties of areal variability seem to be damped out, giving a reasonable overall response. A very simple snowmelt routine, using mean daily temperature as an index of snowmelt, can be used in this forested basin.

The criterion of fit, or error function, must be used with caution, if it does not in detail reflect the intentions of the modeller. Improvements seen with the eye are sometimes not discernible in this function. Therefore the interpretation of its response is difficult, especially as regards small differences. Still, the study of the error function in this study has been a very useful tool in the investigation of the general behaviour of the model. It has shown us that the simplified distribution of the C_0 value and the introduction of C_{eff} do not significantly improve the model. This means that observations made by point measurements at test sites are damped out in this large catchment. Thus the conclusion is that we can use a much simpler model than expected.

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Received: November 1, 1974.

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