

**Hamilton's Principle in Continuum Mechanics**, by A. Bedford. Pitman Publishing Limited, London, England, 1985. 106 pages. Price: £ 12.50.

REVIEWED BY S. L. PASSMAN<sup>1</sup>

An elegant result in the theory of pure mechanics of a particle with conservative external forces is that the equation of conservation of energy is a consequence of the balance of momentum. This result is useful as a pedagogical tool in that it provides convincing motivation to beginning students that kinetic energy in the form usually assumed is a quantity appropriate for study and that, indeed, the energy equation is a significant basic physical principle. The techniques of variational mechanics are not accessible to most beginning students. To those who pursue their studies far enough, however, variational mechanics provides both mystery and reward: Mystery, because the concepts and formal manipulations used are seldom explained well; Reward, because variational principles provide an easy proof of a converse to the result mentioned above, as well as tools for solving a plethora of problems which were not soluble with any tool they had before. Furthermore, there is no mystery for the student who understands functional analysis, because the basis of the calculus of variations is just an application of techniques which he knows. Most of today's students of mechanics, however, lack either the time or the inclination to study functional analysis. Many of them still recognize that variational techniques are potentially useful tools, and want a clear elementary exposition. For such students, even the most inquisitive ones, the path is difficult. The small book, *Hamilton's Principle in Continuum Mechanics*, by A. Bedford, is a most useful guide to part of this path. Bedford has written out the material on Hamilton's principle and some of its generalizations that a skilled teacher, well-versed in the subject, would present to a class of first-year graduate students in mechanics. He has then polished the material, as many teachers plan to but few ever do, so that it is coherent, concise, and smoothly worded. His principal objective is to show some uses of an extension of Hamilton's principle to continuum mechanics, but he helps the student to this goal with preliminary material on Hamilton's principle in particle mechanics and enough basic continuum mechanics so that the student can understand his lectures without having to study that subject separately. Afterward, he provides a sufficient number of examples to motivate the idea that he has not committed that horrible sin, an "academic exercise".

Does the book have flaws? Yes, it does, and they will be noticed readily by readers sophisticated either in variational principles or in continuum mechanics. The flaws are not

serious enough, however, to lead the audience too far astray. Most are far outbalanced by the book's readability and utility. I suspect that many copies will be purchased, and that they will be read and referred to many times over. Speaking to this point, the method of production of the book is significant. It is provided as a paperback with glued bindings. Given the type of use expected of this particular book, one may anticipate that it will fall apart in its youth. To Bedford's very great credit, he has violated the publisher's requirement for typed, double spaced pages, to be produced by photo-lithography. Instead he has provided the printer with a manuscript done with a computer typesetting program, and printed on a laser printer. The result is eminently readable, though there are occasional strange hyphenations, and the line spacing is somewhat more generous than necessary. One difficulty which any author who handles a large manuscript must face is that, no matter how proficient he is at spelling, errors in spelling creep into the manuscript in the inevitable numerous processes of retyping. A side benefit of using a computer is the ability to mitigate this: I found but one word in the whole book which is incorrectly spelled (oddly, the word "functional"), and one other whose spelling is marginally acceptable.

In its present form, many people will find this book worth buying and reading. A hardcover copy, properly printed, would be even more appealing.

**Advanced Dynamics of Rolling Elements**, by P. K. Gupta. Springer-Verlag, New York, 1984. 295 pages. Price: \$42.50.

REVIEWED BY B. PAUL<sup>2</sup>

This book contains a very comprehensive and detailed analysis of the rigid body dynamics of rolling element bearings (e.g., ball and roller bearings). These bearings are mechanical systems consisting of an *inner race*, an *outer race*, the *rolling elements* between the races, and a *cage* which keeps neighboring rolling elements from making direct contact. One of the races is either fixed or has a specified motion with respect to an inertial frame, the two races have a given relative rotational motion, and the space between the races is filled with a lubricant.

The end result of this analysis is a very powerful computer program called ADORE (Advanced Dynamics of Rolling Elements) which is listed in its entirety (38 pages of FORTRAN code, of which approximately 65 percent consists of comment lines). Users may access this code directly from some of the "world-wide computer networks, such as CYBERNET Services." Because rolling element bearings are so ubiquitous,

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and are so frequently the most critical components in mechanical systems, this book and ADORE should prove to be very useful engineering tools. Only rarely is the source code for a major program published, and even more rarely is a complete book devoted to such a program. The author is to be heartily congratulated on the major program which he has produced and the excellent book which complements it.

The book is organized into two main parts. Chapters 1 through 6 provide a thorough discussion of the underlying mathematics and mechanics for the problem. Chapter 1 is a good overview of the range of problems covered, and shows why and when it is desirable to use either a "quasi-static model" or a "dynamic model." The former permits a kinetostatic solution for known relative motions of the races (via a Newton-Raphson type numerical method of the governing system of nonlinear algebraic equations). This solution requires the use of certain assumed kinematical constraints which limit the validity of the results to a restricted, but useful, range of conditions. Chapter 2 describes the various coordinate transformations which will be needed, and gives the appropriate Newton-Euler equations for the system. Chapter 3 is devoted to the geometric relationships between the various components that determine the relative velocity or slip vector between all contacting surface elements, and the shape and extent of the assumed Hertzian contact patches on contacting bodies. The constitutive relations used to convert local slip and pressure into surface tractions are described in Chapter 4, where results from the literature on elastohydrodynamic lubrication are summarized. These results take into account the effects of temperature and pressure on the rheological properties of various lubricants. In Chapter 5, the *drag forces*, and *churning moments* produced on the moving bodies by the lubricant are discussed. In view of the general complex internal geometry involved, the author states that the results introduced here are to be considered as first approximations which establish general trends. Two types of numerical integration algorithms are described in Chapter 6 (Runge-Kutta-Fehlberg, and the author's modification of an Adams type method). In program ADORE, the user may invoke either of these algorithms, based upon some guidelines presented.

The second part of the book (Chapters 7-10) describes features of program ADORE, and includes such topics as: program structure, program capabilities, input/output, performance simulations, experimental validation, and guidelines for rolling bearing design.

The book is a fine illustration of modern techniques for applying analytical and numerical tools to a complex engineering system by an adept utilization of exact theory, experimental results, and some judiciously applied semiempirical results. The author has skillfully integrated concepts from applied mechanics, elastohydrodynamics, and numerical analysis, with a practical command of rolling bearing design. The exposition is very good, if somewhat terse in parts.

Undoubtedly the book, and the ADORE program, will be invaluable for workers in advanced design and applied research on rolling element bearings. It should also be of considerable value to experts in applied mechanics, who are curious about the complicated dynamic processes taking place within rolling element bearings.

**Schalentheori**, by E. L. Axelrad. B. G. Teubner, Stuttgart, West Germany, 1983. 211 pages [In German]

REVIEWED BY E. REISSNER<sup>3</sup>

This relatively brief book is intended for students and engineers with a solid mastery of the mathematics and

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mechanics included in the undergraduate curriculum of a school of engineering, with additional advantages for those having proceeded to the master's degree level. The author is a product of the Lurie-Goldenveizer tradition of research in shell theory and a significant contributor in his own right. There are altogether six chapters in this compactly written work, with more material per page than is often found nowadays.

The first chapter is an introduction to the two-dimensional analysis of stress and strain in thin layers, combined with the three-dimensional fundamentals which are required for the formulation of two-dimensional constitutive relations. The author limits himself at the outset to the classical approach which neglects the effects of transverse shear and normal stresses in the sense of Kirchhoff. On the other hand, the presentation includes the relevant finiteness aspects, subject to an assumption of small strain.

The second chapter describes analytical solution procedures with emphasis on the difference between shells which are "stiff" as against those which are "flexible". An interesting discussion concerns the difference between simplified shell equations based on a stipulation that the direction of the shell normal varies less rapidly than the state of stress, in the sense of Vlasov, and on a stipulation of shallowness, in the sense of Marguerre.

Chapter 3 is devoted to rotationally symmetric problems for shells of revolution. It begins with a discussion of membrane problems and proceeds to the conventional treatment of cylindrical shells, prior to a discussion of the general shell of revolution problem, including finite deflection effects, with a valuable discussion of historical aspects. An example from the linear theory of spherical shells is used to illustrate the relatively small differences between exact results and two different kinds of asymptotic results in other than shallow shells, for which a different treatment is presented (which would have gained in effectiveness if the author had been aware of a comprehensive paper on this subject in the *J. Math. and Phys.*, Vol. 25, pp. 279-300, 1946). Additional material is devoted to conical shells and, in particular, to linear and nonlinear problems of toroidal shells.

Chapter 4 deals with shells of revolution without rotational symmetry of loads, now limited to linear-theory problems.

Chapter 5 brings a sophisticated analysis of unsymmetrical bending of cylindrical shells, with particular emphasis on what is designated as semimembrane theory. On the basis of this semimembrane theory there follows a substantial treatment of the problems of curved tube bending, a subject to which the author himself has contributed significantly.

A final chapter discusses the problem of elastic stability, including the question of imperfection sensitivity. Particular attention is devoted to the effect of prebuckling deformations on critical loads, specifically in connection with the author's analysis of the problem of localized buckles as these modify Brazier's St. Venant type analysis of finite tube bending.

All in all, this is a valuable work with much information not otherwise existing in book form. No serious student of the subject of shells, from the point of view of the foundations of solid mechanics, or from the point of view of computational mechanics, should be unaware of its contents.

**An Introduction to the Theory of Seismology**, by K. E. Bullen and B. A. Bolt. Cambridge University Press, New York, 1985. 499 pages. Price: \$69.50/hardcover; \$24.95/paperback.

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