General Discussion

A Method for Relating Test Track Data to the Real World

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The purpose of this discussion is (a) to develop a method by which test track data can be related directly to any railroad's rock and roll problem, and (b) to provide a means for objectively evaluating the railroad's requirements for rock and roll control devices.

Introduction

The method that is discussed shows how to use the rate of energy buildup in a car that is found on a test track and the distribution of low track joints on any piece of curved track, railroad property, to calculate the probable derailment rate on that railroad for any car equipped with any device. This probable derailment rate is then used to determine the requirements for rock and roll control devices to minimize the rock and roll problem on any railroad.

Statement of the Problem

The problem is based on the interaction between wheel and rail, since this interaction is the mechanism by which energy is added to the car body. In the case of the high cube car, the truck center distance is such that both trucks fall into low joints on one side of the track at the same time. This condition nearly doubles the roll energy input to the high cube car over cars with longer or shorter truck center distances. The high energy input in roll, combined with a high center of gravity, has generated our now infamous "rock and roll" problem.

The Significance of Energy Inputs From the Truck

Energy, and a lot of it, is required to unload wheels and create a potential derailment condition. The amount of energy required has been shown by many to be on the order of 100,000 in-lb. This energy level can be considered the result of a number of smaller inputs which occur when the car is subjected to track irregularities, such as low joints. Each low joint will cause a certain amount of energy to be added to the wheels and axles. The amount added to the car body will be reduced below this input by the suspension system and the snubbing available in the truck. Consecutive low joint inputs of a magnitude greater than the snubbing can dissipate will build up car body energy. If sufficient consecutive low joints are available, the energy input to the car will build up to 100,000 in-lb and a potential derailment condition will exist. This rate of energy buildup has been observed on a number of test tracks. C&O/B&O test track experience has shown that the number of consecutive low joints required to unload wheels varies directly with the amount of energy absorption available in the truck. Each type of rock and roll control device appeared to have a unique rate at which the car body energy would build up as it moved over the track.

While the previously mentioned total energy of 100,000 in-lb required to unload wheels is important, the rate at which this car body energy builds up as it travels over a test track is far more important. It is, in effect, the most significant parameter for our purpose of relating test track data to the real world.

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1 This discussion is in reference to papers by D. Wiebe and K. A. Henderson and J. Johnson, all published in this issue.
2 The Chesapeake and Ohio Railroad Company, Baltimore, Md.
Why is the rate of energy buildup so important? It is important because it provides the bridge between the test track and the real world. For example, if there is a rough piece of track, you might find five consecutive $\frac{1}{4}$-in. cross-level variations easily. But, 10 consecutive variations? Very unlikely. Consequently, if a car will build up to an energy level of 100,000 in-lb in five joints, it will probably derail on that piece of track; but if that same car requires 10 consecutive joints to build up to that energy level, it probably will not derail. Therein lies the reason why the rate at which car body energy builds up on a test track is such a significant factor.

Analysis of Energy Requirements for Rock-Off

The energy input to the wheels and axles of a high cube car of the rock and roll type can be calculated from the following equation, which is derived in Appendix 1:

$$E_t = \frac{h^2}{8} \left( \frac{W}{g} \right) \left( \frac{v}{\lambda} \right)^2 \left( \rho^2 + L^2 + b^2 \right)$$

where $E_t$ is the energy input due to a low joint which creates a cross-level variation of $h$ in., $W$ is the sprung weight of the car, $g$ is the acceleration due to gravity, $v$ is the car speed, $\lambda$ is the distance between joints, $b$ is $1/2$ the distance between spring group centers, $\rho$ is the car body radius of gyration about the CG, and $L$ is the height above the CG above the spring group. This equation was used to calculate the energy input to the wheels and axles from each joint. This energy input minus the energy absorbed by the rock and roll control devices and the car body energy build up. This rate of buildup defines, in turn, the number of consecutive low joints required to unload the wheels and create a potential derailment condition. Figs. 1, 2, and 3 show this condition more clearly.

Fig. 1 shows the effect of snubbing on the number of joints required to lift wheels. The dashed lines show the amount of energy absorbed by two types of snubbing. The solid line shows the cumulative input from each successive joint. The brackets labeled $E_{WL}$ show the point at which 100,000 in-lb of net energy is built up in the car and where unloading can occur. Notice that standard snubbing requires six joints and intermediate snubbing requires eight. Based on this figure, it would be expected that an increase in the theoretical snubbing rate to the level of the cross-level input would solve the problem. This is a dangerous assumption. Fig. 2 shows why.

Fig. 2 shows a comparison between the theoretical and actual performance of friction snubbing. Notice that the difference between theoretical snubbing and the actual snubbing achieved increases as the level of snubbing force is increased. This occurs because smaller spring deflections and stiffer trucks occur with high snubbing forces. Since energy absorption is force times travel, the theoretical snubbing value is never achieved. In fact, the efficiency of the snubbers decreases with increasing snubbing force.

Fig. 3 shows the effect of cross-level deviation on wheel lift. Notice how the number of joints required to develop a net energy input of 100,000 in-lb changes. Only three consecutive 1-in. cross-level deviations are required, while six are required for $\frac{1}{4}$-in. deviations and well over 12 are required for $\frac{1}{2}$-in. deviations.

Thus it can be seen that it takes varying numbers of consecutive low joints to unload the wheels. This number depends on the magnitude of the cross-level deviation and the amount of snubbing or stabilization that has been installed on the car.

Relating Test Track Data to the Real World

From test track data, we have the number of joints required to create a potential derailment of any car equipped with any rock and roll control device. The problem of relating this data to a particular railroad boils down to determining the condition of the vertical track surface on that railroad.

This is done by measuring the low joints on the curved territory over which this equipment must operate. The measurement is most easily accomplished with a track measurement car, such as the C&O/B&O RI-2 or Track Fax cars. Once measured, a
Once the distribution of low joints has been found, the equation given next and derived in Appendix 2 can be used to calculate the probability of finding any particular number of consecutive low joints in the territory measured.

\[ P(x) = \frac{e^{-wM^2}}{x!} \]

Where \( M = xp \), \( P(x) \) is the probability of finding exactly \( X \) number of low joints of a particular depth and \( p \) is the fraction of the total joints measured that are of the particular depth desired.

Since a derailment condition exists when there are \( X \), or more consecutive joints of \( \frac{1}{8} \)-in. or greater depth, the foregoing equation must be solved for exactly \( X \), exactly \( X + 1 \) \( \ldots \) \( X + n \) and for exactly \( \frac{1}{4} \)-in., exactly \( \frac{1}{4} \)-in. \( \ldots \) \( \frac{1}{2} \)-in., or more. The equation can, therefore, be written as follows to define the probability of finding a derailment condition:

\[ P(D) = \sum_{x=i}^{\infty} \sum_{j=1}^{\infty} \frac{e^{-wM^2}}{x!} \]

Where \( P(D) \) is the probability of finding a derailment condition on the measured track, \( j \) is the minimum joint depth required to cause rock off, \( i \) is the minimum number of joints required to cause rock off, and the other terms are as defined before. The results generated by the use of this equation give the probability of finding a condition on the railroad which will cause the derailment of any car equipped with any device as a function of the number of consecutive low joints required to unload the wheels.

Once these probabilities are known, they can be used to determine the probability of rock and roll derailments on the railroad relative to any given standard, such as new \( 3^{11/16} \)-in. travel covered hopper cars. This calculation is based on the situation that the probability of derailment of any car can be directly related to the probability of finding the number of consecutive low joints required to unload that car's wheels. For example, if one car requires five joints to unload the wheels and another car requires 10 joints to unload the wheels, and the probability of finding five joints on the rough-jointed rail is 0.0143 while the probability for 10 joints is 0.0003, then the relative probability of derailment for the latter car is 0.0003 + 0.0143 or 0.021.

Fig. 5 shows the probability of derailment relative to a new \( 3^{11/16} \)-in. travel covered hopper car as a function of the number of joints required to unload the wheels. Since test track data have shown that a new \( 3^{11/16} \)-in. travel covered hopper car requires five consecutive joints to lift the wheels, it is given a relative derailment probability of 1.

The relative probability curve is used as follows. Assume that past history has shown that 10 percent of the new high cube fleet equipped with \( 3^{11/16} \)-in. travel springs derailed during their first year of operation. A device "X" is being considered for installation which has been shown to require eight joints of \( \frac{1}{4} \)-in. cross level to unload the wheels. If the territory over which the car must operate looks like the rough-jointed rail, the relative derailment probability for the equipped car is 0.8. Consequently, a derailment rate of 1 percent of these cars could be expected during their first year of operation. If, however, device "Y" which requires 12 joints to unload the wheels is installed, the relative derailment probability becomes 0.01 and, consequently, a derailment rate of one car out of a thousand would be expected. This latter rate is probably less than the derailment rate by other causes.

Summary and Conclusions

As was noted in the foregoing, it takes roll energy in the amount of about 100,000 in-lb to unload the wheels of a high cube car and create a potential derailment condition. The rate of energy buildup or the number of joints required to unload the wheels defines the car's susceptibility to derailment. The rate of energy buildup also allows test track data to be related to the real world. By determining the distribution of low joints in the curved territory over which the equipment must operate, and the relative probability of derailment thus generated, the test track data can be used to objectively evaluate the needs of any railroad for rock and roll devices.
APPENDIX 1

Energy Input to the Wheels and Axles

For the purpose of this analysis, the cross-level deviation is considered to be continuous and is represented by

\[ Y = \frac{h}{2} \left( 1 - \cos \frac{2\pi x}{\lambda} \right) \]  

(1)

where \( Y \) is the vertical displacement of the lower spring seat, \( h \) is the cross-level deviation, and \( \lambda \) is the distance between consecutive cross-level deviations.

At a constant velocity, \( v = \frac{x}{t} \) and the lower spring seat displacement in time will be

\[ Y = \frac{h}{2} \left( 1 - \cos \frac{2\pi vt}{\lambda} \right) \]  

(2)

Differentiating equation (2) twice to obtain the acceleration of the lower spring seat yields

\[ a = \frac{h}{\lambda} \left( \frac{2\pi v}{\lambda} \right)^2 \cos \frac{2\pi vt}{\lambda} \]  

(3)

This acceleration input to the wheels and axles would be input directly to the car body, were it not for the spring and snubbing. Consequently, it represents the upper limit of acceleration and, hence, energy input to the car from a low joint. Consider the car to be pivoting about the high rail as shown in the sketch.

The torque on the car body then is found from the movements about \( R \) and is

\[ T = 2F \rho - 2W b = 1\alpha \]  

(4)

where \( \alpha \) is the angular acceleration and is given by

\[ \alpha = \frac{\dot{y}}{2b} \]  

(5)

Substituting equation (5) into (4) and rearranging, we get

\[ F = \frac{1}{4b^2} \frac{\dot{y}^2}{\lambda} + W \]  

(6)

where the net force on the lower spring seat, causing car body acceleration, is equal to the first term and is given by

\[ F = \frac{1}{4b^2} \frac{\dot{y}^2}{\lambda} \]  

(7)

The moment of inertia of the car is given by

\[ I = \frac{W}{g} \rho^2 \]  

(8)

where \( \rho \) is the radius of gyration about the point of rotation which is \( R \). Consequently, \( \rho \) can be given as

\[ \rho^2 = \bar{\rho}^2 + l^2 + b^2 \]  

and acceleration force is

\[ F = \frac{h}{8b^2} \left( \frac{2\pi v}{\lambda} \right)^2 \left( \frac{W}{g} \right) \left( \bar{\rho}^2 + l^2 + b^2 \right) \cos \frac{2\pi vt}{\lambda} \]  

(9)

Simplifying and rearranging terms yields

\[ F = \frac{hW}{2g} \left( \frac{\pi v}{\lambda b} \right)^2 \left( \bar{\rho}^2 + l^2 + b^2 \right) \cos \frac{2\pi vt}{\lambda} \]  

(10)

The energy input due to the low joint is given by

\[ E = \int F(y) \, dy \]  

but since both \( F \) and \( Y \) are in terms of time

\[ E = \int F(t) Y(t) \, dt \]  

The time interval over which the energy is added to the car by virtue of the discontinuity is the time required to traverse the rising half of the discontinuity. Therefore

\[ E = \int_{t=0}^{t=\frac{\lambda}{2v}} F(t) Y(t) \, dt \]  

(11)

Substituting equations (2) and (10) into equation (11) yields

\[ E = \int_{t=0}^{t=\frac{\lambda}{2v}} \left[ \frac{hW}{4g} \left( \frac{\pi v}{\lambda b} \right)^2 \left( \bar{\rho}^2 + l^2 + b^2 \right) \cos \frac{2\pi vt}{\lambda} \right] \times \left[ 1 - \cos \frac{2\pi vt}{\lambda} \right] \]  

(12)

To simplify the integration, let

\[ A = \frac{hW}{4g} \left( \frac{\pi v}{\lambda b} \right)^2 \left( \bar{\rho}^2 + l^2 + b^2 \right) \]  

and

\[ a = \frac{2\pi v}{\lambda} \]  

Then

\[ E = \int_{t=0}^{t=\frac{\lambda}{2v}} A \cos at - A \cos^2 at \]  

(13)

which yields upon integration

\[ E = \frac{\pi A}{2} \]  

Consequently, the energy input to the wheels and axles of a car can be given by

\[ E = \frac{h^2}{8} \left( \frac{W}{g} \right) \pi^2 \left( \frac{\pi v}{\lambda b} \right)^2 \left( \bar{\rho}^2 + l^2 + b^2 \right) \]  

(14)

Equation (14), therefore, gives the maximum energy that would be input to a car with solid block springs and no energy absorption. Consequently, the difference between the energy calculated by equation (14) and the net car body energy is the energy absorbed by the springs, snubbers, and rock and roll control devices.

APPENDIX 2

Derivation of the Probability of Encountering a Derailment Condition

For a distribution of discrete occurrences, such as very low joints in a section of railway track, the binomial distribution provides the best fit to the data. Since we are looking for consecutive low joints of relatively large depth which are rare occurrences, Poisson distribution is applied directly to the problem and is derived next.

The binomial distribution is given in standard texts as

\[ P(x) = \frac{n!}{x!(n-x)!} p^x q^{n-x} \]  

(15)

where

\[ P(x) = \text{the probability of an event occurring exactly } X \text{ times} \]
The limit of equation (20) is 1 since \( x = 1 \) from 5 to 18, which is the limit of the test track, and then the occurrence of very low joints in the track, and \( x \) is large enough to keep \( nq \) finite, let \( nq = n \), then equation (15) becomes

\[
P(x) = \frac{e^{-m}}{x!} (1 - \frac{m}{n})^x \left( \frac{m}{n} \right)^n (1 - \frac{m}{n})^{-x}
\]

rearranging (2)

\[
P(x) = \frac{m^n (1 - \frac{m}{n})^n}{n!} \left[ \frac{n!}{n^x (1 - \frac{m}{n})^n} \right]
\]

The limit of \( (1 - \frac{m}{n})^n \) as \( n \) becomes large is: \( e^{-m} \).

Approximating factorials with stirlings approximation

\[
n! \sim (2\pi)^{1/2} n^{n+1/2} e^{-n}
\]

Since the terms in the exponent beyond the first term are negligibly small, they may be neglected and the second term in equation (17) becomes

\[
\frac{\sqrt{2\pi n} e^{-n}}{\sqrt{2\pi(n-x)} e^{-x}} n^{-x/2} (1 - \frac{m}{n})^x
\]

and (19) simplifies to

\[
\left[ \left( \frac{e^{-x}}{1 - \frac{x}{n}} \right)^{x/2} \right] \left[ \left( 1 - \frac{x}{n} \right)^{x/2} \right]
\]

The limit of equation (20) is 1 since

\[
\left( 1 - \frac{x}{n} \right)^x = e^{-x}
\]

and the limit of

\[
\left( 1 - \frac{x}{n} \right)^{x/2} = \left( 1 - \frac{m}{n} \right)^{x/2} = 1
\]

Consequently, equation (17) becomes

\[
P(x) = \frac{e^{-m} m^n}{x!}
\]

for rare occurrences. Equation (21) was, therefore, used to calculate the probability of finding the required number of low joints of \( 7/4 \) in. or greater to generate a derailment condition as found on the test track.

Equation (21), as written, gives you the probability of finding exactly \( X \) number of low joints of a given depth. Therefore, equation (21) must be applied to find \( X \) of low joints of \( 7/4 \) in. \( 7/4 \) in., \( 1 \) in., etc. These probabilities must then be added together to arrive at the probability of finding exactly \( X \) number of low joints of \( 7/4 \) in. or greater. Finally, \( X \) must be varied from 5 to 18, which is the limit of the test track, and then must be added to obtain the probability of \( X \) or more joints of \( 7/4 \) in. or greater depth. This final value is the probability of finding a derailment condition on any piece of track whose distribution of low joints is known. The complete operation is expressed mathematically as

\[
P(x) = \sum_{x=5}^{18} \sum_{x=1/4}^{x} \frac{e^{-m}}{x!}
\]

Where \( m = Xp \)

\( X \) = number of consecutive low joints for which the probability is to be calculated

\( p \) = fraction of total joints of a specified depth occurring in the distribution of low joints

Freight Car Dynamics Session of the 1968 Joint ASME-IEEE Railroad Conference

T. H. Yang and B. Johnstone

We would like to commend all the authors of the four papers presented at the Freight Car Dynamics Session of the 1968 Joint ASME-IEEE Railroad Conference.

Open discussion of the technical problems in railroading is an important part of the technical advancement of the industry. These papers all contribute substantially to the understanding of the rock and roll problem. The development of a usable simulation, and also of a good test method will undoubtedly lead to a valuable solution.

Our discussion will begin with the paper by Messrs. Diboll and Bieniecki. The two equations of motion developed in \( x \) and \( \theta \) contain the lateral rail variation \( u \) and the cross-level variation \( \delta_0 \). These are the two exciting forces on the suspension system. Only \( u \) has been investigated in this paper. It is important for passenger cars, but \( \delta_0 \) has been the more important exciting force in the freight car rock and roll problem.

While we realize that the following are typographical errors, we mention them to dispel misunderstanding. The units of \( k_1 \) in Table 1 should be in lb/rad. In Fig. 1, the \( e_0 \) should be the distance from the truck mass center to the truck spring seat, not the car bottom.

The simulation by Mr. Liepins treats the standard freight car problem. Engineering is of course based on assumptions, but these assumptions should always be carefully examined. Two assumptions (unlimited spring travel and constant friction) are generally made, and they simplify this simulation. A nonlinear spring to produce a “bottoming out” effect would more realistically model the action near resonance when tests generally produce bottoming out with standard trucks, but would be quite difficult to include in the simulation, and becomes unnecessary if the snubbing device does prevent bottoming out.

Changing the direction of the friction force abruptly with a change in the sense of the relative velocity of the surfaces produces a step discontinuity in the displacement which is unnatural. If another integration step could be introduced at the maximum displacement, when the velocity reaches zero, the discontinuity is eliminated. The importance of this step is of course dependent on the relative importance of the friction force in the car motion. The results of the simulation do show good agreement for the cases presented.

The approach taken by Mr. Henderson is purely empirical and can be used only for final evaluation since testing is so expensive as a means of development. The simulation procedures in the two above papers should be used for design development in conjunction with judgment gained through testing.

It is unfortunate that the papers contained the typographical errors mentioned. In the future, the authors are encouraged to carefully proofread their manuscripts before submission.